IB HL Math
Block 2
12/09/08
Bonmin Koo

Parabola Investigation: IB HL Math Type I Portfolio

Description

In this task, you will investigate the patterns in the intersections of parabolas and the lines y = x and y = 2x. Then you will be asked to prove your conjectures and to broaden the scope of the investigation to include other lines and other types of polynomials.

Background

Functions: 1) $ax + b, a \neq 0$	- Linear Equation
2) $ax^2 + bx+c$	- Quadratic
3) $ax^3 + bx^2+cx+d$	- Cubic
4) $ax^4 + bx^3+cx^2+dx+e$	- Quartic

Introduction

The investigation of parabola brings me to think and promote my knowledge about quadratic functions. Also, it helps me to find the 'patterns' and 'rules' of parabola which was another great source to improve my mathematic skills. In the HL type I, it asks about thoughtful questions such as relationship between parabolas and lines. There are six questions in this portfolio and each question requires rumination. I will try to focus on the member of the family of polynomials precisely since they are the basic functions of these problems and it will help me to 'link' some ideas. Equally important, interestingly, this portfolio wants the student to find their 'own' conjecture. Thus, I will try to focus on the patterns the questions have in order to find accuracy conjectures.

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Part One

1) Consider the parabola $\mathbf{y = (x-3)^2 + 2 = x^2 - 6x + 11}$ and the lines y = x and y = 2x.

   a. Using the technology find the four intersections illustrated on the right
      i. Intersection points #1: (1.76, 3.53), (6.24, 12.47) – these points are intersection points with y = 2x
      ii. Intersection points #2: (2.38, 2.38), (4.62, 4.62) – these points are intersection points with y = x
   b. Label the x – values of these intersections as they appear from left to right on the x – axis as $\mathbf{x_1, x_2, x_3}$**, and** $\mathbf{x_4}$

c. Find the values of $x_2 - x_1$ **and** $x_4 - x_3$ and name them respectively S_L and S_R.
   i. Since, $x_2 - x_1 = 2.38 – 1.76 = 0.62$. Hence, $S_L = 0.62$
   ii. In addition, $x_4 - x_3 = 6.24 – 4.62 = 1.62$. Thus, $S_R = 1.62$

d. Finally, calculate D = $| S_L - S_R |$
   i. So, $| S_L - S_R |$ = $| \mathbf{0.62 – 1.62} |$ = $\underline{| \mathbf{-1} |}$
   ii. Therefore, D equals to **1** because of the absolute value, -1 becomes to 1.
   iii. Equally important, it is clear that when the number is greater than 0. In other words, when D > 0, it intersects with two different points.

Part Two

2) Find the values of D for other parabolas of the form **$y = ax^2 + bx + c$, $a>0$,** with vertices in quadrant 1, intersected by the lines $y = x$ and $y = 2x$. Consider various values of a, beginning with a =1. Make a conjecture about the value of D for these parabolas.

   a. To find the conjecture ($a = 1$)
      i. Equation: **$y = ax^2 + bx + c$**
      ii. So let say $a=1$, $b = -4$ and $c = 5$
      iii. **$y = x^2-4x+5$**

Process

- $x_2 - x_1 = 1.39 – 1 = 0.39 = S_L$
- $x_4 - x_3 = 5 – 3.61 = 1.39 = S_R$
- So, the $|S_L - S_R| = |0.39 – 1.39| = |-1| = 1$, in other words**, D = 1.**

b. To find the conjecture (a = 2)
   i. Equation: $\mathbf{y = ax^2 + bx + c}$
   ii. let a =2, b = -10, and c = 14
   iii. $\mathbf{y = 2x^2-10x+14}$

Process

- $x_2 - x_1 = 2 - 1.58 = 0.41 = S_L$
- $x_4 - x_3 = 4.41 - 3.5 = 0.91 = S_R$
- So, the $\lceil S_L - S_R \rceil = \lceil 0.41 - 0.91 \rceil = \lceil -0.5 \rceil = 0.5$, in other words, **D = 0.5**

c. To find the conjecture (a = 3)
   i. let a = 3, b = -11 and c = 11
   ii. **$y = 3x^2-11x+11$**

