International Baccalaureate

Math Standard Level Internal Assessment

Portfolio Type: I

# Portfolio Title: Infinite Summation

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**Introduction:**

A series is a sum of terms of a sequence. A finite series, has its first and the last term defined, and the infinite series, or in other words infinite summation [3] is a series which continues indefinitely. The Taylor's theorem [1] and the Euler-Maclaurin's formula [2] will help us solve our given infinite summation, which is:

$$t_0 = 1,\quad t_1 = \frac{(x \ln a)}{1},\quad t_2 = \frac{(x \ln a)^2}{2x1},\quad t_3 = \frac{(x \ln a)^3}{3x2x1},\ \ldots,\quad t_n = \frac{(x \ln a)^n}{n!}$$

And by adding different values for $x$ and $a$, we will be able to find a general pattern in which the sequences tends to move with. And this is mainly what this portfolio will ask us to do.

**Method:**

For our sequence, which is:

$$t_n = \frac{(x \ln a)^n}{n!}$$

, we have to substitute in the case where x = 1 and a = 2. After that, we have to calculate the first $n$ terms which happen to be eleven to fulfill the given condition $0 \leq n \leq 10$ .

So after substitution we get $t_n = \frac{(\ln 2)^n}{n!}$

Now let's calculate for $n$, when $0 \leq n \leq 10$ :

$$t_0 = 1$$

$$t_1 = \frac{(\ln 2)^1}{1!} = 0.693147$$

$$t_2 = \frac{(\ln 2)^2}{2!} = 0.240226$$

$$t_3 = \frac{(\ln 2)^3}{3!} = 0.055504$$

$$t_4 = \frac{(\ln 2)^4}{4!} = 0.009618$$

$$t_5 = \frac{(\ln 2)^5}{5!} = 0.001333$$

$$t_6 = \frac{(\ln 2)^6}{6!} = 0.000154$$

$$t_7 = \frac{(\ln 2)^7}{7!} = 0.000015$$

$$t_8 = \frac{(\ln 2)^8}{8!} = 0.000001$$

$$t_9 = \frac{(\ln 2)^9}{9!} = 0.000000 \quad 0 = 0$$

$$t_{10} = \frac{(\ln 2)^{10}}{10!} = 0.000000 \quad = 0$$

In fact, $t_9$ and $t_{10}$ are not equal to 0, but since we have to take our answers correct to six decimal places, we can't see the real values. However, the numbers become so small, that they become insignificant, or in other words they are equal to 0.

Now, we need to find the sum of $S_n$ :

$$S_0 = 1$$

$$S_1 = 1 + \frac{(\ln 2)^1}{1!} = 1.693147$$

$$S_2 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} = 1.933373$$

$$S_3 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} = 1.988877$$

$$S_4 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} = 1.998495$$

$$S_5 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} = 1.999829$$

$$S_6 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} = 1.999983$$

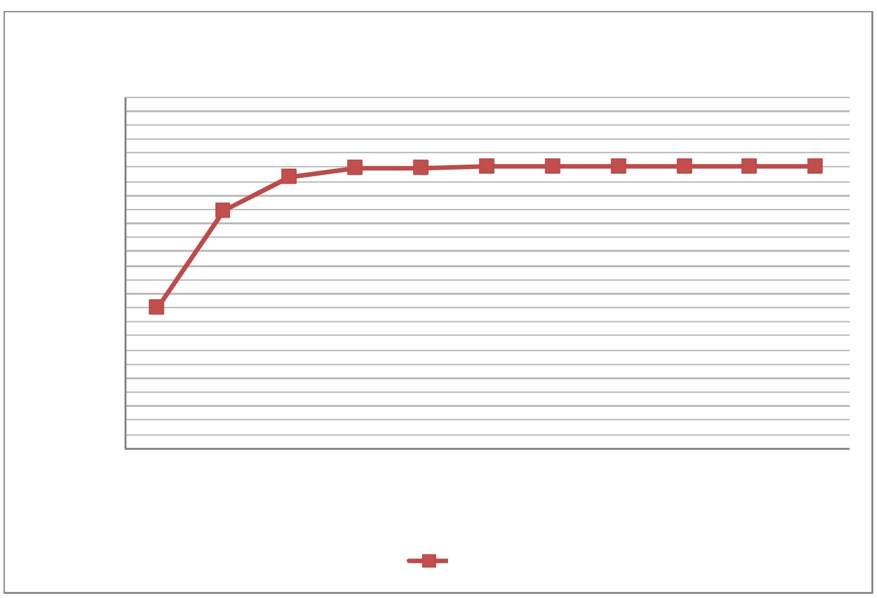
$$S_7 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} = 1.999998$$

$$S_8 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} + \frac{(\ln 2)^8}{8!} = 1.999999$$

$$S_9 = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} + \frac{(\ln 2)^8}{8!} + \frac{(\ln 2)^9}{9!} = 1.999999$$

$$S_{10} = 1 + \frac{(\ln 2)^1}{1!} + \frac{(\ln 2)^2}{2!} + \frac{(\ln 2)^3}{3!} + \frac{(\ln 2)^4}{4!} + \frac{(\ln 2)^5}{5!} + \frac{(\ln 2)^6}{6!} + \frac{(\ln 2)^7}{7!} + \frac{(\ln 2)^8}{8!} + \frac{(\ln 2)^9}{9!} + \frac{(\ln 2)^{10}}{10!} \approx 2$$

Now, using Excel 2010, let's plot the relation between $S_n$ and $n$ :



Looking at the graph, we can notice that $S_n$ increases rapidly at first, and then it evens out when it reaches 2, which seems like an asymptote. The same happens with the terms' values. They decrease rapidly until they reach the 0, which if we plot will seem like its asymptote. Therefore, we can see that both move a maximum of 1 unit away from their first point, and then even out to the mentioned asymptote.

For $S_n$, the asymptote is $x = 2$.
For the terms' calculation for given $n$, the asymptote is $x = 0$.

Therefore:
As $n$ approaches infinity, $S_n$ approaches 2:

$$n \to \infty,\ S_n \to 2$$

Now, we do the same thing as before, but for $a = 3$ with the same condition for $n$ ($0 \le n \le 10$):

$$t_0 = 1$$

$$t_1 = \frac{\ln 3^{1}}{1!} = 1.09862$$

$$t_2 = \frac{\ln 3^{2}}{2!} = 0.60347$$

$$t_3 = \frac{\ln 3^{3}}{3!} = 0.22094$$

$$t_4 = \frac{\ln 3^{4}}{4!} = 0.06069$$

$$t_5 = \frac{\ln 3^{5}}{5!} = 0.01336$$

$$t_6 = \frac{\ln 3^{6}}{6!} = 0.002441$$

$$t_7 = \frac{\ln 3^{7}}{7!} = 0.000383$$

$$t_8 = \frac{\ln 3^{8}}{8!} = 0.000052$$

$$t_9 = \frac{\ln 2^{9}}{9!} = 0.0000006$$

$$t_{10} = \frac{\ln 2^{10}}{10!} = 0.000000 = 0$$

Now we have to calculate $S_n$ again, but for $a = 3$:

$$S_0 = 1$$

$$S_1 = 1 + \frac{\ln 3^{1}}{1!} = 2.09862$$

$$S_2 = 1 + \frac{\ln 3^{1}}{1!} + \frac{\ln 3^{2}}{2!} = 2.70208$$

$$S_3 = 1 + \frac{\ln 3^{1}}{1!} + \frac{\ln 3^{2}}{2!} + \frac{\ln 3^{3}}{3!} = 2.92308$$

$$S_4 = 1 + \frac{\ln 3^{1}}{1!} + \frac{\ln 3^{2}}{2!} + \frac{\ln 3^{3}}{3!} + \frac{\ln 3^{4}}{4!} = 2.98378$$

$$S_5 = 1 + \frac{\ln 3^{1}}{1!} + \frac{\ln 3^{2}}{2!} + \frac{\ln 3^{3}}{3!} + \frac{\ln 3^{4}}{4!} + \frac{\ln 3^{5}}{5!} = 2.997149$$

$$S_6 = 1 + \frac{(\ln 3)^1}{1!} + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \frac{(\ln 3)^5}{5!} + \frac{(\ln 3)^6}{6!} = 2.99956$$

