Portfolio assignment:

Infinite Surds

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IB Math
MHF 4UB

First of all, roots that are irrational are called surds.

The following expresion is an example of an infinite surd:

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$$

From the previous expresion, generraly know as an **infinite surd**, we can create the following sequence:

$$a_1 = \sqrt{1+\sqrt{1}} = 1,414213$$

$$a_2 = \sqrt{1+\sqrt{1+\sqrt{1}}} = 1,553773$$

$$a_3 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}} = 1,598053$$

$$a_5 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}} = 1,616121$$

$$a_4 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}} = 1,611847$$

$$a_7 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}} = 1,617851$$

$$a_8 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}} = 1,617977$$

$$a_9 = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}}} = 1,618016$$

$$a_{10} = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}}}}}}}} = 1,618028$$

ETC…..

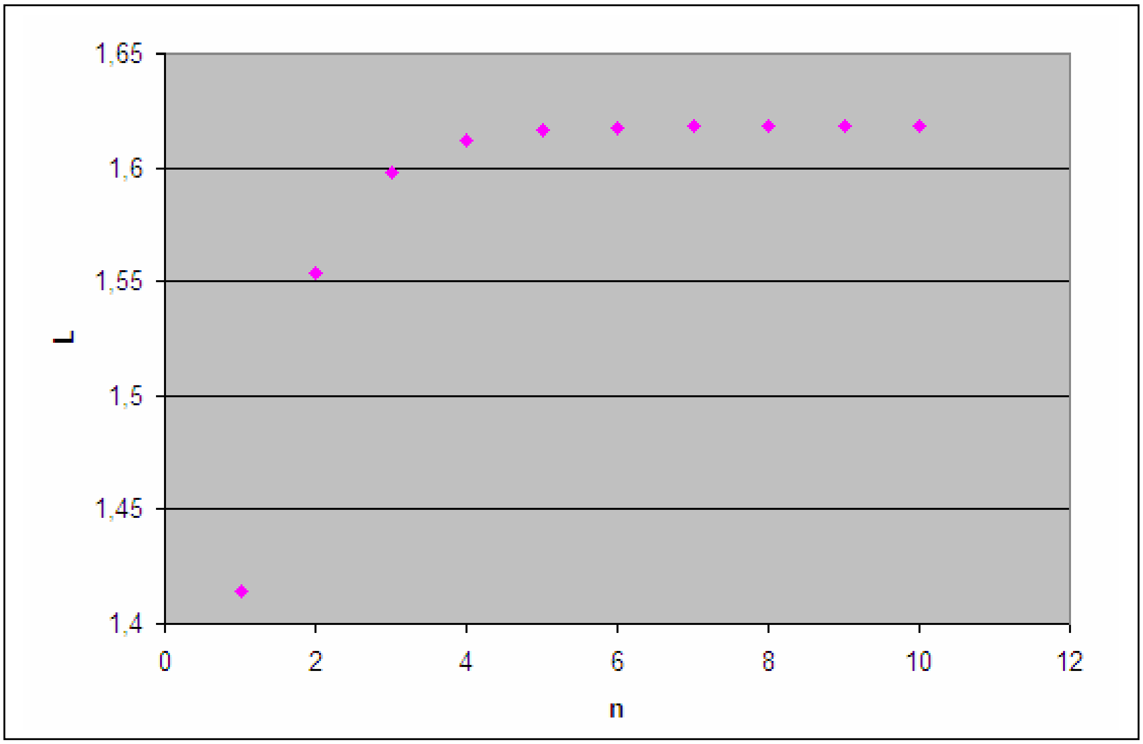
From theses first ten terms of the sequence we can derive a formula for a_{n+1}in terms of a_n. The formula is:

$a_{n+1} = \sqrt{1 + a_n}$

By plotting the first ten terms of the sequence on a graph, we can study the relation between$a_n - a_{n+1}$.

Graph 1

This illustrates the relation between *n* and *L* in the case of $L = a_n$.



From this graph we can see that the value of L is slowly moving towards the value of aprox. 1,618, but the value of L will never attain this value. If we look at the relation between a_n and a_{n+1} we can establish that in the case of$a_n - a_{n+1}$.

$a_n - a_{n+1}$, when *n* approaches infinity,

lim($a_n - a_{n+1}$) → 0

We can expand this and arrive at another infinite surd with a sequence of:

$b_1 = \sqrt{2 + \sqrt{2}}$ ...