Process

- $x_2 - x_1 = 1.42 – 1.14 = 0.28 = S_L$
- $x_4 - x_3 = 3.18 – 2.57 = 0.61 = S_R$
- So, the $\rceil S_L - S_R \rceil = \rceil 0.28 - 0.61 \rceil = \rceil -0.33 \rceil = 0.33$, in other words, **D = 0.33**

d. To find the conjectures (a = 4)
   i. let a = 4, b = -9 and c = 6
   ii. **$y = 4x^2-9x+6$**

Process

- $x_2 - x_1 = 1 - 0.75 = 0.25 = S_L$
- $x_4 - x_3 = 2 - 1.5 = 0.5 = S_R$
- So, the $| S_L - S_R | = | 0.25 - 0.5 | = | -0.25 | = 0.25$, in other words, **D =0.25**

e. To find the conjectures (a =5)

   i. let a = 5, b =-14 and c = 11

   ii. **$5x^2$-14x+11**

Process

- x_2 - x_1 = 1.27 – 1 = 0.27 = S_L
- x_4 - x_3 = 2.2 – 1.72 = 0.47 = S_R
- So, the ⌉ S_L - S_R ⌉ = ⌉ 0.27 – 0.47 ⌉ = ⌉ -0.20 ⌉ = 0.20, in other words, **D = 0.20**

f. Patterns

i. $a = 1, D = 1$
ii. $a = 2, D = 0.5$ or $1/2$
iii. $a = 3, D = 0.33$ or $1/3$
iv. $a = 4, D = 0.25$ or $1/4$
v. $a = 5, D = 0.20$ or $1/5$

Clearly, there is pattern. According to the results that I have, I can make the conjecture.

Since $(D)(a) = 1$, D has to be equal to $1/a$.
Consequently, the conjecture is $D = 1/a$

It is not about the conjecture but I am introducing this rule in order to demonstrate the exceptions.

g. Basic rule
i. Relationship

$y = ax^2 + bx + c$ **(a is not equal to 0)**	$D = b^2 - 4ac$
$D > 0$	Intersects with two different points
$D = 0$	Intersects with point
$D < 0$	No point

h. According to the answer from part 1, when $D = 1$, it clearly depicts that graph intersects with two different points.

i. In fact, the vertex of graph is equal to ($-b/2a$, $(-(b^2- 4ac)/ 4a)$)
   i. In case the value of x is equal to **$-b/2a$,** let's figure out the value of y for parabola and lines.
   ii. For instance, the value of y for parabola is greater than the value of y for lines. In other words, $D < 0$.
   iii. Moreover, when the value of y for parabola is same with the number of y for lines, then $D = 0$.
   iv. Finally, when the value of y for parabola is less than the value of y for lines, then $D > 0$.

j. So, in this question we can divide the conjecture into five cases.

i. When the parabola doesn't touch any points

ii. When the parabola touches only one point. (line $y = 2x$)

iii. When the parabola touches two points (line $y = 2x$)

iv. When the parabola touches three points (lines: $y = 2x$ and $y = x$)

v. When the parabola touches four points (two intersection points for lines: $y = x$ and $y = 2x$)

k. In detail, I will try to graph number I in order to prove my conjecture

i. $y = ax^2 + bx + c$

ii. I put actual number of a, b, and c

iii. let $a = 2$, $b = -3$, and $c = 4$

iv. Discriminant $(D) = b^2 - 4ac$

v. $9 – 4 (2) (4) = -23$ which is less than 0.

vi. Since the number is $D < 0$ which clarifies “no points” like graph #1.

Consequently, since all the graphs are in the quadrant 1, the y has to be greater than 0 (positive numbers). In addition, the graphs shown above are the exceptions which mean that they do NOT have four intersections. Basically, they are not suitable to use D = $\lceil \mathbf{S_L} - \mathbf{S_R} \rceil$.