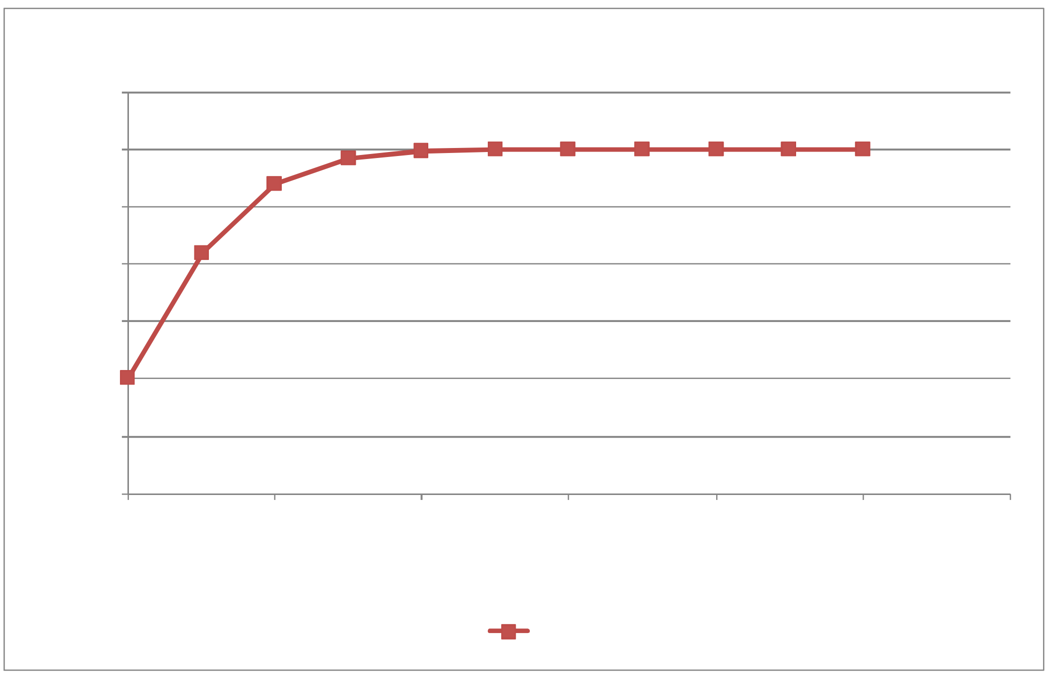
$$S_7 = 1 + \frac{(\ln 3)^1}{1!} + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \frac{(\ln 3)^5}{5!} + \frac{(\ln 3)^6}{6!} + \frac{(\ln 3)^7}{7!} = 2.99994$$

$$S_8 = 1 + \frac{(\ln 3)^1}{1!} + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \frac{(\ln 3)^5}{5!} + \frac{(\ln 3)^6}{6!} + \frac{(\ln 3)^7}{7!} + \frac{(\ln 3)^8}{8!} = 2.999992$$

$$S_9 = 1 + \frac{(\ln 3)^1}{1!} + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \frac{(\ln 3)^5}{5!} + \frac{(\ln 3)^6}{6!} + \frac{(\ln 3)^7}{7!} + \frac{(\ln 3)^8}{8!} + \frac{(\ln 3)^9}{9!} = 2.99999$$

$$S_{10} = 1 + \frac{(\ln 3)^1}{1!} + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \frac{(\ln 3)^4}{4!} + \frac{(\ln 3)^5}{5!} + \frac{(\ln 3)^6}{6!} + \frac{(\ln 3)^7}{7!} + \frac{(\ln 3)^8}{8!} + \frac{(\ln 3)^9}{9!} + \frac{(\ln 3)^{10}}{10!} \approx 3$$

We plot the relation between $S_n$ and *n* for this case (using Excel 2010)



Looking at the graph, we can see the same happening as in the previous graph, however $S_n$'s asymptote was moved 1 unit further upwards till it reached $x = 3$. The terms calculated for *n* have an asymptote $x = 0$ again (seen from the calculations), however the 2nd term 'jumps' upwards before it starts decreasing.

Therefore, in our case where $a = 3$, as *n* approaches infinity, $S_n$ approaches 3:

$$n \to \infty, \; S_n \to 3$$

Now, we observe the same behavior with $x = 1$, however we choose different values of *a* to put instead, having in mind that *n* should be, again, $0 \le n \le 10$.

Since *a* cannot be a negative number (limitation by the *ln*), we can only choose values for *a* which are positive, therefore we will try 5, 7, and 10.

Starting with $a = 5$:

$$t_0 = 1$$

$$t_1 = \frac{(\ln 5)^1}{1!} = 1.609437$$

$$t_2 = \frac{(\ln 5)^2}{2!} = 1.295145$$

$$t_3 = \frac{(\ln 5)^3}{3!} = 0.694818$$

$$t_4 = \frac{(\ln 5)^4}{4!} = 0.279566$$

$$t_5 = \frac{(\ln 5)^5}{5!} = 0.089989$$

$$t_6 = \frac{(\ln 5)^6}{6!} = 0.024138$$

$$t_7 = \frac{(\ln 5)^7}{7!} = 0.005549$$

$$t_8 = \frac{(\ln 5)^8}{8!} = 0.001116$$

$$t_9 = \frac{(\ln 5)^9}{9!} = 0.000199$$

$$t_{10} = \frac{(\ln 5)^{10}}{10!} = 0.000032$$

Now we calculate $S_{10}$ for $a = 5$:

$$S_0 = 1$$

$$S_1 = 1 + \frac{(\ln 5)^1}{1!} = 2.609437$$

$$S_2 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} = 3.904583$$

$$S_3 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} = 4.599401$$

$$S_4 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} = 4.878968$$

$$S_5 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \frac{(\ln 5)^5}{5!} = 4.968957$$

$$S_6 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \frac{(\ln 5)^5}{5!} + \frac{(\ln 5)^6}{6!} = 4.993096$$

$$S_7 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \frac{(\ln 5)^5}{5!} + \frac{(\ln 5)^6}{6!} + \frac{(\ln 5)^7}{7!} = 4.998646$$

