

Logan's Logo IA type II

In this assignment, I will investigate the mathematical functions which model these curves and determine Logan's logo.

For this, I will use an appropriate set of axis with a scale of 1 centimetre per unit and record a number of different data points on the curves which will then create model functions.

Logan's logo represents two curves: the lower curve and the upper curve. The variables used are the horizontal-axis and the vertical-axis.

The measured numbers of data points for the lower and upper curve are represented in this table.

1. The various numbers of data points from the curve are entered in a table as shown below. L1 being the horizontal-axis, L2 the lower curve and L3 the upper curve.

L1	L2	L3	1
0	.8	2.7	
.5	.4	2	
1	.4	1.2	
1.5	.6	1	
2	1	1.3	
2.5	1.4	1.7	
3	2	2.4	

$L1(1) = 0$

The window parameters for the graph are then programmed for both the upper and the lower curve to respect the scale on Logan's logo:

```

WINDOW
Xmin=0
Xmax=8
Xscl=1
Ymin=0
Ymax=8
Yscl=1
Xres=1

```

Then a

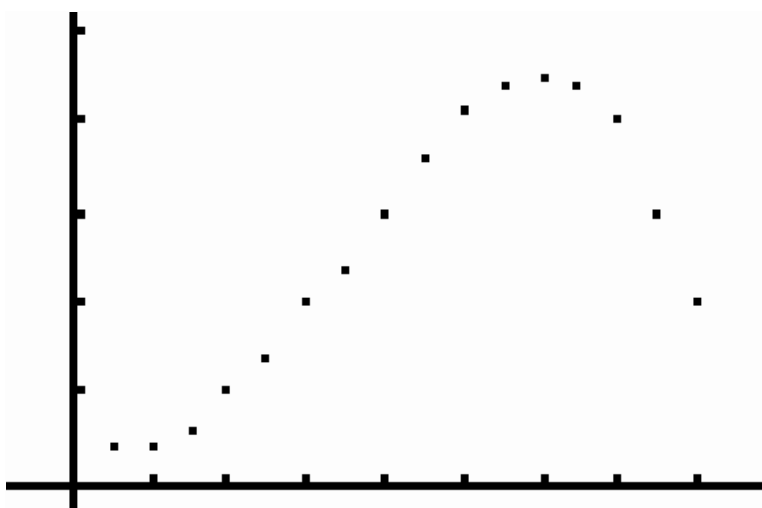
scatterplot is obtained for

both curves after putting the points of each curve into a table.

```

Plot1 Plot2 Plot3
Off Off
Type: [Box Plot] [Line Plot] [Bar Plot]
      [Histogram] [Scatter Plot] [Line Plot]
Xlist:L1
Ylist:L2
Mark: [Box Plot] + .
    
```

Graph of the lower curve:

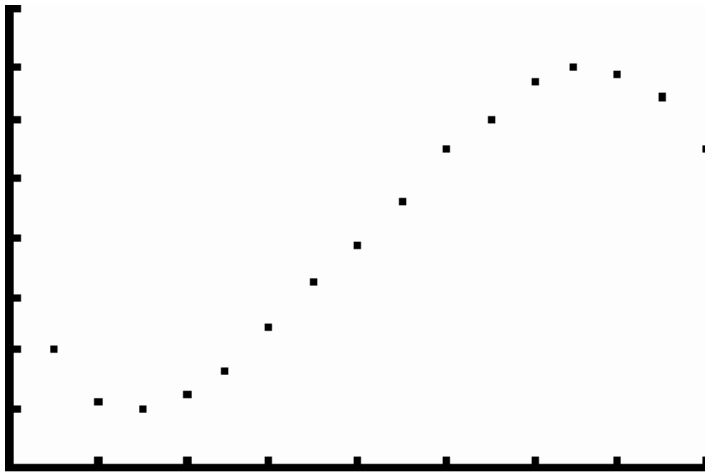


Scatter plot for the upper curve:

```

Plot1 Plot2 Plot3
On Off
Type: [Box Plot] [Line Plot] [Bar Plot]
      [Histogram] [Scatter Plot] [Line Plot]
Xlist:L3
Ylist:L4
Mark: [Box Plot] + [Scatter Plot]
    
```