Part Three

3) investigate your conjecture for any real value of a and any placement of the vertex. Refine your conjecture as necessary, and prove it. Maintain the labeling convention used in parts 1 and 2 by having the intersections of the first line to be x_2 and x_3 and the intersections with the second line to be x_1 and x_4

a. Unlike the part two, part three is asking about the 'real value of a' which means a can be a negative number
   i. Since I focused on positive value of a, I am going to use negative numbers in order to investigate the conjecture.
b. Through the part one and part two, my conjecture is a D = 1/a.
c. However, there are some possibilities to change my conjecture.

c. Investigating conjecture (a = -1)
   A. $y = ax^2 + bx + c$
   B. let a = -1, b = 6 and c = -3
   C. $\mathbf{y = -x^2+6x-3}$

Process

i. $x_2 - x_1 = 0.70 – 1 = -0.30 = S_L$
ii. $x_4 - x_3 = 3 – 4.3 = - 1.30 = S_R$
iii. So, the $\rceil S_L - S_R \rceil = \rceil -0.30 - -1.30 \rceil = \rceil 1.00 \rceil = 1.$, in other words, **D = 1**
iv. However, there are some problems because my conjecture is **D = 1/a** but it seems not working in this problem. Since a = -1, if I use my conjecture, then answer is 1 = -1 which is incorrect.
v. Therefore, let's try another one in order to figure out another conjecture.

d. Investigating Conjecture (a = -2)
   a. let a = -2, b = -6 and c = -5.
   b. **$-2x^2-6x-5$**

Process

i. $x_2 - x_1 = -3.2 + 2.5 = =- 0.7 = S_L$
ii. $x_4 - x_3 = -1 + 0.80= - 0.20 = S_R$

- So, the $| S_L - S_R | = | -0.7 - -0.20 | = | -0.50 | =$ 0.50, in other words, **D = 0.50**

- Even if the graph is on the quadrant 3, the conjecture still works.

- Again, if I use the conjecture (D = 1/a, then it gives 0.50 = - 0.50 which is not correct.

c. Investigating Conjecture (a = -3)

A. let a = -3, b = -9 and c =-2

B. **$-3x^2-9x-2$**

Process

i. $x_2 - x_1 = -3.47 + 3.12 = -0.35 = S_L$
ii. $x_4 - x_3 = -0.21 + 0.19 = -0.02 = S_R$

Hence, the $|S_L - S_R| = |-0.35 - -0.02| = |-0.33| = 0.33$, thus, **D = 0.33**

Again, D = 1/a is not working for this problem.

f. Patterns (for a<0)

i. So, I listed all the answer.

A. a = -1, D = 1
B. a = -2, D = 0.5 or 1/2
C. a = -3, D = 0.33 or 1/3

It seems like when a = negative number, then the conjecture (D = 1./a) doesn't work. Therefore, what makes the value always **positive?**
The answer is **absolute value**
Therefore, it is clear that the conjecture has to be **D = 1/ lal**

Thus, let's try other one in order to investigate new conjecture.

g. a = -5, b = 3 and c =4

b. $-5x^2+3x+4$

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Process

i. $x_2 - x_1 = -0.8 - -0.71 = -0.09 = S_L$

ii. . $x_4 - x_3 = 1 – 1.11 = =-0.11 = S_R$

iii. $\rceil S_L - S_R \rceil = \rceil -0.09 – -0.11 \rceil = \rceil 0.20 \rceil = 0.2$

So, **D = 0.2**

Therefore, since the conjecture is D = 1/lal and a is -5.

Equation will look like 0.2 = 1/l-5l

0.2 = 1/5 or 0.2, which is correct.

Finally, through part two, I realized the D = 1/lal and it works clearly and absolutely when the a is negative numbers

In deed, it works when a = positive numbers too because absolute value always makes the number to positive.

Again, this is not about the conjecture yet I am proposing this rules in order to depict the exceptions.

a. Different question but same concept

i. In part two, I proposed the vertex of graph

ii. Same as part 1 and part 2, this question is asking about the value of a and placement of the vertex.

iii. Nevertheless, the question is asking about the "real value of a." In other words, a can be a negative. Since I analyzed and listed the graph about a >0. In this part, I am going to introduce and investigate a < 0 (not a=0 because if a = 0, the parabola cannot exist).

b. Again, the vertex of graph is equal to (**$-b/2a$, $(-(b^2- 4ac)/ 4a)$**)

i. Same concept

ii. **$y = -ax^2 + bx + c$**

c. **Exceptions:** Clearly, if there is no four points for intersection, then it is impossible to investigate and calculate the conjecture. Hence in this question, the graph can also divided into four cases like part two. (I didn't label the intersection points like x_1 and x_2 because these graphs are shown because they don't have the intersection points or miss some points.

