April 13, 2008

Investigating Matrices

1.) Calculate M^n : *n* = 2, 3, 4, 5, 10, 20, 50

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 2\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = 4\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^4 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} = 8\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^5 = \begin{bmatrix} 32 & 0 \\ 0 & 32 \end{bmatrix} = 16\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^{10} = \begin{bmatrix} 1024 & 0 \\ 0 & 1024 \end{bmatrix} = 512\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^{20} = \begin{bmatrix} 1048576 & 0 \\ 0 & 1048576 \end{bmatrix} = 524288\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$M^{50} = \begin{bmatrix} 1.125899907E15 & 0 \\ 0 & 1.125899907E15 \end{bmatrix}$$

- Pattern I found was that $M^n = x\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ as *n* increases *x* is doubled the previous value while the matrix remains as $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

- Example:

$$A_1 = x = 2[I]$$
$$A_2 = 2[I] + 2[I] = 4[I]$$
$$A_3 = 4[I] + 4[I] = 8[I]$$

- Another pattern with the matrices was that $M^n = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ and as *n* increases *a* is doubled $M^n = \begin{bmatrix} a \bullet 2 & 0 \\ 0 & a \bullet 2 \end{bmatrix}$ as *zero "0"* remains unchanged.

  Example:

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 2 \bullet 2 & 0 \\ 0 & 2 \bullet 2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 4 \bullet 2 & 0 \\ 0 & 4 \bullet 2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$
$$A_4 = \begin{bmatrix} 8 \bullet 2 & 0 \\ 0 & 8 \bullet 2 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

2.) Consider the matrices :

$$P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$
$$and$$
$$S = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

a.)

$$P^2 = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \bullet \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 \bullet 3 + 1 \bullet 1 & 3 \bullet 1 + 1 \bullet 3 \\ 1 \bullet 3 + 3 \bullet 1 & 1 \bullet 1 + 3 \bullet 3 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} = 2\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$
$$P^3 = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \bullet \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 10 \bullet 3 + 6 \bullet 1 & 10 \bullet 1 + 6 \bullet 3 \\ 6 \bullet 3 + 10 \bullet 1 & 6 \bullet 1 + 10 \bullet 3 \end{bmatrix} = \begin{bmatrix} 36 & 28 \\ 28 & 36 \end{bmatrix} = 4\begin{bmatrix} 9 & 7 \\ 7 & 9 \end{bmatrix}$$
$$P^4 = \begin{bmatrix} 36 & 28 \\ 28 & 36 \end{bmatrix} \bullet \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 136 & 120 \\ 120 & 136 \end{bmatrix} = 8\begin{bmatrix} 17 & 15 \\ 15 & 17 \end{bmatrix}$$
$$P^5 = \begin{bmatrix} 136 & 120 \\ 120 & 136 \end{bmatrix} \bullet \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 136 \bullet 3 + 120 \bullet 1 & 496 \\ 120 \bullet 3 + 136 \bullet 1 & 528 \end{bmatrix} = \begin{bmatrix} 528 & 496 \\ 496 & 528 \end{bmatrix} = 16\begin{bmatrix} 33 & 31 \\ 31 & 33 \end{bmatrix}$$

- $P^n = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ where $a = a \bullet 3 + b \bullet 1$ and $b = b \bullet 3 + a \bullet 1$ => observation 1

***Note:** that they (a and b)are multiplied by the matrix (3 and 1)of P^1 *

Example:

$$P^6 = \begin{bmatrix} 528 & 496 \\ 496 & 528 \end{bmatrix} = \begin{bmatrix} 528 \bullet 3 + 496 \bullet 1 & b \\ 496 \bullet 3 + 528 \bullet 1 & a \end{bmatrix} = \begin{bmatrix} 2080 & 2016 \\ 2016 & 2080 \end{bmatrix} = 32\begin{bmatrix} 65 & 63 \\ 63 & 65 \end{bmatrix}$$

b.)

