

Trapezium Portfolio


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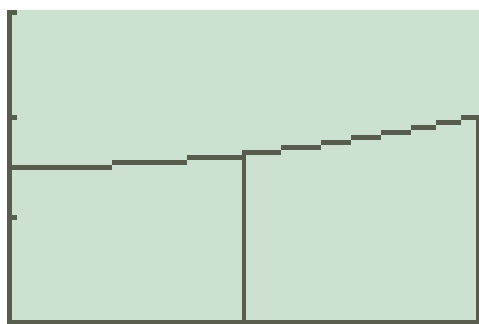
Goal

In this investigation, I will attempt to find a rule to approximate the area under a curve (i.e. between the curve and the x-axis) using trapeziums (trapezoids). Also throughout this investigation, I will be using all sorts of technology varying from an online version of a TI-83 to online graphs.

First Function and 5 Trapeziums

To start, I will take a look at the function $g(x) = x^2 + 3$

The diagram below shows the graph of g . The area under this curve from $x=0$ to $x=1$ is approximated by the sum of the area of the two trapeziums. Below is the graph.



To find this approximation, I will be using the trapezoid rule:

$$(0.5 * length) * [(b_1) + 2(b_2) + 2(b_3) + ... + 2(b_{n-1}) + (b_n)]$$

So, I just use algebra to plug in numbers for these variables...

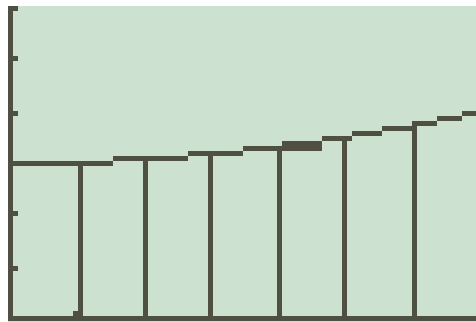
$$\begin{aligned} & (0.5 * 0.5) * [g(0) + 2g(0.5) + g(1)] \\ &= (0.25) * [3 + 6.50 + 4] \\ &= 3.375 \end{aligned}$$

Now, I will increase the number of trapeziums to five and find a second approximation for the area.

$$\begin{aligned} & (0.5 * 0.2) * [g(0) + 2g(0.2) + 2g(0.4) + 2g(0.6) + 2g(0.8) + g(1)] \\ &= (0.1) * [3 + 6.08 + 6.32 + 6.72 + 7.28 + 4] \\ &= 3.34 \end{aligned}$$

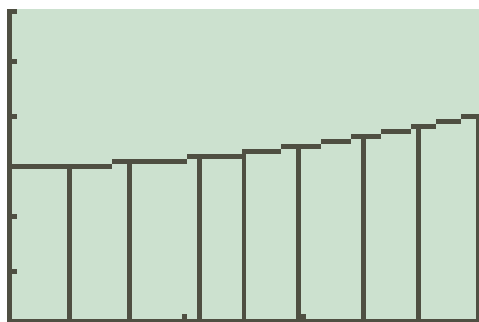
7 Trapeziums

With the help of technology, I have created three diagrams showing an increasing number of trapeziums. Alongside each graph, I will also find the approximation for the area.



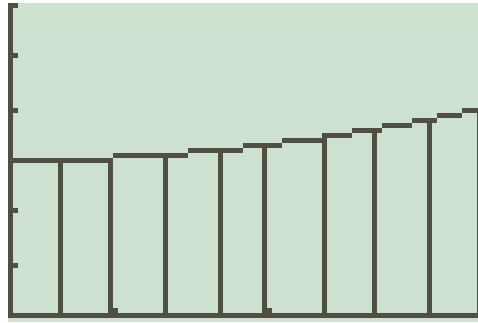
$$\begin{aligned}
 & (0.5 * 0.14286) * [g(0.14286) + 2g(0.28571) + 2g(0.42857) + 2g(0.57143) + 2g(0.71429) + 2g(0.85714) + g(1)] \\
 & = (0.2857) * [3 + 6.0408 + 6.1633 + 6.3673 + 6.6531 + 7.0204 + 7.4694 + 4) \\
 & = 3.336568878
 \end{aligned}$$

8 Trapeziums



$$\begin{aligned}
 & (0.5 * 0.125) * [g(0.125) + 2g(0.25) + 2g(0.375) + 2g(0.5) + 2g(0.625) + 2g(0.75) + 2g(0.875) + g(1)] \\
 &= (0.0625) * [3 + 6.0313 + 6.125 + 6.2813 + 6.5 + 6.7813 + 7.125 + 7.5313 + 4] \\
 &= 3.33595
 \end{aligned}$$

9 Trapeziums



$$\begin{aligned}
 & (0.5 * 0.111) * [g(0.111) + 2g(0.222) + 2g(0.333) + 2g(0.444) + 2g(0.556) + 2g(0.667) + 2g(0.778) + 2g(0.889) + g(1)] \\
 & = (0.0555) * [3 + 6.0247 + 6.0988 + 6.2222 + 6.3951 + 6.6173 + 6.8889 + 7.2099 + 7.5802 + 4] \\
 & = 3.332
 \end{aligned}$$

With the help of technology, my online version of a TI-83, I found the exact integral to be 3.333333. So, I noticed that the more trapezoids I use, the closer the approximation for the area under the curve.