i. When the graph touches three points (line y=2x: 1 point and y=x: 2 points)

ii. When the graph touches two points (line y = x)

iii. When the graph touches only one point

iv. When the graph does not touch any points

d. Now, I will put the actual numbers to the equation in order to determine and check the results.

i. $y = ax^2 + bx + c$

ii. let $a = -9$, $b = 18$ and $c = -3$

iii. Discriminant (D) = $b^2 - 4ac$.

iv. $(18)^2 - 4(-9)(-3) = 432$

V. $216>0$, since the number is greater than 0, it is clear that the graph should intersects two points for each line.

In conclusion, through the part three, I realized that the D = 1/lal not D = 1/a because if we have negative number for **a** then it is impossible to get correct answer with D = 1/a.

Part Four

3) Does your conjecture hold if the intersecting lines are changed? Modify your conjecture, if necessary; and prove it.

   a. Since we had y = x and y = 2x until now, I will try other lines such as y = 3x.and y = 4x
   b. When the line is y = 3x and y = 4x
      i. Clearly, we know that conjecture is **D = 1/lal**.
      ii. So, let a = 1, b = 4 and c = -5
      iii. **x^2+4x-5**

Process

$x_2 - x_1 = -2.8 + 2.2 = -0.6 = S_L$

$x_4 - x_3 = 2.2 - 1.8 = 0.4 = S_R$

So, the $|S_L - S_R|$ $|-0.6 - 0.4| = |-1|$

In other words, D = 1.

Correct? D = 1/lal, and a = 1.

Since D =1, 1= 1/1 = Correct.

- **Clearly, it still works even if it has different lines.**

c. Let's try other numbers, $y = 7x$ and $y = 8x$
   i. Let $a = 7$, $b = 5$ and $c = -11$
   ii. $7x^2 + 5x - 11$

Process

- $x_2 - x_1 = -1.12 + 1.06 = -0.06 = S_L$
- $x_4 - x_3 = 1.5 - 1.42 = 0.08 = S_R$
- So, the $|S_L - S_R| = 0.14$
- Since, $a = 7$, $0.14 = 1/7$
- Without doubt, even though, the parabola has different intersection lines, the conjecture never changes.

Through these two graphs, even if the parabola has different numbers of lines like y = 4x, 5x, 6x …. *n*x. It really doesn't matter to conjecture. Then what's going to happen when the intersection lines are "negative?"

d. When the intersection lines are negative (y = -2x)
   i. let a = 1, b = 3 and c = 4
   ii. **x^2+3x+4**

Process

$x_2 - x_1 = -4 + 5.23 = 1.23 = S_L$

$x_4 - x_3 = -0.77 + 1 = 0.23 = S_R$

So, the $|S_L - S_R| = |1.23 – 0.23| = |1| = 1$

Hence, **D = 1. Since a= 1: 1 = 1/1 (Correct)**

Again, it's working perfectly with negative lines

e. When the intersection lines are $y = x$ and $y = 2x$

i. let $a = 1$, $b = -3$ and $c = 3$

Process

$$x_2 - x_1 = 1 - 0.70 = 0.30 = S_L$$

$$x_4 - x_3 = 4.30 - 3 = 1.30 = S_R$$

So, the $|S_L - S_R| = |0.30 - 1.30| = |-1| = 1$

Therefore, D = 1 which is correct answer. However, interestingly, I found that (m_2 - m_1) can be replacing the position of 1. So, the equation could be $D = (m_2 - m_1)/|a|$

Just make sure, let's try with new conjecture

f. Investigating Conjecture: $(m_2 - m_1)/|a|$

i. let a = 1, b = 4 and c= 5

Process

$x_2 - x_1 = -5 - -6.2 = 1.2 = S_L$

$x_4 - x_3 = -0.8 - -1 = 0.2 = S_R$

So, the $| S_L - S_R | = | 1.2 – 0.2 | = 1$

Thus, D = 1. So let's try the answer using new conjecture

Since y = -2x and -3x. Equation $((m_2 - m_1)/|a|)$ will look like $(-2 - -3)/1 = 1/1 = 1$