$$S^1 = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$S^2 = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \bullet \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 4\bullet 4 + 2\bullet 2 & 4\bullet 2 + 2\bullet 4 \\ 2\bullet 4 + 4\bullet 2 & 2\bullet 2 + 2\bullet 4 \end{bmatrix} = \begin{bmatrix} 20 & 16 \\ 16 & 20 \end{bmatrix} = 2\begin{bmatrix} 10 & 8 \\ 8 & 10 \end{bmatrix}$$

$$S^3 = \begin{bmatrix} 20 & 16 \\ 16 & 20 \end{bmatrix} \bullet \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 20\bullet 4 + 16\bullet 2 & d\bullet 4 + c\bullet 2 \\ 16\bullet 4 + 20\bullet 2 & c\bullet 4 + d\bullet 2 \end{bmatrix} = \begin{bmatrix} 112 & 104 \\ 104 & 112 \end{bmatrix} = 8\begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

$$S^4 = \begin{bmatrix} 112 & 104 \\ 104 & 112 \end{bmatrix} \bullet \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 656 & 640 \\ 640 & 656 \end{bmatrix} = 16\begin{bmatrix} 41 & 40 \\ 40 & 41 \end{bmatrix}$$

$$S^5 = \begin{bmatrix} 656 & 640 \\ 640 & 656 \end{bmatrix} \bullet \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3904 & 3872 \\ 3872 & 3904 \end{bmatrix} = 32\begin{bmatrix} 122 & 121 \\ 121 & 122 \end{bmatrix}$$

- $S^n = \begin{bmatrix} c & d \\ d & c \end{bmatrix}$ Where $c = c \bullet 4 + b \bullet 2$ and $d = d \bullet 4 + c \bullet 2$ => observation 2

***Note:** *They too are multiplied by 4 and 2 the matrix of S^1 **

- For P^n I have observed the pattern "observation 1" and $x\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ in where *x* is doubled the previous value the pattern of *x= 2, 4, 8, 16, 32…* and "a" would always be twice bigger/higher and "b". where *b=a-2*.

- For S^1 I have also observed a pattern "observation 2" and $x\begin{bmatrix} c & d \\ d & c \end{bmatrix}$ in where x is again doubled the previous value except for the fact that it skips *x=4* but then it does continue on as normal having the pattern of *x= 2, 8, 16, 32…* Also "c" would always be 1 number higher than "d". where *d= c-1.*

3.) Now consider the matrices of the form:

$$\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$$

$$\begin{aligned}
&where:\\
&k=0=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\\
&k=1=\begin{bmatrix} 1+1 & 1-1 \\ 1-1 & 1+1 \end{bmatrix}=\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\\
&k=2=\begin{bmatrix} 2+1 & 2-1 \\ 2-1 & 2+1 \end{bmatrix}=\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\\
&k=3=\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\\
&k=4=\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}\\
&k=6=\begin{bmatrix} 7 & 5 \\ 5 & 7 \end{bmatrix}\\
&k=10=\begin{bmatrix} 11 & 9 \\ 9 & 11 \end{bmatrix}
\end{aligned}$$

- I have observed a pattern that $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ as *k* increases "a" always remain twice higher than "b" where *b=a-2;* $k = \begin{bmatrix} a & a-2 \\ a-2 & a \end{bmatrix}$. "a" and "b" increases as in a sequence, the previous "a" matrix is 1 number less than the next "a" in the next matrix; same goes for "b" and so the pattern goes:

  a = 2, 3, 4, 5, 6, 7, 8, 9…

  b = 0, 1, 2, 3, 4, 5, 6, 7…

4.) Use Technology to investigate what happens with the further values of *k* and *n*.

$$\begin{aligned}
&\textit{where}: \\
&k = 10 = \begin{bmatrix} 11 & 9 \\ 9 & 11 \end{bmatrix} \\
&k^2 = \begin{bmatrix} 11 & 9 \\ 9 & 11 \end{bmatrix}^2 = \begin{bmatrix} 202 & 198 \\ 198 & 202 \end{bmatrix} \\
&k^3 = \begin{bmatrix} 11 & 9 \\ 9 & 11 \end{bmatrix}^3 = \begin{bmatrix} 4004 & 3996 \\ 3996 & 4004 \end{bmatrix} \\
&k^4 = \begin{bmatrix} 80008 & 79992 \\ 79992 & 8008 \end{bmatrix} \\
&k^5 = \begin{bmatrix} 1600016 & 1599984 \\ 1599984 & 1600016 \end{bmatrix}
\end{aligned}$$

- For k=10:

$k^n = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ in where $a = a \bullet 11 + b \bullet 9$ and $b = b \bullet 11 + a \bullet 9$.

There is no limit to *k* and *n.*

5.) Explain why your results hold true in general:

Well I believe that my results hold true in general because the pattern I have been studying works will all the matrices I have tried it with. I have also used technology to see if my answers and observations were correct, in fact I was not mistaken.