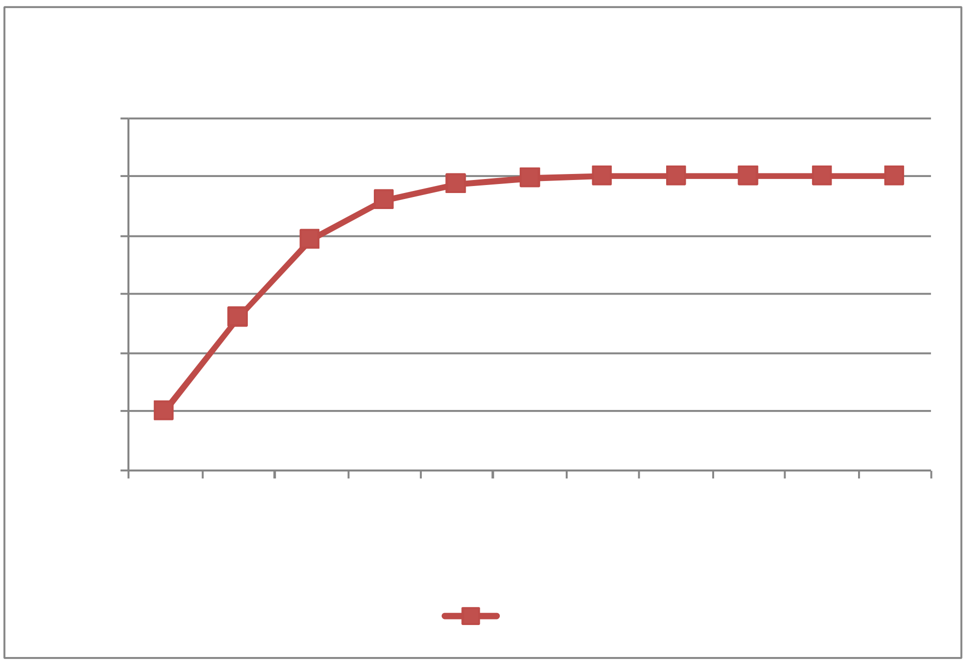
$$S_8 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \frac{(\ln 5)^5}{5!} + \frac{(\ln 5)^6}{6!} + \frac{(\ln 5)^7}{7!} + \frac{(\ln 5)^8}{8!} = 4.999762$$

$$S_9 = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \frac{(\ln 5)^5}{5!} + \frac{(\ln 5)^6}{6!} + \frac{(\ln 5)^7}{7!} + \frac{(\ln 5)^8}{8!} + \frac{(\ln 5)^9}{9!} = 4.99962$$

$$S_{10} = 1 + \frac{(\ln 5)^1}{1!} + \frac{(\ln 5)^2}{2!} + \frac{(\ln 5)^3}{3!} + \frac{(\ln 5)^4}{4!} + \frac{(\ln 5)^5}{5!} + \frac{(\ln 5)^6}{6!} + \frac{(\ln 5)^7}{7!} + \frac{(\ln 5)^8}{8!} + \frac{(\ln 5)^9}{9!} + \frac{(\ln 5)^{10}}{10!} =$$

$$= 4.99994 \qquad \approx 5$$

We will also draw the graphs for each different value of *a* that we chose, so we can check if there is some pattern (using Excel 2010):



We notice that when $a = 5$, the asymptote of $S_n$ (which is also approximately the value of $S_{10}$) is 5.

Also, as $n \to \infty$, $S_n \to 5$

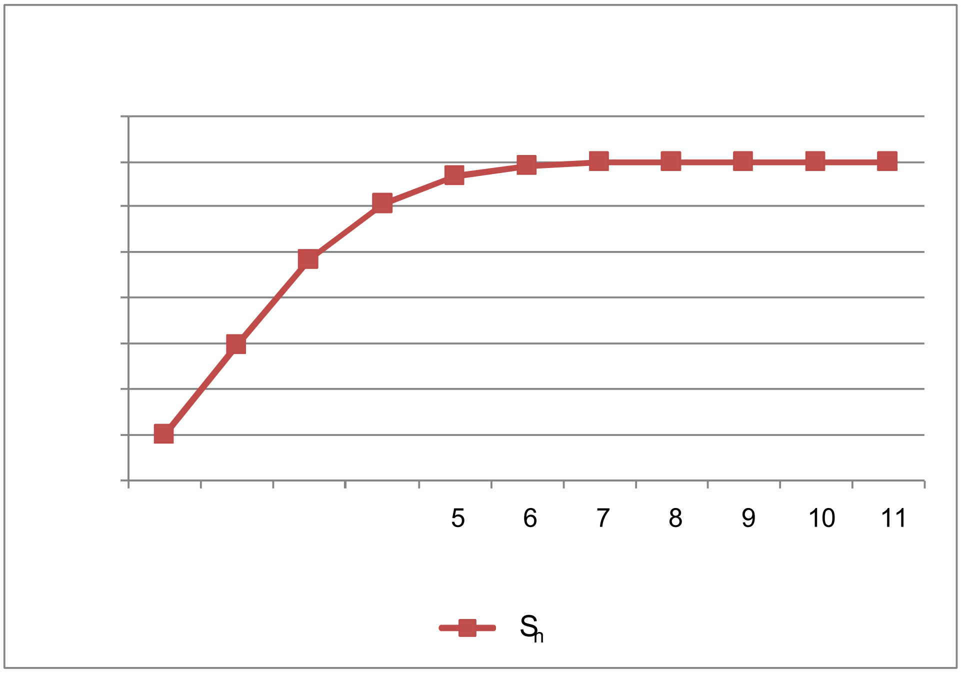
We go on with $a = 7$ :

$$t_0 = 1$$
$$t_1 = \frac{(\ln 7)^1}{1!} = 1.945910$$
$$t_2 = \frac{(\ln 7)^2}{2!} = 1.893283$$
$$t_3 = \frac{(\ln 7)^3}{3!} = 1.228052$$
$$t_4 = \frac{(\ln 7)^4}{4!} = 0.597420$$
$$t_5 = \frac{(\ln 7)^5}{5!} = 0.232505$$
$$t_6 = \frac{(\ln 7)^6}{6!} = 0.075405$$
$$t_7 = \frac{(\ln 7)^7}{7!} = 0.020961$$
$$t_8 = \frac{(\ln 7)^8}{8!} = 0.005098$$
$$t_9 = \frac{(\ln 7)^9}{9!} = 0.001102$$
$$t_{10} = \frac{(\ln 7)^{10}}{10!} = 0.000214$$

And now, to find $S_{10}$ for $a = 7$ :

$$S_0 = 1$$
$$S_1 = 1 + \frac{(\ln 7)^1}{1!} = 2.945910$$
$$S_2 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} = 4.839193$$
$$S_3 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} = 6.067246$$
$$S_4 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} = 6.664666$$
$$S_5 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} + \frac{(\ln 7)^5}{5!} = 6.897171$$
$$S_6 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} + \frac{(\ln 7)^5}{5!} + \frac{(\ln 7)^6}{6!} = 6.972577$$
$$S_7 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} + \frac{(\ln 7)^5}{5!} + \frac{(\ln 7)^6}{6!} + \frac{(\ln 7)^7}{7!} = 6.993539$$
$$S_8 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} + \frac{(\ln 7)^5}{5!} + \frac{(\ln 7)^6}{6!} + \frac{(\ln 7)^7}{7!} + \frac{(\ln 7)^8}{8!} = 6.998637$$
$$S_9 = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} + \frac{(\ln 7)^5}{5!} + \frac{(\ln 7)^6}{6!} + \frac{(\ln 7)^7}{7!} + \frac{(\ln 7)^8}{8!} + \frac{(\ln 7)^9}{9!} = 6.999740$$
$$S_{10} = 1 + \frac{(\ln 7)^1}{1!} + \frac{(\ln 7)^2}{2!} + \frac{(\ln 7)^3}{3!} + \frac{(\ln 7)^4}{4!} + \frac{(\ln 7)^5}{5!} + \frac{(\ln 7)^6}{6!} + \frac{(\ln 7)^7}{7!} + \frac{(\ln 7)^8}{8!} + \frac{(\ln 7)^9}{9!} + \frac{(\ln 7)^{10}}{10!} =$$
$$= 6.999954 \qquad \approx 7$$