General Expression

I will now find a general expression for the area under the curve of g , from $x = 0$ to $x = 1$, using n trapeziums.

$$\left[\frac{1}{n} * \frac{1}{2}\right] * [g(0) + 2g(0 + \frac{1}{n}) + 2g(0 + 2(\frac{1}{n})) + \dots + 2g(1 - \frac{1}{n}) + g(1)]$$

So basically, all I did was use the trapezoid rule as my base and just replace the

b_1, b_2, b_n , and x 's with actual numbers and n for the amount of trapeziums.

The General Statement

So to find **the** general statement what will estimate the area under any curve

$y = f(x)$ from $x = a$ to $x = b$ using n trapeziums, all I will need to do is change the variables and numbers given to me.

Hence...

$$\int_a^b f(x)dx = \left[\frac{1}{2} * \frac{b-a}{n}\right] * \left[f(a) + 2f\left(a + \frac{b-a}{n}\right) + 2f\left(a + 2\frac{b-a}{n}\right) + \dots + 2f\left(b - 2\frac{b-a}{n}\right) + 2f\left(b - \frac{b-a}{n}\right) + f(b)\right]$$

Three New Curves

Here are three more curves that I will be considering from $x = 1$ to $x = 3$

$$y_1 = \left(\frac{x}{2}\right)^{\frac{2}{3}}$$

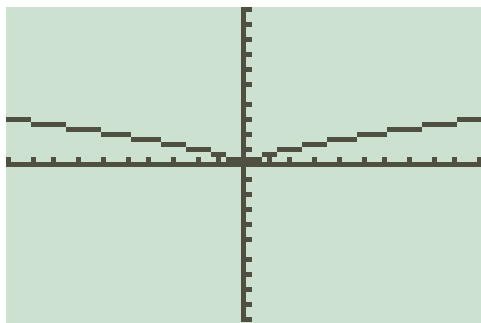
$$y_2 = \frac{9x}{\sqrt{x^3 + 9}}$$

$$y_3 = 4x^3 - 23x^2 + 40x - 18$$

Using my general statement, with eight trapeziums, I will find approximations for each of these areas.

Curve # 1

$$y_1 = \left(\frac{x}{2}\right)^{\frac{2}{3}}$$



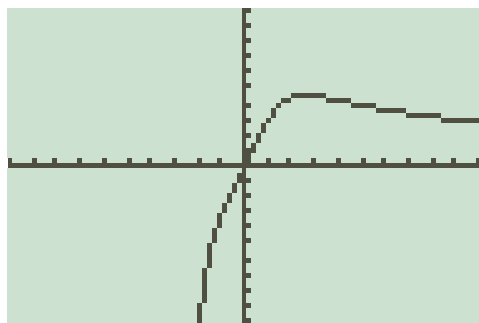
$$\int_1^3 f(x) dx = \left[\frac{1}{2} * \frac{3-1}{8}\right] * [f(1) + 2f(1 + \frac{3-1}{8}) + 2f(1 + 2\frac{3-1}{8}) + 2f(1 + 3\frac{3-1}{8}) + 2f(1 + 4\frac{3-1}{8}) + 2f(1 + 5\frac{3-1}{8}) + 2f(3 - 2\frac{3-1}{8}) + 2f(3 - \frac{3-1}{8}) + f(3)]$$

$$\int_1^3 f(x) dx = \left[\frac{1}{8}\right] * [0.62995 + 1.462 + 1.651 + 1.8297 + 2 + 2.1634 + 2.3208 + 2.473 + 1.31035]$$

$$\int_1^3 f(x) dx = 1.980025$$

Curve # 2

$$y_2 = \left(\frac{9x}{\sqrt{x^3 + 9}} \right)$$



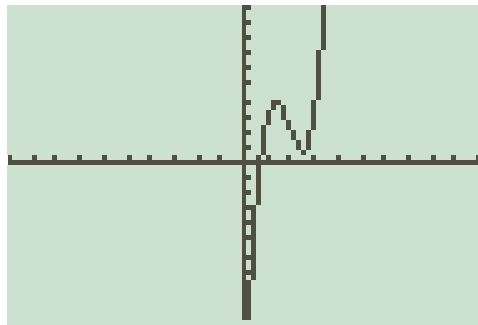
$$\int_1^3 f(x) dx = \left[\frac{1}{2} * \frac{3-1}{8} \right] * \left[f(1) + 2f\left(1 + \frac{3-1}{8}\right) + 2f\left(1 + 2\frac{3-1}{8}\right) + 2f\left(1 + 3\frac{3-1}{8}\right) + 2f\left(1 + 4\frac{3-1}{8}\right) + 2f\left(1 + 5\frac{3-1}{8}\right) + 2f\left(3 - 2\frac{3-1}{8}\right) + 2f\left(3 - \frac{3-1}{8}\right) + f(3) \right]$$

