Internal Assessment number 1
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IB Mathematics SL (year 2)
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Infinite Surds

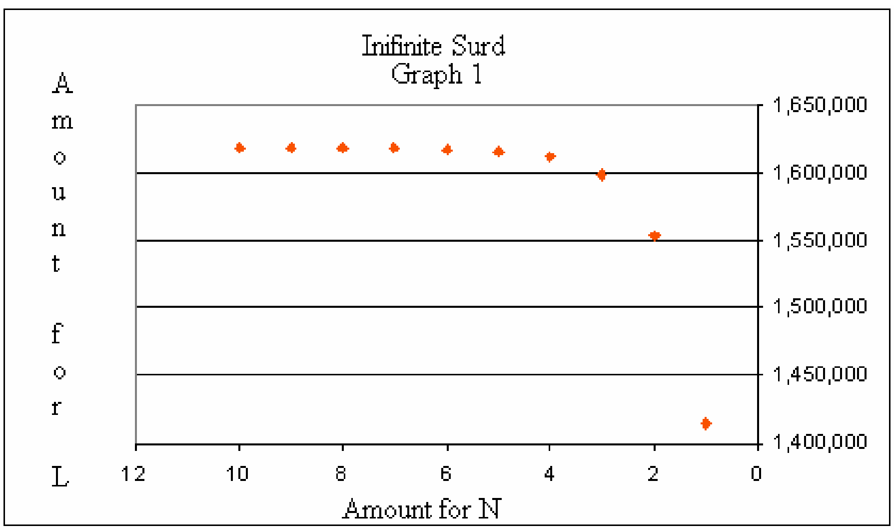
This following expression is known as an infinite surd.

$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\dots}}}}$

The previous infinite surd can be changed into the following sequence:
$a1= \sqrt{1+\sqrt{1}}= 1,414213$
$a2= \sqrt{1+\sqrt{1+\sqrt{1}}}= 1,553773$
$a3= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}= 1,598053$
$a4= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}= 1,611847$
$a5= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}= 1,616121$
$a6= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}= 1,617442$
$a7= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}= 1,617851$
$a8= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}}= 1,617977$
$a9=\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}}}= 1,618016$
$a10= \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}}}}= 1,618028$

The first 10 terms can be represented by:
$a_{n+1}= \sqrt{1 + a_n}$

If we graph the first 10 terms of this sequence we can show that the relationship between n and L can be represented by $L= a_n$

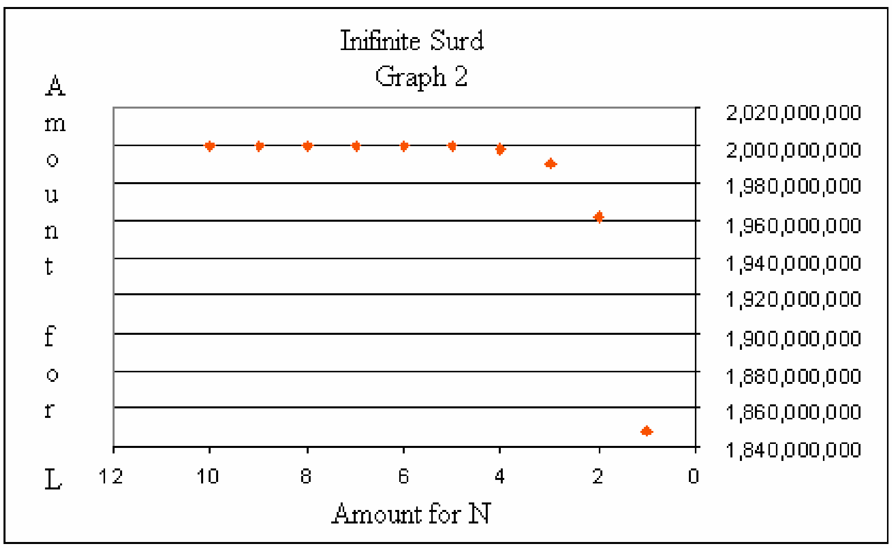


The data begins to increase by a smaller amount about each consecutive n, suggesting that the data may be approaching as asymptote. As these values get very large, they will probably not get much higher than the value of a10, because there already appears to be almost horizontal trend. The data also suggests that the asymptote is between the value of 6 and seven, although to find the exact value requires a different approach

The graph clearly shows that the value of L gradually moves approximately towards 1,618, but it will never reach that number. Furthermore, the relationship shows that as "an" approaches infinity we can deduce that:
$a_n – a_{n+1}$

lim(an – an+1) approaches zero.

Another sequence to prove the previous point is:
b1 = $\sqrt{2}+\sqrt{2}$ = 1,847759056
b2 = $\sqrt{2}+\sqrt{2}+\sqrt{2}$ = 1,961570561
b3= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,990369453
b4= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,997590912
b5= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,999397637
b6= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,9998494404
b7= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,9999849404
b8= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$ = 1,999990588
b9= $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,9999997647
b10=$\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{2}$= 1,9999999412



From this graph we can easily deduce that the values approach 2,0000000000 but it never reaches is it. It acts as an asymptote to the sequence, where the numbers approach the 2 but never quite reach it.

x= $\sqrt{k}+\sqrt{k}+\sqrt{k}\ldots$
x^2= $\sqrt{k}+\sqrt{k}+\sqrt{k}\ldots^2$
x^2= k+ $\sqrt{k}+\sqrt{k}+\sqrt{k}\ldots$

Because we are working with an infinite surd we can deduce that:
$x^2 = k + x$
$0= k + x – x^2$
$0 = (x+k)(x-k)$

The null factor law can be used to portray that any value of k represents an integer.

$(x + 4)(x - 4) = 0$

$\rightarrow x^2 - 4x + 4x - 16 = 0$
$\rightarrow x^2 - 16 = 0$
$\rightarrow x^2 = 16$
$\rightarrow x = 4$

As we compare this result to the general statement we provided we can easily establish that our general statement is valid.