Infinite summation – SL Portfolio type I

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The infinite series is almost considered as the main tool in calculus, it has different utilizes. It guesses the behavior of functions, investigates differential

equations and also it's used in numerical analysis. Beside these uses in math, the infinite series may be used in physics and economics as well.

t_nConcerning the portfolio, the evaluation is composed on the sum of the series below, where:

$$\ldots \frac{(x \ln a)^n}{n!}, \ldots, t_n = \frac{(x \ln a)^3}{3\times2\times1}, t_3 = \frac{(x \ln a)^2}{2\times1}, t_2 = \frac{(x \ln a)}{1} \quad t_0 = 1, t_1 =$$

, $n!$ in the series above, the task gave a notation for the factorial n for the value described as :

$$n! = n(n-1)(n-2)\cdots(3)\times(2)\times(1)$$

$$5! = 5\times4\times3\times2\times1 = 120$$

(Given)$0! = 1$

To find out the general statement that represents the infinite sum of this terms of n of the first $S_n$sequence, it's always required to determine the sum $x = 1, a = 2$, for where $0 \le n \le 10$the infinite sequence for

$$1, \frac{(ln2)}{1}, \frac{(ln2)^2}{2!}, \frac{(ln2)^3}{3!}, \ldots$$

Using the notations given mentioned and the sequence values, we concluded S_n and it's effect on the n this table containing the variation of

Note for ALL tables and graphs:

n*The only controlled variable in the table is the*

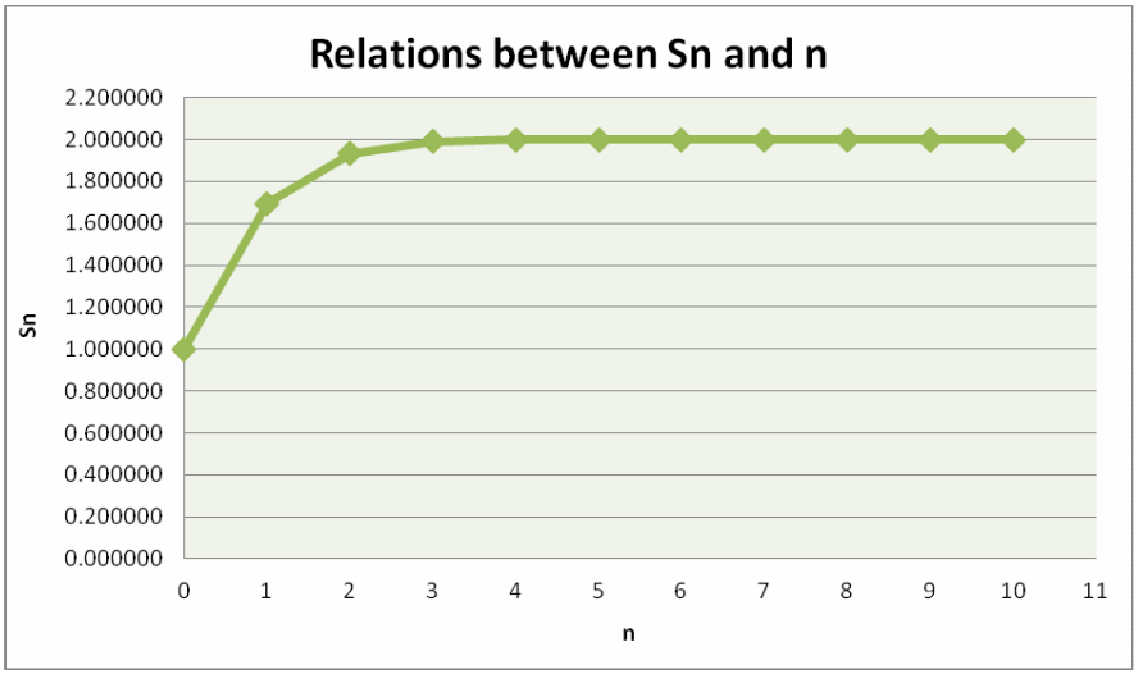
All of the answers below will be corrected to 6 decimal places

$0 \le n \le 10$ *Domain of n is*

Graphs are all done using Microsoft Excel

$x = 1, and\ a = 2)($ **n and S_nRelation between**

n	x	a	S_n
0	1	2	1.000000
1	1	2	1.693147
2	1	2	1.933374
3	1	2	1.988878
4	1	2	1.998496
5	1	2	1.999829
6	1	2	1.999983
7	1	2	1.999999
8	1	2	2.000000
9	1	2	2.000000
10	1	2	2.000000



n value is increasing when the S_nI can notice right now from this plot, that the value increases, so simply they are positively correlated .