$$\int_1^3 f(x) dx = \left[\frac{1}{8} \right] * [2.84605 + 6.7985 + 7.6752 + 8.3127 + 8.7313 + 8.9689 + 9.0683 + 9.0682 + 4.5]$$

$$\int_1^3 f(x) dx = 8.2462$$

Curve # 3

$$y_3 = 4x^3 - 23x^2 + 40x - 18$$



$$\int_1^3 f(x)dx = \left[\frac{1}{2} * \frac{3-1}{8}\right] * [f(1) + 2f(1+\frac{3-1}{8}) + 2f(1+2\frac{3-1}{8}) + 2f(1+3\frac{3-1}{8}) + 2f(1+4\frac{3-1}{8}) + 2f(1+5\frac{3-1}{8}) + 2f(3-2\frac{3-1}{8}) + 2f(3-\frac{3-1}{8}) + f(3)]$$

$$\int_1^3 f(x)dx = \left[\frac{1}{8}\right] * [3 + 7.75 + 7.5 + 6 + 4 + 2.25 + 1.5 + 2.5 + 3]$$

$$\int_1^3 f(x)dx = 4.6875$$

Actual Areas

With the use of technology (A.K.A. the online version of a TI-83), I found the actual areas under these three curves to be...

$$\int_1^3 \left(\frac{x}{2}\right)^{\frac{2}{3}} dx = 1.9806909$$

$$\int_1^3 \left(\frac{9x}{\sqrt{x^3+9}}\right) dx = 8.2597312$$

$$\int_1^3 (4x^3 - 23x^2 + 40x - 18) dx = 4.6666667$$

Actual Areas vs. Approximation

I found an average error by taking the differences between the actual and estimated areas and taking its average i.e....

$$\frac{[(1.9806909 - 1.980025) + (8.2597312 - 8.2462) + (4.6875 - 4.6666667)]}{3}$$
$$= 0.0116768$$

My approximations were actually very close. On average, my approximations had an error of ± 0.0116768 . This very small error helps convey that my approximations were very accurate.

Trapezium Rule vs. Riemann Sums

Just to test out the accuracy of the Trapezium rule with other equations, I have decided to test

it against the upper, lower, and middle Riemann sums. I will be using the

$$\text{function } f(x) = x^2 + 3$$

From earlier on, we figured out that using the Trapezium rule, the area under the curve is approximately 3.332 (using 9 Trapeziums)

I will now try to estimate the area under the curve using 9 trapeziums again but I will use Riemann sums to figure it out.

$$L_9 = (0.1111) * [f(0) + f(0.1111) + f(0.2222) + f(0.3333) + f(0.4444) + f(0.5556) + f(0.6667) + f(0.7778) + f(0.8889)]$$

$$L_9 = (0.1111) * [29.51833]$$

$$L_9 = 3.2794$$

$$U_9 = (0.1111) * [f(0.1111) + f(0.2222) + f(0.3333) + f(0.4444) + f(0.5556) + f(0.6667) + f(0.7778) + f(0.8889) + f(1)]$$

$$U_9 = (0.1111) * [30.51833]$$

$$U_9 = 3.3905$$

$$M_9 = (0.1111) * [f(0.05555) + f(0.16665) + f(0.27775) + f(0.38885) + f(0.5) + f(0.61115) + f(0.72225) + f(0.83335) + f(0.94445)]$$

$$M_9 = (0.1111) * [29.9903]$$

$$M_9 = 3.319$$

Though these Riemann sums are somewhat close the real area under the curve they are not as accurate as the approximation given by the Trapezium rule.

Scope and Limitations

Throughout this investigation, we derived the general statement which will estimate the area under any curve. Obviously, there are some limitations for this statement. The curvier a function is, the less approximate the estimate will be. This is so because a trapezium relies on a line at the top rather than a curve. Thus the curvier the top of the trapezium, the harder it will be to accurately find the area under the curve. However this accuracy can be increased by the increased use of trapeziums. The other major problem of the general statement is that its accuracy relies on the number of trapeziums. (e.g. 2 trapeziums vs. 5) The last major limitation that I found was a function with a hole. This reduces the accuracy of the area under the curve because it increases the area by a little because it adds that little area under the curve which is, in reality, discontinuous.

In conclusion, this method of finding the area under the curve is quite accurate at finding the area under the curve. It is more accurate than the upper, lower, and middle Riemann sums but it is still less accurate than just mathematically calculating it with an integral.

Disclaimer:

This document is submitted as my own work. I do not seek or receive any unauthorized assistance.

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Date: January 21, 2009

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