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March 9, 08

Infinite Surds

1.) $(\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}})$

A_1	$\sqrt{1+\sqrt{1}}$	1.4142
A_2	$\sqrt{(1+A_1)}$	1.5537
A_3	$\sqrt{(1+A_2)}$	1.5980
A_4	$\sqrt{(1+A_3)}$	1.6118
A_5	$\sqrt{(1+A_4)}$	1.6161
A_6	$\sqrt{(1+A_5)}$	1.6174
A_7	$\sqrt{(1+A_6)}$	1.6178
A_8	$\sqrt{(1+A_7)}$	1.6179
A_9	$\sqrt{(1+A_8)}$	1.61801
A_{10}	$\sqrt{(1+A_9)}$	1.61802

- As n increases, A_n also increases, however when n gets very large $A_n - A_n + 1$

- $A_n = \sqrt{(1+A_{n-1})}$
- $A_{n+1} = \sqrt{(1+A_n)}$

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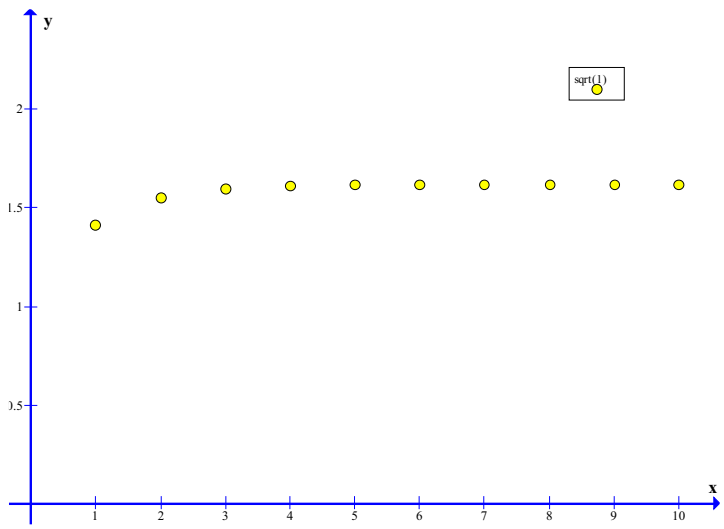

$A_n = \sqrt{(1+ A_{n-1})}$

$A_n = \sqrt{(1+ A_n)}$

$A_n^2 = 1 + A_n$

Quadratic formula :

$a = 1 \quad b = -1 \quad c = -1$

$A_n = -b \ \pm\sqrt{(b^2-4ac)} / 2a$

$A_n = 1 \ \pm\sqrt{(-1)^2-4(1)(-1))} / 2(1)$

$A_n = 1 \ \pm\sqrt{(5)} / 2$

a.) $1 \ +\sqrt{(5)} / 2$

b.) $1 \ -\sqrt{(5)} / 2 < 0$

c.) answer: $1 \ +\sqrt{(5)} / 2$

$A_n^2 - A_n - 1 = 0$: As n gets very large $A_n - A_{n-1} = 0$; $A_n = A_{n-1}$

2.) $\sqrt{(2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}})}$

A_1	$\sqrt{2+\sqrt{2}}$	1.8477
A_2	$\sqrt{(2+ A_1)}$	1.9615
A_3	$\sqrt{(2+ A_2)}$	1.9903
A_4	$\sqrt{(2+ A_3)}$	1.9975
A_5	$\sqrt{(2+ A_4)}$	1.9993

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A_6	$\sqrt{(2+ A_5)}$	1.9998
A_7	$\sqrt{(2+ A_6)}$	1.99996
A_8	$\sqrt{(2+ A_7)}$	1.99999
A_9	$\sqrt{(2+ A_8)}$	1.999997
A_{10}	$\sqrt{(2+ A_9)}$	1.999999

- As n increases, A_n also increases, however when n gets very large

  $A_n - A_n + 2$

- $A_n = \sqrt{(2+ A_{n-1})}$

Therefore:

- $A_{n+1} = \sqrt{(2+ A_n)}$

$A_n = \sqrt{(2+ A_{n-1})}$

$A_n = \sqrt{(2+ A_n)}$

$A_n^2 = 2+ A_n$

Quadratic formula :

$a = 1 \quad b = -1 \quad c = -2$

$A_n = -b \pm\sqrt{(b^2-4ac)} / 2a$

$A_n = 1 \pm\sqrt{(-1)^2-4(1)(-2))} / 2(1)$

$A_n = 1 \pm\sqrt{(9)} / 2$

$1 \pm 3 / 2 = 4/2$

Answer : $A_n = 2$

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$A_n^2 - A_n - 2 = 0$: As n gets very large $A_n - A_{n-1} = 0$; $A_n = A_{n-1}$

2.) $\sqrt{(k+\sqrt{k+\sqrt{k+\sqrt{k+\ldots}}})}$

$A_1 = \sqrt{(k+\sqrt{k})}$

$A_n = \sqrt{(k+ A_{n-1})}$

Therefore:

$A_{n+1} = \sqrt{(k+ A_n)}$

$A_n = \sqrt{(k+ A_n)}$

$A_n^2 = 2+ A_n$

$A_n^2 - A_n - k = 0$

Quadratic formula :

$a = 1 \quad b = -1 \quad c = -k$

$A_n = -b \ \pm\sqrt{(b^2-4ac)} / 2a$

$A_n = 1 \ \pm\sqrt{(-1)^2-4(1)(-k)} / 2(1)$

$A_n = 1 \ \pm\sqrt{(1 + 4k)} / 2$

$1+ \sqrt{(1 + 4k)} / 2$

- In a sequence as *n* increases by jumping even numbers, it equals to an integer. Sequence value of *k:*
  - $k = 2$
  - $k = 6$
  - $k = 12$
  - $k = 20$
  - $k = 30$

The difference between each value is 2, 4, 6, 8, 10

- $1 + 4k$ needs to be a square number and it has to be odd.

So therefore if *k* is 2 in $1+4k$ then:

- When k = 2
  1+4k = 1 + 4(2) = 9
- When k = 6
  1+4k = 1+4(6) = 25

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- 4, *9,* 16, *25 ,* 36…
    - *k* can not be below 0 and you can not square root a negative number so therefore.
        - $k : \{ \boldsymbol{x} \in \boldsymbol{R} \geq \mathbf{0} \}$