Graph for the upper curve:



It can be presumed and therefore concluded when looking at both the lower curve and the upper curve of Logan's model that the behaviour of these curves are modelled by sinusoidal functions or sine waveform with alterations between minimum and maximum points. It's most basic form is:

$$f(x) = a * \sin(bx + c) + d$$

2. Now we will try to find the first equation of the presumed lower sine curve using the model equation (SinReg) in the Calc menu of the graphical calculator and rounding off the results to two decimals.

```

EDIT  [2ND] [F5] TESTS
8↑LinReg(a+bx)
9:LnReg
0:ExpReg
H:PwrReg
B:Logistic
[2ND] SinReg
0:Manual-Fit
    
```

```
SinReg
y=a*sin(bx+c)+d
a=2.01
b=.67
c=-2.27
d=2.39
```

Therefore the values of a, b, c and d are the following:

$$f(x) = a \times \sin(bx + c) + d$$

$$a \approx 2.01$$

$$b \approx 0.67$$

$$c \approx -2.27$$

$$d \approx 2.39$$

Now the same method for the upper sinusoidal curve using the sinusoidal regression formula (SinReg):

```
EDIT [2nd] [MODE] TESTS
8:LinReg(a+bx)
9:LnReg
0:ExpReg
A:PwrReg
B:Logistic
[2nd] SinReg
D:Manual-Fit
```

Using the sinusoidal regression formula the values of a, b, c and d for the upper sine curve are found:

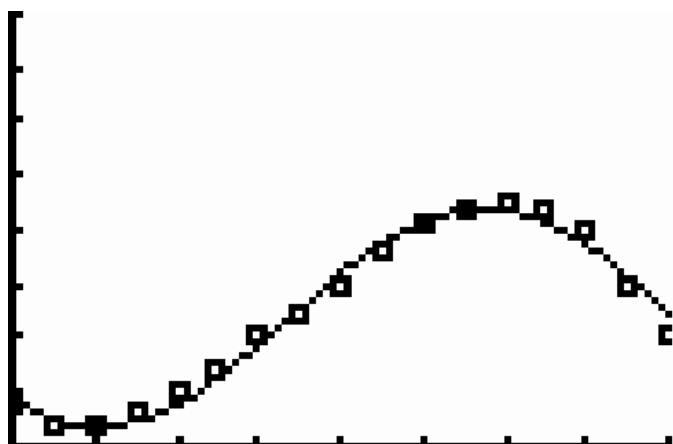
```
SinReg
y=a*sin(bx+c)+d
a=2.85
b=.64
c=-2.60
d=4.05
```

Following the sine formula, the letters are replaced with the values found with the sinusoidal regression formula in the function plot to then create a graph for each curve.

```

Plot1 Plot2 Plot3
\Y1=2.01*sin(0.6
7X-2.27)+2.39
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
    
```

The graph of the lower sine curve resembles this:

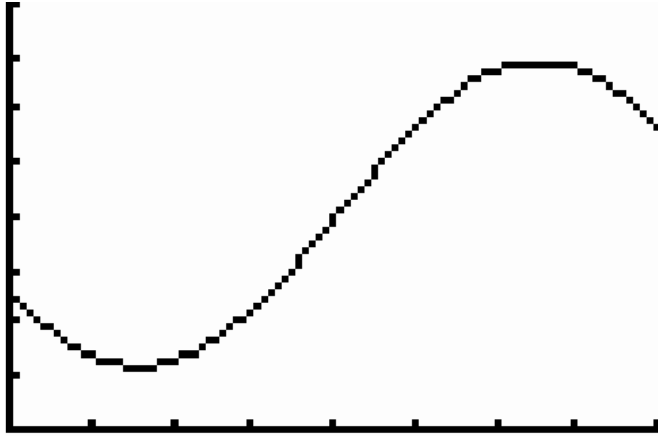


The same method is replicated for the second upper sine curve:

```

Plot1 Plot2 Plot3
\Y1=
\Y2=2.85*sin(0.6
4X-2.60)+4.05
\Y3=
\Y4=
\Y5=
\Y6=
    
```