. So now can completely agree $y = 2$ however, the graph does not go beyond moreover, it is noticeable that as $y = 2.$ that there is a kind of asymptote at . Even though on the table, the data shows that $S_n \to 2$, the values of $n \to \infty$. In this case we can call what i s happening a $S_n = 2.000000$when n=10, , while on the $S_{n\,=}\ asymtote$limitation, because now in th e table we see the

doesn't intersect the asymptote, this has occurred because $n = 10$ graph the we can only take six decimal places . So when we expended the value, It $n = 10, S_n = 1.9999999995$shows when

The investigation will be clea rer and more will result more relevant if we change and add a new variable on the investigation . This time the variable and have a more S_n will be shifted into 3, to see a clearer impact on the $\boldsymbol{a}$ developed observation …

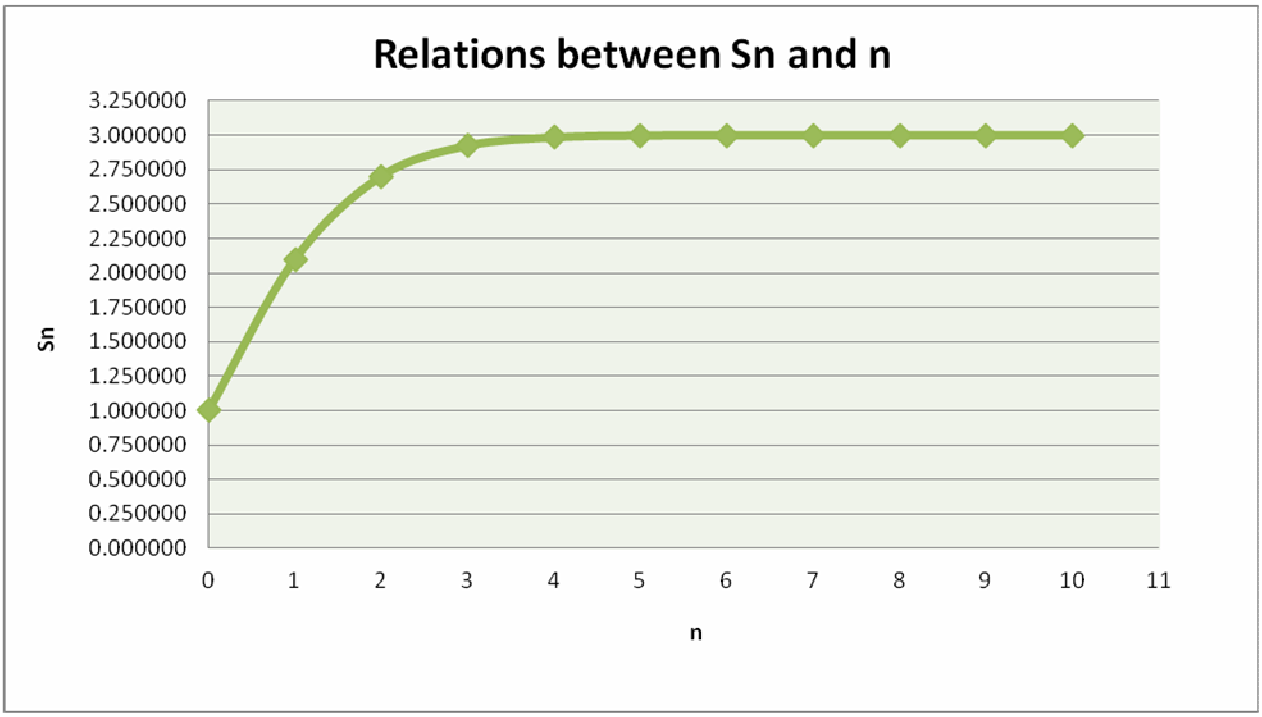
$x = 1, and\ a = 3$

$$1, \frac{(ln3)}{1}, \frac{(ln3)^2}{2!}, \frac{(ln3)^3}{3!}, \ldots$$

$x = 1, and\ a = 3$)($n$and S_nRelation between

$\boldsymbol{n}$	$\boldsymbol{x}$	$\boldsymbol{a}$	$\boldsymbol{S_n}$
0	1	3	1.000000