After drawing the relation between $S_n$ and $n$ for $a = 7$ (with Excel 2010) we get:




We see that as $n \to \infty$, $S_n \to 7$

We see that $a = 7$ and the asymptote (which is also approximately the value of $S_{10}$) of $S_n$ are the same (in this case the asymptote is 7). It seems that $S_n = a$, however we will try with our last value, and see if this is true:

For $a = 10$ :

$$t_0 = 1$$

$$t_1 = \frac{(\ln 10)^1}{1!} = 2.302585$$

$$t_2 = \frac{(\ln 10)^2}{2!} = 2.650949$$

$$t_3 = \frac{(\ln 10)^3}{3!} = 2.034678$$

$$t_4 = \frac{(\ln 10)^4}{4!} = 1.171255$$

$$t_5 = \frac{(\ln 10)^5}{5!} = 0.539382$$

$$t_6 = \frac{(\ln 10)^6}{6!} = 0.206995$$

$$t_7 = \frac{(\ln 10)^7}{7!} = 0.068089$$

$$t_8 = \frac{(\ln 10)^8}{8!} = 0.019597$$

$$t_9 = \frac{(\ln 10)^9}{9!} = 0.005013$$

$$t_{10} = \frac{(\ln 10)^{10}}{10!} = 0.001154$$

Now we find $S_{10}$ for $a = 10$

$$S_0 = 1$$

$$S_1 = 1 + \frac{(\ln 10)^1}{1!} = 3.302585$$

$$S_2 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} = 5.953534$$

$$S_3 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} = 7.988212$$

$$S_4 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} = 9.159467$$

$$S_5 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} + \frac{(\ln 10)^5}{5!} = 9.698850$$

$$S_6 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} + \frac{(\ln 10)^5}{5!} + \frac{(\ln 10)^6}{6!} = 9.905846$$

$$S_7 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} + \frac{(\ln 10)^5}{5!} + \frac{(\ln 10)^6}{6!} + \frac{(\ln 10)^7}{7!} = 9.973926$$

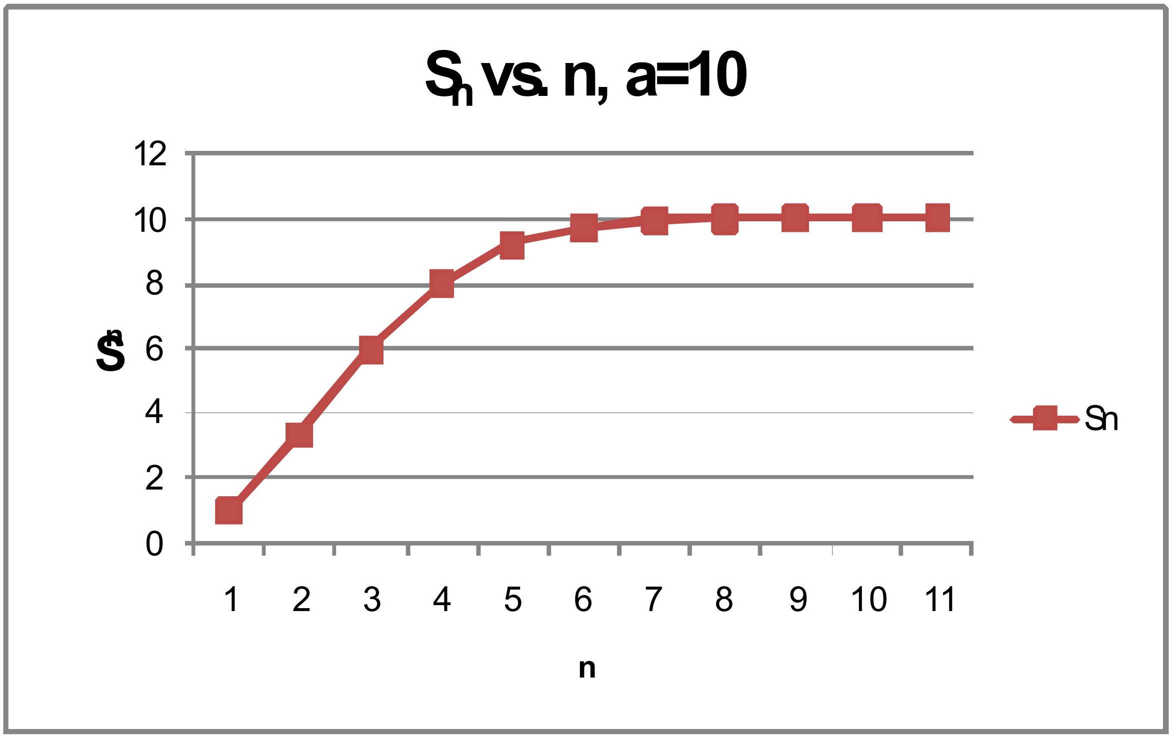
$$S_8 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} + \frac{(\ln 10)^5}{5!} + \frac{(\ln 10)^6}{6!} + \frac{(\ln 10)^7}{7!} + \frac{(\ln 10)^8}{8!} = 9.993533$$

$$S_9 = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} + \frac{(\ln 10)^5}{5!} + \frac{(\ln 10)^6}{6!} + \frac{(\ln 10)^7}{7!} + \frac{(\ln 10)^8}{8!} + \frac{(\ln 10)^9}{9!} =$$
$$= 9.998547$$

$$S_{10} = 1 + \frac{(\ln 10)^1}{1!} + \frac{(\ln 10)^2}{2!} + \frac{(\ln 10)^3}{3!} + \frac{(\ln 10)^4}{4!} + \frac{(\ln 10)^5}{5!} + \frac{(\ln 10)^6}{6!} + \frac{(\ln 10)^7}{7!} + \frac{(\ln 10)^8}{8!} + \frac{(\ln 10)^9}{9!} + \frac{(\ln 10)^{10}}{10!} =$$
$$= 9.999702 \qquad \approx 10$$

Now, we draw for the last time the relation between $S_n$ and $n$ for $a = 10$ (with Excel 2010 again):

$S_n$




We note that as $n \to \infty$, $S_n \to 10$

As we were observing all of those different values for *a*, we've noticed that as $n \to \infty$, $S_n \to a$, and also $S_n \approx a$, and if we make an approximate estimation of the $S_n$, we could easily say that $S_n = a$. And this is the general statement that applies in our case. However, theoretically, this only holds true when $x = 1$.

We will examine for different values of *x*, and then tell what is the true general statement for any values for *x* and *a*. For now, it is $S_n = a$.

Now, in order to get a general statement for $T_n$ we will have to change the values for *a* and *x*, and then find a pattern.

We will start with $a = 2$, and for *x* we will try various positive values.