It is clear that it is working perfectly with equation $(m_2 - m_1)/|a|$

g. Investigating Conjecture: $(m_2 - m_1)/|a|$

i. $a = 2$, $b = 5$, and $c = 5$

Process

$x_2 - x_1 = -2.5 - -3.2 = 0.7 = S_L$

$x_4 - x_3 = -0.8 - -1 = 0.2 = S_R$

So, the $|S_L - S_R| = |0.7 - 0.2| = 0.5$

D = 0.5

Check the solution with new conjecture: $(m_2 - m_1)/|a|$

$a = 2$, $m_2 = -2$, and $m_1 = -3$

So the equation will look like $(-2 - -3)/2 = 1/2$

D = 0.5 or 1/2. Conjecture answer = 1/2

Proved

In conclusion, through the process, part three makes me to think about the new conjecture using other lines (like y = 3x). Without doubt, even if the lines are negative or positive, it really doesn't give impacts to D because, through the process, it has no problem to face with it (it always gets the right result). However, the most important point in this part is the new conjecture $(m_2 - m_1)/|a|$. Through this equation, now I have another conjecture can use for investigating parabola.

Part Five

5) Determine whether a similar conjecture can be made for cubic polynomials.

   a. Start with the basic graph

      i. $y = ax^3 + bx^2 + cx + d$

      ii. let a = 3, b = 1, c = -4 and d = -3.

Unlike the part four, part five proposed the cubic polynomials. Therefore, there are another pair of intersection points, in other words, there are six intersection points in the graph.

Like part four, I have to modify my conjecture that is suitable to cubic polynomials.

Since there are six intersection points, I have to make $\mathbf{x_5}$ **and** $\mathbf{x_6}$ for this graph.
And make $\mathbf{S_m} = \mathbf{x_6} - \mathbf{x_5}$

Process

$x_2 - x_1 = 1.47 - 1.39 = 0.08 = S_L$
$x_4 - x_3 = -0.72 + 0.52 = 0.20 = S_R$
$\mathbf{x_6 - x_5} = -1 + 1.28 = 0.28 = S_m$

Since there are three solutions, the equation has to be $D = |S_L - S_R - S_m|$

$|0.08 - 0.20 - 0.28| = |0| = 0$

Interestingly, this time, D is equal to 0.

b. When the a is equal to negative number?
   i. let a = 1 b = -2 c = 4 d = 7
   ii. $ax^3 + bx^2 + cx + d$

Process

$x_2 - x_1 = \quad 3.19 - 2.89 = 0.30 = S_L$

$x_4 - x_3 = \quad 1 - 1.17 = 0.17 = S_R$

$\mathbf{x_6 - x_5} = \quad -2.05 + 2.18 = 0.13 = S_m$

$D = |S_L - S_R - S_m| = 0.30 - 0.17 - 0.13 = 0$

Since, $D = |S_L - S_R - S_m|$ is working perfectly. I can also try other conjecture like $D = |S_R - S_m| - S_L$

c. Investigating Conjecture: $D = | S_R - S_m | - S_L$

i. Let a = -1, b = 2, c = 3, d = -3

ii. $ax^3 + bx^2 + cx + d$

Process

$$x_2 - x_1 = 2.3 - 3 = -0.7 = S_L$$

$$x_4 - x_3 = 1 - 0.62 = 0.38 = S_R$$

$$\mathbf{x_6 - x_5} = -1.62 - -1.3 = -0.32 = S_m$$

$$D = | S_R - S_m | - S_L = | 0.38 + 0.32 | - 0.7 = 0$$

Surprisingly, through this process, there are some possibility that equation $D = | S_L - S_R - S_m |$ could be wrong conjecture. Since, if D is $| S_L - S_R - S_m |$.

The equation will look as -0.7 –0.38 - -0.32 = -0.76 which is NOT 0. In addition, this happens when the one of the line has negative **a** (like y = -x)

Therefore, let's try another one to get precise answer.

i. Let a = 2, b = 9, c= 3, and d= 4
ii. $\mathbf{2x^3+9x^2+3x-4}$

Process

$x_2 - x_1 = 0.6 – 0.45 = 0.15 = S_R$

$x_4 - x_3 = -1.12 - -0.8 = -0.32 = S_L$

$\mathbf{x_6 - x_5} = -3.8 - -4.27 = 0.47 = S_m$

$D = \rceil\ S_m – S_R\ \rceil - S_L = \rceil\ 0.47 – 0.15\ \rceil - 0.32 = 0$

Without doubt, the equation always ends with 0 even with the negative line.