1	1	3	2.098612
2	1	3	2.702087
3	1	3	2.923082
4	1	3	2.983779
5	1	3	2.997115
6	1	3	2.999557
7	1	3	2.999940
8	1	3	2.999993
9	1	3	2.999999
10	1	3	3.000000

.n and S_n Again, let's plot a graph representing the relation between



value is increasing while S_nThis plot insure the proof that the
. $y = 3$ is also increasing. This time, the graph would not go further$n$ $value$the
The limitation that I $y = 3$.It's noticeable that the asymptote of this graph is
, the values $n \to \infty$found in the first trial can be valid, by the statement that as
, in this case it is also 3. $S_n \to a$of

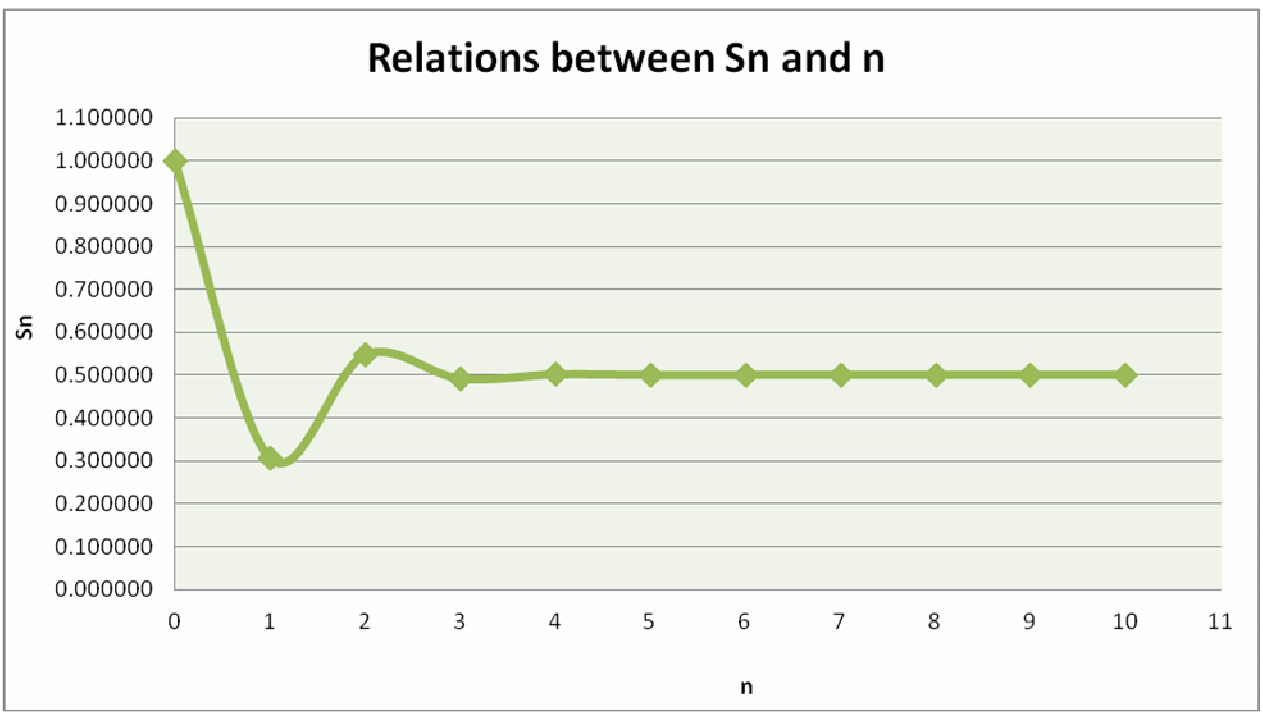
, and different $x = 1$, when $S_n$at the moment, I will evaluate the sum
will be as following 0.5, 0.1, 25. First two a. The different values of $a$values of
trials with rational numbers then the third with a greater number than all the
a valueothers trials. Before starting this evaluation, we can agree that the
is for $a$cannot be a zero or a negative value, , because the domain of
),also we cannot take a zero beside (ln) as ln −10positive numbers only as (
simply ln 0for instance:

as aLet's try now the what we planned for, so first try the variable

$$a = 0.5$$

$(x = 1, and\ a = 0.5)$ **n and S_nRelation between**

n	x	a	S_n
0	1	0.5	1.000000
1	1	0.5	0.306853
2	1	0.5	0.547079
3	1	0.5	0.491575
4	1	0.5	0.501193
5	1	0.5	0.499860
6	1	0.5	0.500014
7	1	0.5	0.499999
8	1	0.5	0.500000
9	1	0.5	0.500000
10	1	0.5	0.500000



isn't behaving exponentially , the relation $y = a$From this trial, the

. $y = a$ rise and fall up and down, and then it reaches $s_n\ and\ n$between

, the values of $n \to \infty$However, my notice is that it still stands as when as

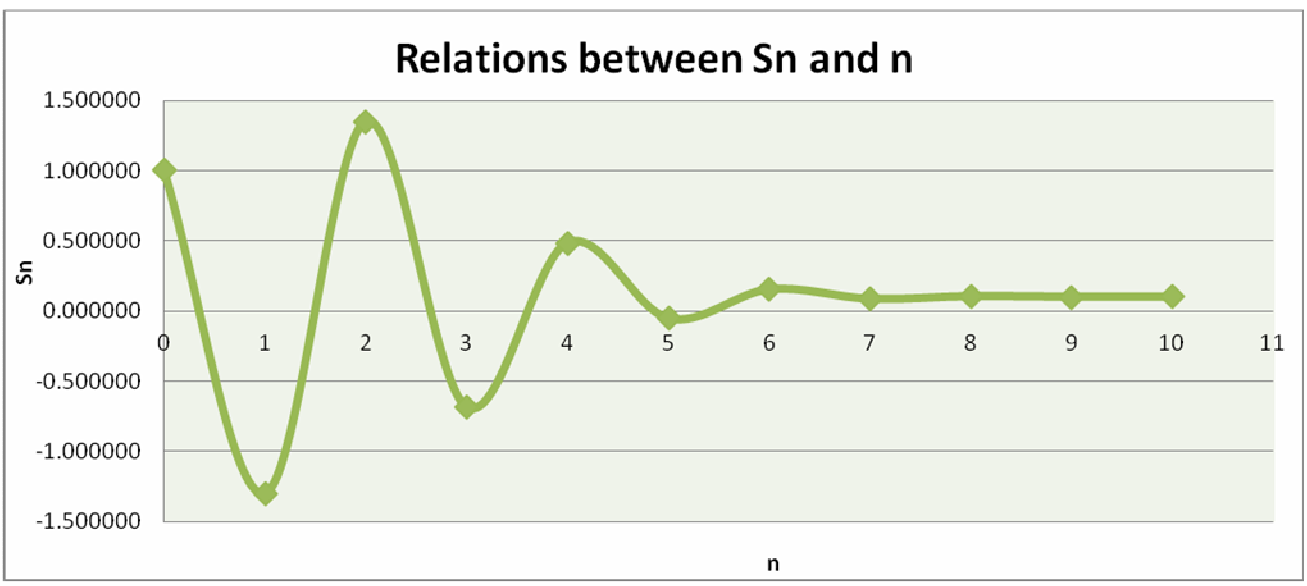
, the graph swing $= positive\ number$. In this graph, also I noticed that a$S_n \to a$ up and down. The graph may fluctuate up and down and varies below the asymptote, but will never intersect with the asymptote .