To compare right the results we'll look correct to 6 decimal places again:

$x = 2$:

$t_0 = 1$

$$t_1 = \frac{(2\ln 2)^1}{1!} = 1.386294$$

$$t_2 = \frac{(2\ln 2)^2}{2!} = 0.960906$$

$$t_3 = \frac{(2\ln 2)^3}{3!} = 0.444032$$

$$t_4 = \frac{(2\ln 2)^4}{4!} = 0.153890$$

$$t_5 = \frac{(2\ln 2)^5}{5!} = 0.042667$$

$$t_6 = \frac{(2\ln 2)^6}{6!} = 0.009858$$

$$t_7 = \frac{(2\ln 2)^7}{7!} = 0.001952$$

$$t_8 = \frac{(2\ln 2)^8}{8!} = 0.000338$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{(2\ln 2)^1}{1!} = 2.386294$$

$$T_2 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} = 3.347200$$

$$T_3 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} + \frac{(2\ln 2)^3}{3!} = 3.791233$$

$$T_4 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} + \frac{(2\ln 2)^3}{3!} + \frac{(2\ln 2)^4}{4!} = 3.945123$$

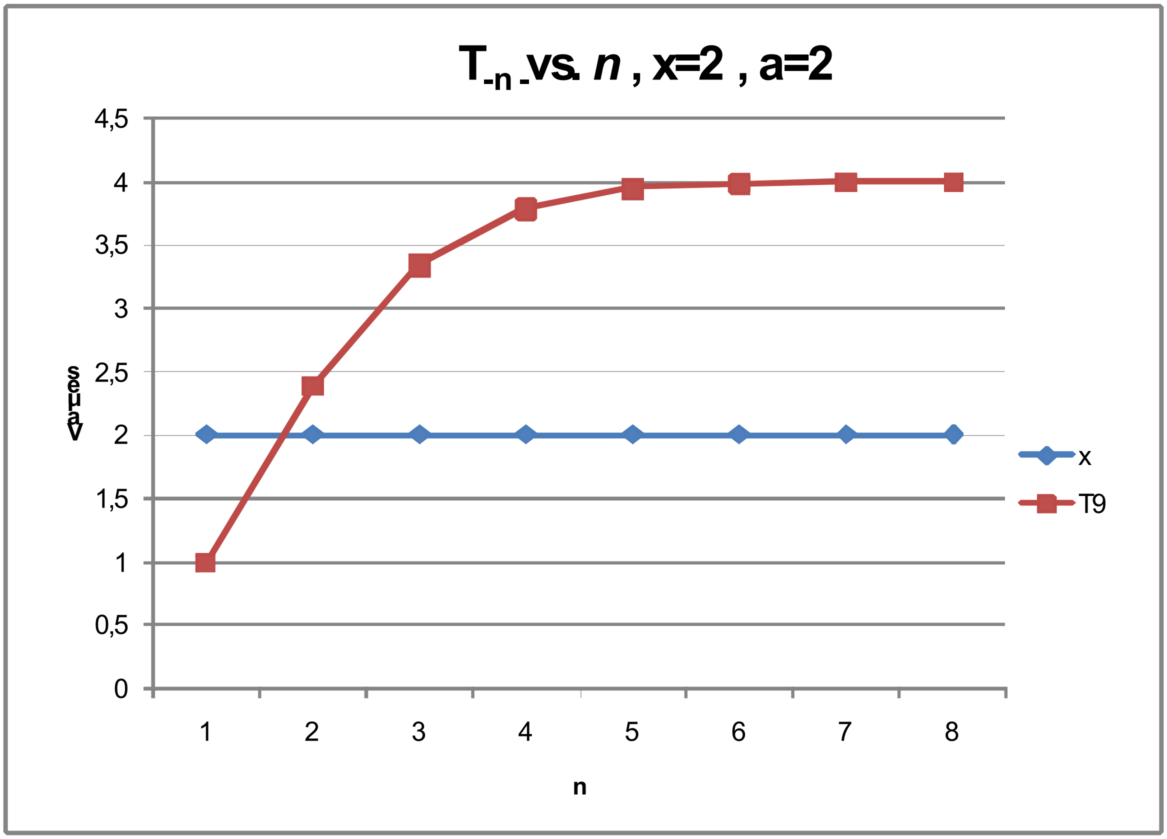
$$T_5 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} + \frac{(2\ln 2)^3}{3!} + \frac{(2\ln 2)^4}{4!} + \frac{(2\ln 2)^5}{5!} = 3.987790$$

$$T_6 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} + \frac{(2\ln 2)^3}{3!} + \frac{(2\ln 2)^4}{4!} + \frac{(2\ln 2)^5}{5!} + \frac{(2\ln 2)^6}{6!} = 3.997648$$

$$T_7 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} + \frac{(2\ln 2)^3}{3!} + \frac{(2\ln 2)^4}{4!} + \frac{(2\ln 2)^5}{5!} + \frac{(2\ln 2)^6}{6!} + \frac{(2\ln 2)^7}{7!} = 3.999601$$

$$T_8 = 1 + \frac{(2\ln 2)^1}{1!} + \frac{(2\ln 2)^2}{2!} + \frac{(2\ln 2)^3}{3!} + \frac{(2\ln 2)^4}{4!} + \frac{(2\ln 2)^5}{5!} + \frac{(2\ln 2)^6}{6!} + \frac{(2\ln 2)^7}{7!} + \frac{(2\ln 2)^8}{8!} = 3.999939$$

$T_8$




It's obvious that the graph keeps the same way as the one described in all previous graphs. We can see that at the $5^{th}$ term the line starts following a kind of asymptote which equals 4 in this case. So it is clear that the general statement $S_n = a$ is absolutely incorrect. However $S_n$ seems to equal $a^2$ and as $x = 2$ we can say that in this exact case $S_n = a^x$.

We can check if this statement is true with other values of $a$ and $x$. For example we can take a decimal number and keep the same value of $a$ so we can compare with the preceding.

For $x = 0.56$

$$t_0 = 1$$

$$t_1 = \frac{\left(0.56 \ln 2\right)^1}{1!} = 0.388162$$

$$t_2 = \frac{\left(0.56 \ln 2\right)^2}{2!} = 0.075335$$

$$t_3 = \frac{(0.56 \ln 2)^3}{3!} = 0.009747$$

$$t_4 = \frac{(0.56 \ln 2)^4}{4!} = 0.000945$$

$$t_5 = \frac{(0.56 \ln 2)^5}{5!} = 0.000073$$

$$t_6 = \frac{(0.56 \ln 2)^6}{6!} = 0.000004$$

$$t_7 = \frac{(0.56 \ln 2)^7}{7!} = 0.000000$$

$$t_8 = \frac{(0.56 \ln 2)^8}{8!} = 0.000000$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{(0.56 \ln 2)^1}{1!} = 1.388162$$

$$T_2 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} = 1.463497$$

$$T_3 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} + \frac{(0.56 \ln 2)^3}{3!} = 1.473244$$

$$T_4 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} + \frac{(0.56 \ln 2)^3}{3!} + \frac{(0.56 \ln 2)^4}{4!} = 1.474190$$