Consequently, through part five, I found another conjecture can be use in cubic polynomial. Since, unlike the quadratic formula, cubic polynomial has six intersection points which led the conjecture to $D = \rceil S_L - S_R - S_m \rceil$. However, after I tried the lines that have negative value for a, it clarified that $D = \rceil S_L - S_R - S_m \rceil$ is wrong conjecture. Therefore, I tried another graph in order to accuracy result and conjecture. Conclusively, my conjecture ends with $D = \rceil S_m - S_R \rceil - S_L$ and it works perfectly for any numbers.

Part Six

6) Consider whether the conjecture might be modified to include higher order polynomials.

a. Again start with the basic graph
b. a = -1, b =9, c= 3, d= -11, and e = -3
c. **$-x^4+9x^3+3x^2-11x-3$**

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Since this graph has eight intersection points which is really confusing. We need to organize. (Group them)

Group: $x_2 - x_1 = 9.16 – 9.18 = - 0.02$

$x_4 - x_3 = 1.23 – 1.20 = 0.03$

$\mathbf{x_6 - x_5} = -0.25 - -0.23 = -0.02$

Now make another one for $\mathbf{x_7}$ **and** $\mathbf{x_8}$**.**

$\mathbf{x_8 - x_7} = -1.13 - -1.16 = 0.03$

Therefore, similar to Part Five, the equation starts with

$D = \left| \left| S_m - S_R \right| - \left| S_L - S_n \right| \right|$. Since, there are eight intersection points (4 pairs).

$$\mathbf{|-0.02 - 0.03| - |-0.02 - 0.03| = |-0.05| - |0.05| = 0}$$

Surprisingly, $\mathbf{x^4}$ equation ends with 0.

d. Then let's try y = $\mathbf{x^5}$
   i. Let a = 1, b = 3, c= 1, d= 1, e=3 and f = 2
   ii. $\mathbf{x^5+3x^4+x^3+x^2+3x+2}$

<u>Process</u>

Since there are ten intersection points, the

$$D = \left|\,|S_m - S_R| - |S_L - S_n|\,\right| - S_o = 0$$

$$x_{10} - x_9 = 6.10 - 6.05 = 0.05 = S_o$$

$$\mathbf{x_8 - x_7} = 5.0 - 4.9 = 0.1 = S_n$$

$$\mathbf{x_6 - x_5} = 0.9 - 0.85 = 0.05 = S_L$$

$$x_4 - x_3 = -3.39 + 3.29 = -0.1 = S_R$$

$$x_2 - x_1 = -3.9 + 3.8 = -0.1 = S_m$$

Hence, equation will look like

$$D = \left|\,|-0.1 - -0.1| - |0.05 - 0.1|\,\right| - 0.05$$

$$\left|\,|0| - |-0.05|\,\right| - 0.05$$

0.05−0.05 = 0, in other words, D = 0

Conclusively, through this part six, it is clear that D is always equal to 0. In addition, through this part, I learned that even if the numbers are getting bigger like x^5 and x^6, there is no change to D. Since the numbers are getting bigger, then the equation is also changing. For instance, when is the cubic polynomials, equation will be $D = | S_m - S_R | - S_L$. Also, when the equation is quadric polynomials, equation looks like $D = | | S_m - S_R | - | S_L - S_n | |$. Without doubt, D is always equal to 0 because when the numbers are changing, equations are changing also.

Conclusion

Finally, the portfolio ends with enormous amounts of graph and equations. Through theses parts, I found the new concept that can't be found in class period. I never had time to think seriously about the conjecture so I am actually glad that I experienced and finished this portfolio. First of all, I needed to make the conjecture in order to understand the problems and patterns. Therefore, in part one, two, three and four. I tried to come up with the conjecture that is suitable to problems. Moreover, part five and six were challenging to me. Since, I never thought that there are some problems about cubic polynomials. All in all, this portfolio was hard. However, it was good experiences to me. The Investigation of Parabola helped me a lot to understand the concept of parabolas, lines and intersections. Through this portfolio, I found enormous sources and information that were not in the HL Math book, and solve many problems that I concerned during the class period. Equally important, I am really thanks to Dr. Hall, who is the conductor of our class, for improving the student to learn and fathom the mathematic concepts using this portfolio.