Now let's go for a small rational number to see what happens …

$(x = 1, and\ a = 0.1)$ **n and S_nRelation between**

n	x	a	S_n
0	1	0.1	1
1	1	0.1	-1.302585093
2	1	0.1	1.348363962
3	1	0.1	-0.68631463
4	1	0.1	0.484940519
5	1	0.1	-0.05444241
6	1	0.1	0.152553438
7	1	0.1	0.084464073
8	1	0.1	0.104061768
9	1	0.1	0.099047839
10	1	0.1	0.100202339

Then we plot it using technology (Microsoft excel)



This sequence of data has lots more of fluctuation than the previous

we found for the first time units under $a = 0.1$, on this trial where $a = 0.5$trial when

the zero, I have observed that the values less than zero are when (n) is an odd number, the

with different variation, the more (n) is, the less $n = 10$graph fluctuates until

the fluctuation is. This graph follows the constant general statement found

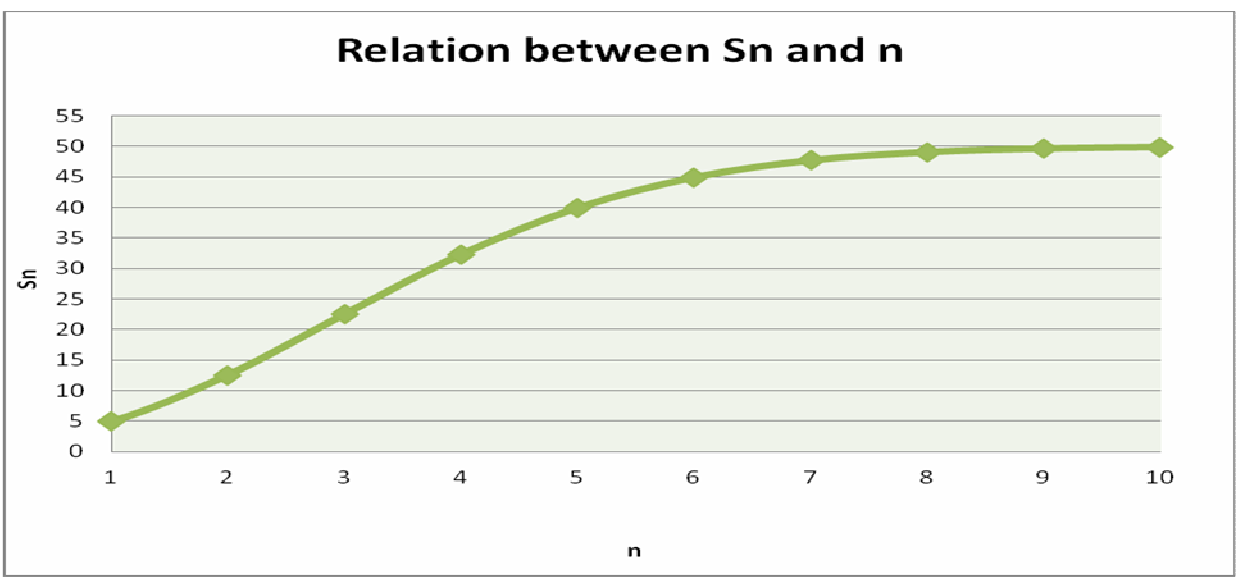
$S_n \to a, which\ is\ 0.5\ in\ this\ case.$, the values of $n \to \infty$previously that says

Now and finally let's try a quite big number in proportion with the previous trials, so now we have *a=50, while*

Here is the table:

n	x	a	S_n
0	1	50	1
1	1	50	4.912023
2	1	50	12.563985
3	1	50	22.542202
4	1	50	32.300956
5	1	50	39.93625
6	1	50	44.914491
7	1	50	47.696632
8	1	50	49.057108
9	1	50	49.648464
10	1	50	49.879805

Then we plot this set of data using technology (Excel):



From this set of data using $a=50$ we now are completely sure that there is always an asymptote on the value of a, also that if we have infinity of n, we will find the sum equals the a at the end...