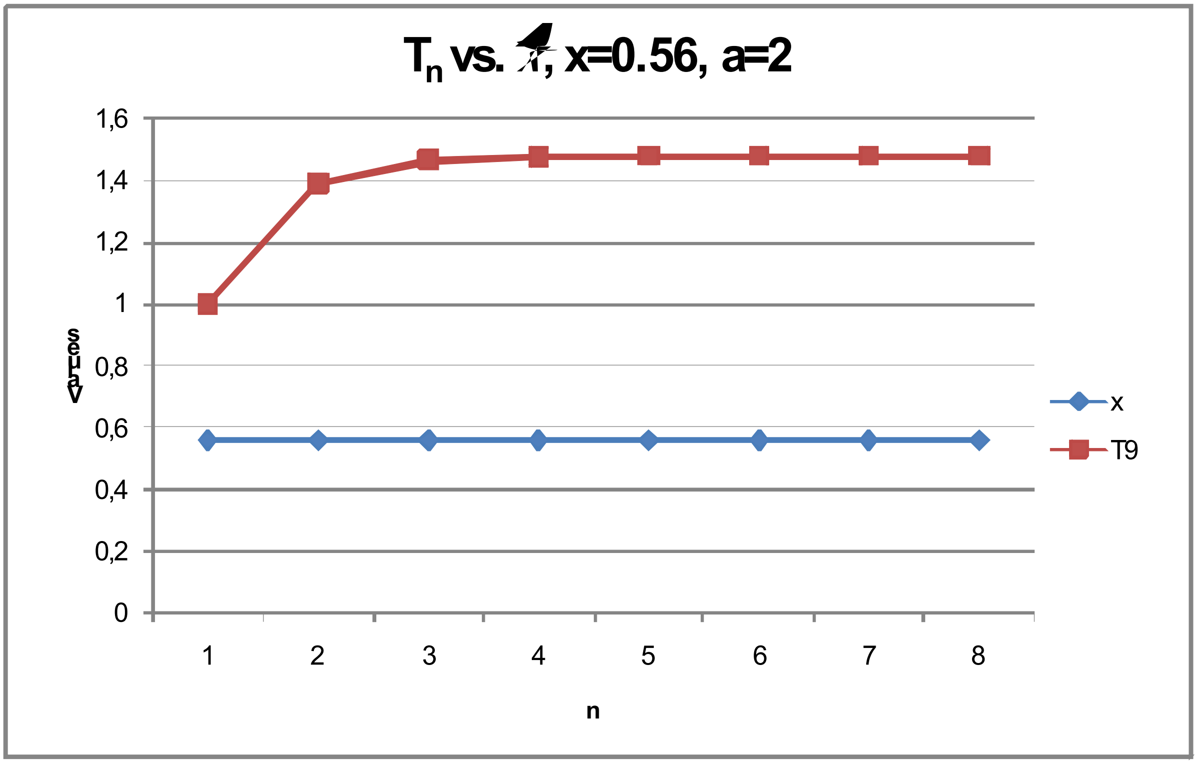
$$T_5 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} + \frac{(0.56 \ln 2)^3}{3!} + \frac{(0.56 \ln 2)^4}{4!} + \frac{(0.56 \ln 2)^5}{5!} = 1.474264$$

$$T_6 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} + \frac{(0.56 \ln 2)^3}{3!} + \frac{(0.56 \ln 2)^4}{4!} + \frac{(0.56 \ln 2)^5}{5!} + \frac{(0.56 \ln 2)^6}{6!} = 1.474268$$

$$T_7 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} + \frac{(0.56 \ln 2)^3}{3!} + \frac{(0.56 \ln 2)^4}{4!} + \frac{(0.56 \ln 2)^5}{5!} + \frac{(0.56 \ln 2)^6}{6!} + \frac{(0.56 \ln 2)^7}{7!} = 1.474269$$

$$T_8 = 1 + \frac{(0.56 \ln 2)^1}{1!} + \frac{(0.56 \ln 2)^2}{2!} + \frac{(0.56 \ln 2)^3}{3!} + \frac{(0.56 \ln 2)^4}{4!} + \frac{(0.56 \ln 2)^5}{5!} + \frac{(0.56 \ln 2)^6}{6!} + \frac{(0.56 \ln 2)^7}{7!} + \frac{(0.56 \ln 2)^8}{8!} = 1.474269$$

$T_8$




As it is seen this graph differs only in that there is no crossing between the $x$ and the $T_8$ lines. It's due to the fact that the decimal number we've taken is less than 1, and it could not be a limitation for the statement that seems to be again $S_n = a^x$.

To be sure we can check the statement with choosing a square root for $x$.

For $x = \sqrt{2}$

$t_0 = 1$

$$t_1 = \frac{(\sqrt{2}\ln 2)^1}{1!} = 0.980258$$

$$t_2 = \frac{(\sqrt{2}\ln 2)^2}{2!} = 0.480453$$

$$t_3 = \frac{(\sqrt{2}\ln 2)^3}{3!} = 0.156989$$

$$t_4 = \frac{(\sqrt{2}\ln 2)^4}{4!} = 0.038472$$

$$t_5 = \frac{(\sqrt{2}\ln 2)^5}{5!} = 0.007542$$

$$t_6 = \frac{(\sqrt{2}\ln 2)^6}{6!} = 0.001232$$

$$t_7 = \frac{\left(\sqrt{2}\ln 2\right)^7}{7!} = 0.000172$$

$$t_8 = \frac{\left(\sqrt{2}\ln 2\right)^8}{8!} = 0.000021$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} = 1.980258$$

$$T_2 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} = 2.460711$$

$$T_3 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 2\right)^3}{3!} = 2.617700$$

$$T_4 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 2\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 2\right)^4}{4!} = 2.656173$$

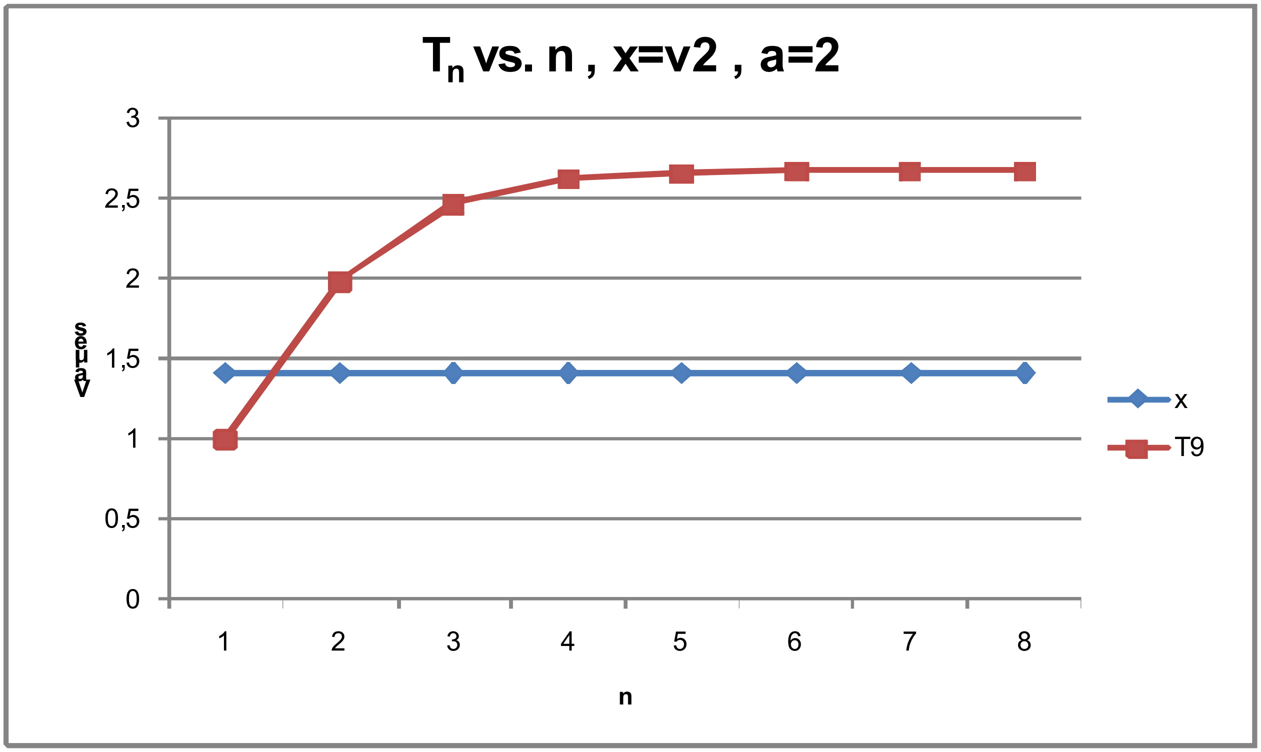
$$T_5 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 2\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 2\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 2\right)^5}{5!} = 2.663715$$