The graph of the upper sine graph is represented as the following:



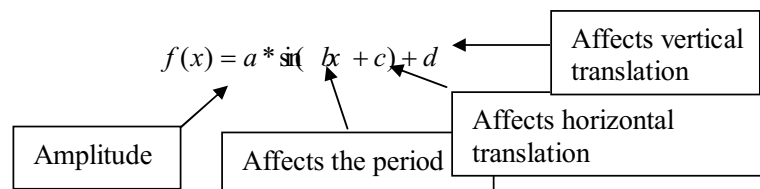
The amplitude of the upper sine curve is calculated with the following formula:

$$\frac{\max - \min}{2}$$

However, the sine curves of Logan's logo carry some limitations.

First of all, it is not certain that this curve follows the sinusoidal behaviour. It is only a possible hypothesis. Furthermore, the values are not extremely precise therefore the results of the sine equations are not very accurate. Thirdly, as it was graphically impossible to obtain the correlation coefficient (r), it could not be determined if the curve fitted the data exactly by giving a result approximately equal to 1. If the result = 1, the data fits the curve perfectly. However, as the results were not very precise the correlation coefficient could never have been equal to 1.

Following the general formula of the sine curve, this belongs to the class of a periodic function which repeats itself over and over again:



3. When doubling the dimensions of the logo in order to print T-shirts with logo on the back, the functions must be modified.

As the dimensions of the logo, must be doubled: the result of the y-axis must be doubled as well as the x-axis. The amplitude (a) and the vertical translation (d) the period both widen the sine curves (the upper and the lower one).

The amplitude widens the format of the logo on the y-axis, a times and the period widens the logo on the x-axis by b^{-1} . However, C (the horizontal translation remains the same, if it is double it will only move to the side but the dimensions of the logo will remain unchanged).

In order to double the dimensions of the vertical-axis, we have to multiply the amplitude of the two curves by two in order to obtain the t-shirt dimensions:

$$f(x) = 2a \times \sin(bx + c) + d$$

$$a \approx 2,01 \rightarrow 2a \approx 4.02$$

$$f(x) = 2a \times \sin(bx + c) + d$$

$$a \approx 2.85 \rightarrow 2a \approx 5.70$$

The amplitude is measured with the following formula:

$$\text{Amplitude} = \frac{\max - \min}{2} =$$

```
WINDOW
Xmin=0
Xmax=16
Xscl=1
Ymin=0
Ymax=16
Yscl=1
Xres=1
```

ower limit=0
Upper limit=8

3. For the business card, we must multiply the amplitude by $\frac{5}{8}$ and to find the area between two curves, we find the area under the curve which is above and subtract the area of the curve which is below. The lower limit is 0 and the upper one is 8. (screenshot 2nd calc , 7)

In order to find the area between the two curves, we have to subtract the area of the lower curve to the area of the upper curve.

$$32.13 - 20.15 \approx 12 \text{ cm}^2$$

The business card's area is 45cm². The amplitude of the two sine functions must be multiplied by $\frac{5}{8}$ to reduce the height (y-axis) to 5 cm

Logan's logo occupies $\frac{3}{16}$ of the business card.

Pour la 1ère fonction A has to be multiplied by $\frac{5}{8}$ and d which affects vertical translation ce qui donne 1.27 et $d = 2.42 * \frac{5}{8} = 1.51$

Pour la 2ème fonction a (amplitude) and the d by $\frac{5}{8}$ $2.93 * \frac{5}{8} = 1.89$. et $d = 4.03 * \frac{5}{8} = 2.52$

Screenshots of business-card format, sine functions :

Then calculate the integrals and area

Sine function 1: $a \times \sin(bx + c) + d$

$$\int f(x) dx = 12 \text{ cm}^2 \text{ for the lower curve}$$

For the upper curve $f(x) = 2a \sin(bx + c) + d$

$$\int f(x) dx = 32.13 \text{ cm}^2 \text{ for the upper curve.}$$