After investigating all the above trials and had a lot of observation about the relation between *Sn and n*, we finally found a general statement that
, the $n \to \infty$represents the infinite sum of the general sequence which is
. Which can be expressed as :$S_n \to a$values of

$$S_n = \sum_{n=0}^{\infty} \frac{(lna)^n}{n!} \to a$$

After getting the general statement that represents the infinite sum of the general sequence, we now have a next challenge concerning the expend of the investigation, let's see how:

$$t_0 = 1, t_1 = \frac{(xlna)^1}{1}, t_2 = \frac{(xlna)^2}{2 \times 1}, t_3 = \frac{(xlna)^3}{3 \times 2 \times 1}, \ldots$$

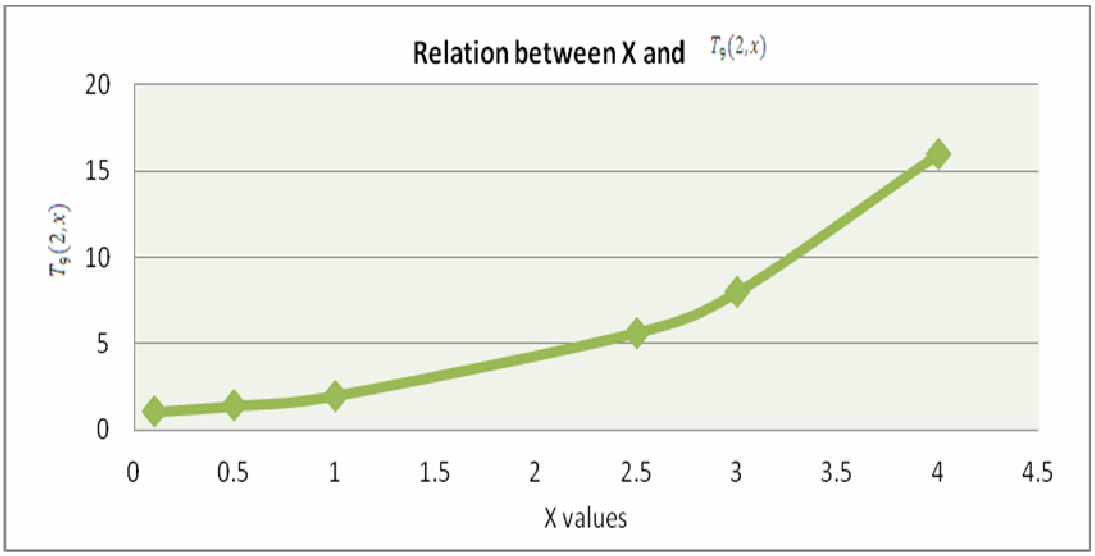
is the sum of the first n terms, for various values $T_n(a,x)$It's given that that
. a and $x$of

let's calculate it with different $T_9(2,x)$Let a =2. We now have the unknown
. $x$positive values of

$(a = 2)$ n and S_nRelation between

x	$T_9(2,x)$	$f(x) = 2^x$
0.1	1.071773	1.071773
1	2.000000	2.000000
2.5	5.656775	5.656854
3	7.999488	8.000000
4	15.990193	16.000000