$$T_6 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 2\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 2\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 2\right)^5}{5!} + \frac{\left(\sqrt{2}\ln 2\right)^6}{6!} = 2.664947$$

$$T_7 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 2\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 2\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 2\right)^5}{5!} + \frac{\left(\sqrt{2}\ln 2\right)^6}{6!} + \frac{\left(\sqrt{2}\ln 2\right)^7}{7!} = 2.665120$$

$$T_8 = 1 + \frac{\left(\sqrt{2}\ln 2\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 2\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 2\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 2\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 2\right)^5}{5!} + \frac{\left(\sqrt{2}\ln 2\right)^6}{6!} + \frac{\left(\sqrt{2}\ln 2\right)^7}{7!} + \frac{\left(\sqrt{2}\ln 2\right)^8}{8!} = 2.665141$$

$T_8$




Once more the graph is moving towards a certain asymptote and the statement is again

$S_n = a^x$. As a special case we can chose $x = \pi$.

For $x = \pi$

$t_0 = 1$

$$t_1 = \frac{\left(\pi \ln 2\right)^1}{1!} = 2.177586$$

$$t_2 = \frac{\left(\pi \ln 2\right)^2}{2!} = 2.370940$$

$$t_3 = \frac{\left(\pi \ln 2\right)^3}{3!} = 1.720975$$

$$t_4 = \frac{\left(\pi \ln 2\right)^4}{4!} = 0.936893$$

$$t_5 = \frac{\left(\pi \ln 2\right)^5}{5!} = 0.408033$$

$$t_6 = \frac{\left(\pi \ln 2\right)^6}{6!} = 0.148087$$

$$t_7 = \frac{\left(\pi \ln 2\right)^7}{7!} = 0.046067$$

$$t_8 = \frac{\left(\pi \ln 2\right)^8}{8!} = 0.012539$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} = 2.177586$$

$$T_2 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} = 2.37094$$

$$T_3 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} + \frac{\left(\pi \ln 2\right)^3}{3!} = 1.720975$$

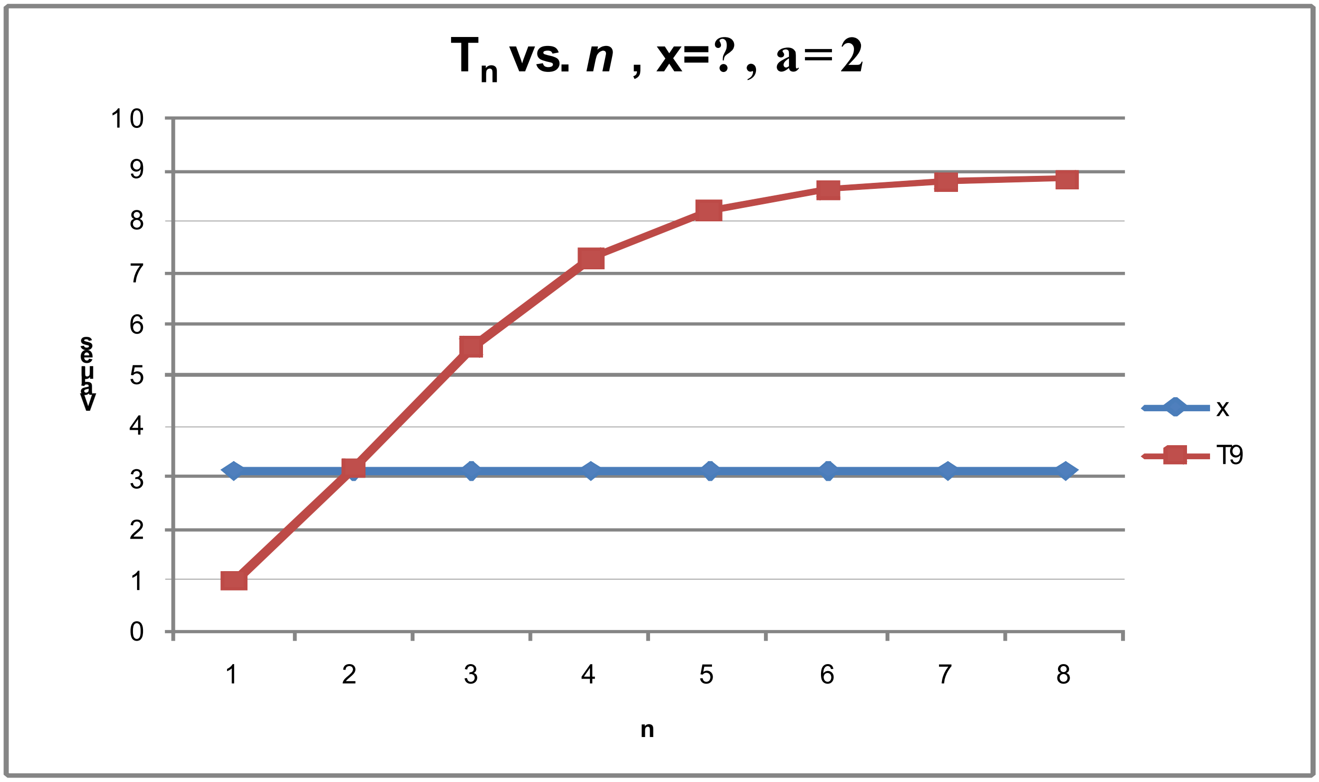
$$T_4 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} + \frac{\left(\pi \ln 2\right)^3}{3!} + \frac{\left(\pi \ln 2\right)^4}{4!} = 0.936893$$

$$T_5 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} + \frac{\left(\pi \ln 2\right)^3}{3!} + \frac{\left(\pi \ln 2\right)^4}{4!} + \frac{\left(\pi \ln 2\right)^5}{5!} = 0.408033$$

$$T_6 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} + \frac{\left(\pi \ln 2\right)^3}{3!} + \frac{\left(\pi \ln 2\right)^4}{4!} + \frac{\left(\pi \ln 2\right)^5}{5!} + \frac{\left(\pi \ln 2\right)^6}{6!} = 0.148087$$

$$T_7 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} + \frac{\left(\pi \ln 2\right)^3}{3!} + \frac{\left(\pi \ln 2\right)^4}{4!} + \frac{\left(\pi \ln 2\right)^5}{5!} + \frac{\left(\pi \ln 2\right)^6}{6!} + \frac{\left(\pi \ln 2\right)^7}{7!} = 0.046067$$

$$T_8 = 1 + \frac{\left(\pi \ln 2\right)^1}{1!} + \frac{\left(\pi \ln 2\right)^2}{2!} + \frac{\left(\pi \ln 2\right)^3}{3!} + \frac{\left(\pi \ln 2\right)^4}{4!} + \frac{\left(\pi \ln 2\right)^5}{5!} + \frac{\left(\pi \ln 2\right)^6}{6!} + \frac{\left(\pi \ln 2\right)^7}{7!} + \frac{\left(\pi \ln 2\right)^8}{8!} = 0.0125395$$




$T_8$

The graph shows no difference with the preceding ones and the statement is again $S_n = a^x$. Now we can see if the statement and the graph are similar if we change $a$.