We can take this sequence and expand it:

$b_1 = \sqrt{2 + \sqrt{2}} = 1,847759065$

$b_2 = \sqrt{2 + \sqrt{2 + \sqrt{2}}} = 1,961570561$

$$b_3 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}} = 1,990369453$$

$$b_4 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}} = 1,997590912$$

$$b_5 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}} = 1,999397637$$

$$b_6 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}} = 1,9998494404$$

$$b_7 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}} = 1,9999849404$$

$$b_8 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}} = 1,999990588$$

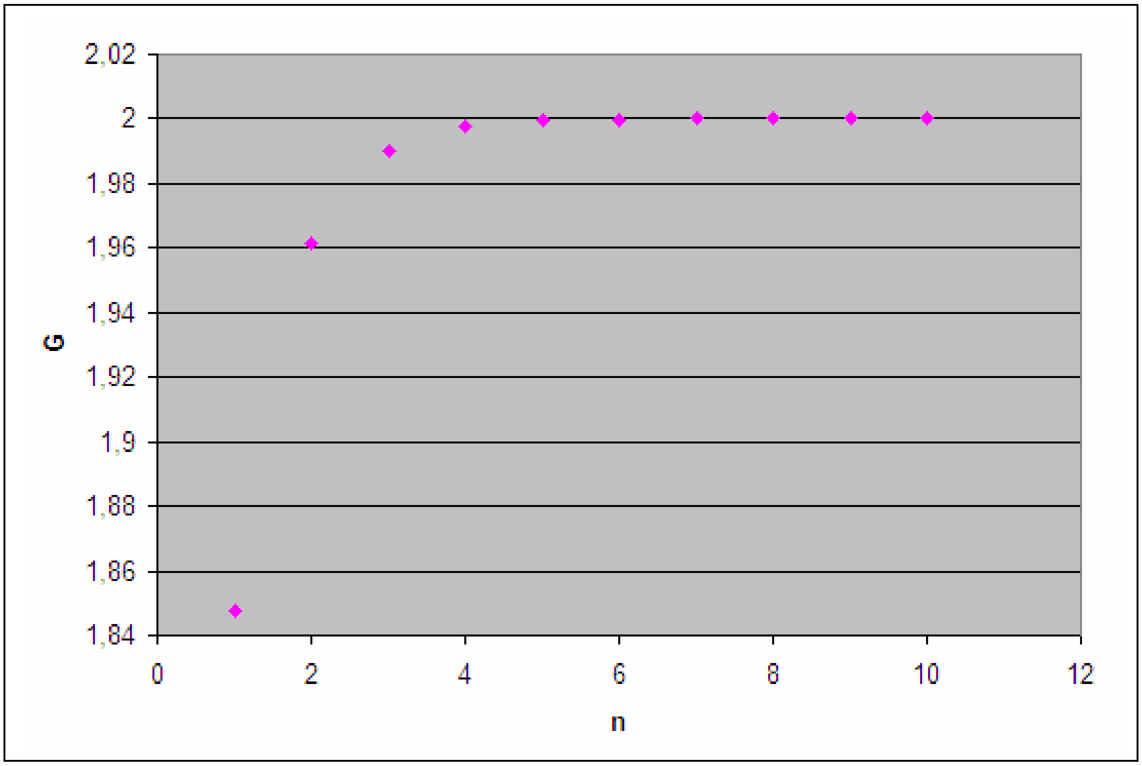
$$b_9 = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}} = 1,999997647$$

$$b_{10} = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}}}}} = 1,999999412$$

ETC…

Graph 2

This graph illustrates *G* in the case of *an.*



From this graph we can clearly see that a_n approaches the value of 2, but it never arrives at 2.

Now consider:

$$x = \sqrt{k + \sqrt{k + \sqrt{k..}}}$$

$$x^2 = \sqrt{k + \sqrt{k + \sqrt{k..}}}^2$$

$$x^2 = k + \sqrt{k + \sqrt{k + \sqrt{k..}}}$$

As we are dealing with an infinite surd, we can expand this out to get:

$$x^2 = k + x$$

$$0 = k + x - x^2$$

$$0 = (x + k)(x - k)$$

This can be expressed as:

$$(x + 2)(x - 2) = 0$$

From this we can determine:

$$x - 2 = 0$$

$$x = 2$$

Comparing this with our graph, we can conclude that our general term is correct!