And now let's see this set of data on the graph…

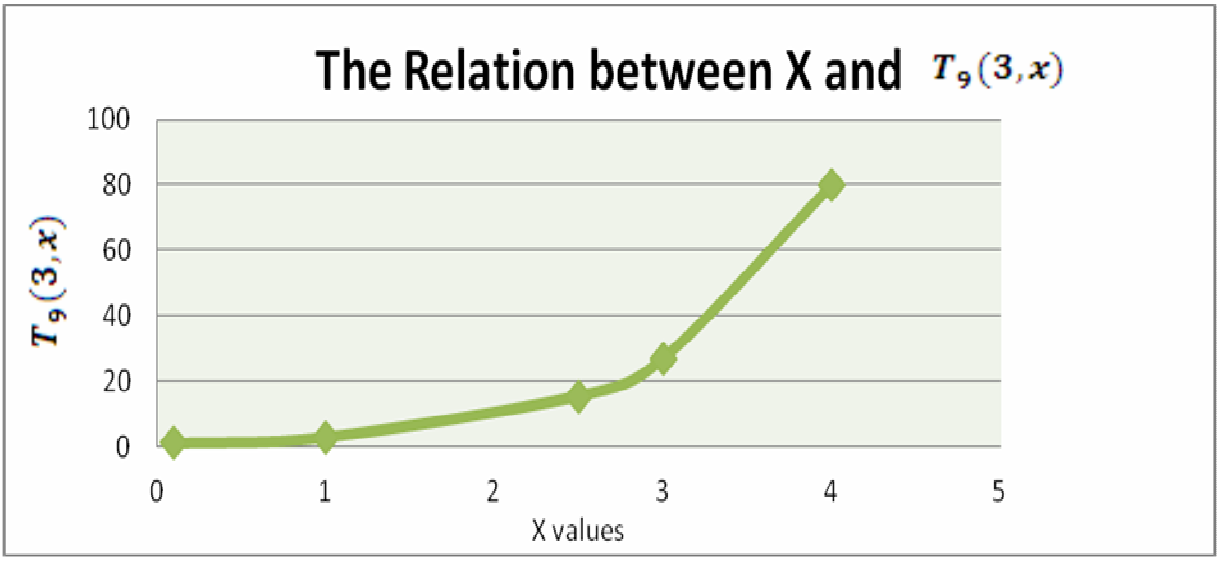


Now we can see how the graph behave exponentially, which was predicted by me before investigating because the function was $f(x) = a^x$ which resemble the exponential function, Beside that we can agree also that the two variables evaluated are positively correlated, as when the x values increases, the values of *T9(2,x)* will increase as a result. Another observation that enhance the investigation evaluation, we , the values of $n \to \infty$came on the previous investigation with a general rule , which we have insured its validity by finding in the graph the base 2, $S_n \to a$ and 2 is the *x which equals a, So now we can write it a* $n \to \infty$,$x = 2$, $Sn \to a^2$....

$T_9(3,x)$Now let's try a=3, meaning that

x	$T_9(3,x)$	$f(x) = 3^x$
0.10	1.116123	1.116123
1.00	2.999999	3.000000
2.50	15.579560	15.588457
3.00	26.941276	27.000000
4.00	79.803290	81.000000

By using Excel we came with this graph …



In this second trial on the extended investigation , it is noticeable that the . The only variable and change is the value, also as I $f(x) = a^x$ function is *will* increase at $T_9(3,x)$mentioned above, the x values increases, the values of an exponential rate. So we can now be sure of our statement saying that :
. $y = a^x, \quad a = 3, x = unknown\ value, T_9\,(3,x) should\ approach \quad y = 3^x$

To more insure this statement and go for it in more depth, I tried another trial using $x = 5, T_9\,(3,x)$, I observed that there is a quite big difference concerning the result, between the algebraically and geographical ly ones or it appears also on the tables, as by looking at the graph, $T_9\,(3,5) = 230.019331$, but on the calculator I found 3 at the power of 5 equals 243.000000. This may be considered as a limitation of the investigation. But the situation was treated by the fact that we are only using the 9th term of the sequence, but if we increase the n value the difference will decrease until it reaches the same result, and the n that equals the algebraically result, is the final term in the sequence...

Now finally getting the final general statement s discovered in the investigation, by all the trials above analysis we can finally say that:

Firstly as the n value approaches ∞, the values of $T_n(a,x)$ will approach a^x. This relation can be represented as:

$$T_n(a,x) = \sum_{n=0}^{\infty} \frac{(xlna)^n}{n!} \rightarrow a^x$$

The domain of the general statement is:

$x \neq 0, \quad a > 0 \; a \neq 1, \quad n = positive\ integers \quad x \neq 0, \; a \neq 1,$

So now we can say that:

$$a^x = 1 + \frac{(xlna)^1}{1} + \frac{(xlna)^2}{2!} + \frac{(xlna)^3}{3!} + \frac{(xlna)^n}{n!} + \cdots$$

With respect to domain listed above...

Finally let's test the validity of the general statement by using variables *a* and *x*…

$T_5(0.5,6)$

n	a	x	$T_5(a,x)$	$y = 0.5^6$
1.000000	0.500000	6.000000	0.895887	
2.000000	0.500000	6.000000	0.447946	
3.000000	0.500000	6.000000	0.149313	
4.000000	0.500000	6.000000	0.037328	
5.000000	0.500000	6.000000	0.007466	0.007466

END OF THE INVESTIGATION...