Now we start with $a = 3$ (we look correct to 6 decimal places again)
Same values for $x$ as before:

$x = 2$:

$t_0 = 1$

$$t_1 = \frac{(2\ln 3)^1}{1!} = 2.197224$$

$$t_2 = \frac{(2\ln 3)^2}{2!} = 2.413897$$

$$t_3 = \frac{(2\ln 3)^3}{3!} = 1.767958$$

$$t_4 = \frac{(2\ln 3)^4}{4!} = 0.971150$$

$$t_5 = \frac{(2\ln 3)^5}{5!} = 0.426767$$

$$t_6 = \frac{(2\ln 3)^6}{6!} = 0.156283$$

$$t_7 = \frac{(2\ln 3)^7}{7!} = 0.049055$$

$$t_8 = \frac{(2\ln 3)^8}{8!} = 0.013473$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{(2\ln 3)^1}{1!} = 3.19724$$

$$T_2 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} = 5.61122$$

$$T_3 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} + \frac{(2\ln 3)^3}{3!} = 7.37981$$

$$T_4 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} + \frac{(2\ln 3)^3}{3!} + \frac{(2\ln 3)^4}{4!} = 8.35023$$

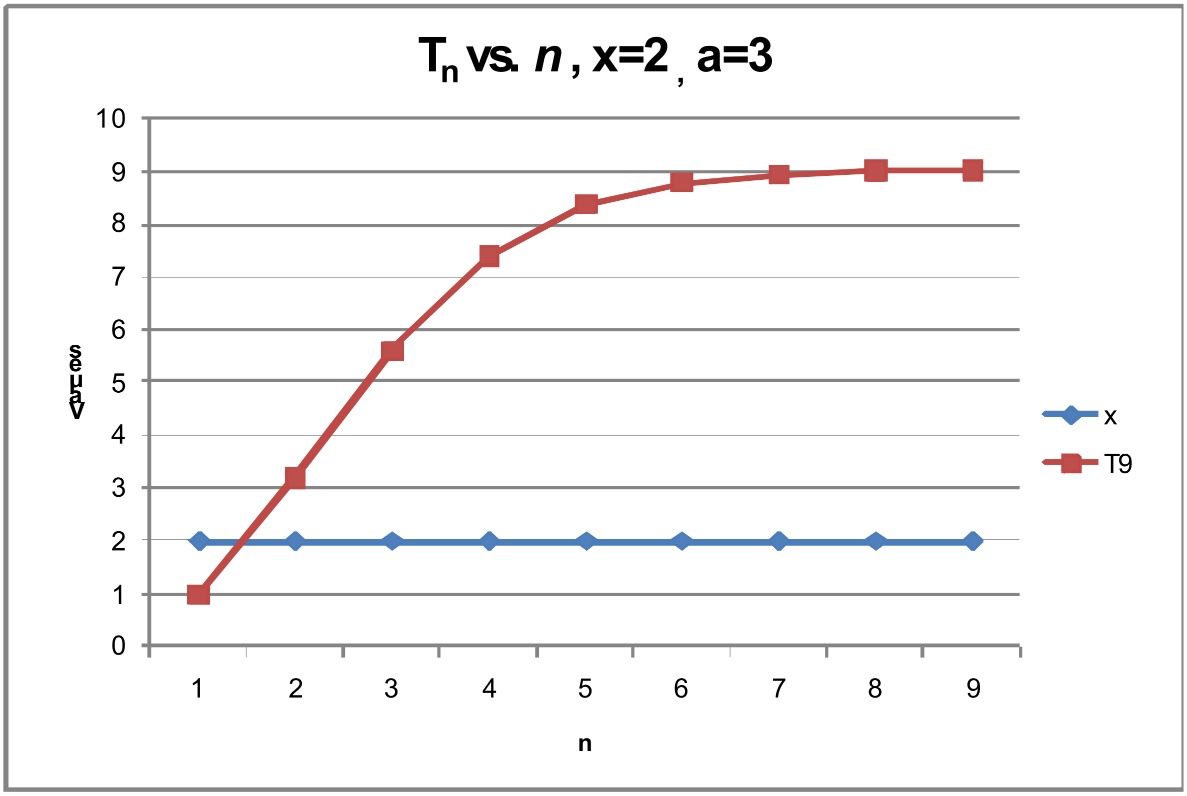
$$T_5 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} + \frac{(2\ln 3)^3}{3!} + \frac{(2\ln 3)^4}{4!} + \frac{(2\ln 3)^5}{5!} = 8.77698$$

$$T_6 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} + \frac{(2\ln 3)^3}{3!} + \frac{(2\ln 3)^4}{4!} + \frac{(2\ln 3)^5}{5!} + \frac{(2\ln 3)^6}{6!} = 8.93282$$

$$T_7 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} + \frac{(2\ln 3)^3}{3!} + \frac{(2\ln 3)^4}{4!} + \frac{(2\ln 3)^5}{5!} + \frac{(2\ln 3)^6}{6!} + \frac{(2\ln 3)^7}{7!} = 8.98238$$

$$T_8 = 1 + \frac{(2\ln 3)^1}{1!} + \frac{(2\ln 3)^2}{2!} + \frac{(2\ln 3)^3}{3!} + \frac{(2\ln 3)^4}{4!} + \frac{(2\ln 3)^5}{5!} + \frac{(2\ln 3)^6}{6!} + \frac{(2\ln 3)^7}{7!} + \frac{(2\ln 3)^8}{8!} = 8.99581$$

$T_8$




Changing the value of $a$ we can see that the two lines get further but it doesn't change the common behavior and statement.

For $x = 0.56$

$$t_0 = 1$$

$$t_1 = \frac{\left(0.56\ \ln 3\right)^1}{1!} = 0.61522$$

$$t_2 = \frac{\left(0.56\ \ln 3\right)^2}{2!} = 0.189249$$

$$t_3 = \frac{\left(0.56\ \ln 3\right)^3}{3!} = 0.038810$$

$$t_4 = \frac{\left(0.56\ \ln 3\right)^4}{4!} = 0.005969$$

$$t_5 = \frac{\left(0.56\ \ln 3\right)^5}{5!} = 0.000734$$

$$t_6 = \frac{\left(0.56\ \ln 3\right)^6}{6!} = 0.000075$$

$$t_7 = \frac{\left(0.56 \ln 3\right)^7}{7!} = 0.00000\quad 6$$

$$t_8 = \frac{\left(0.56 \ln 3\right)^8}{8!} = 0.000000$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} = 1.615222$$

$$T_2 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} = 1.804472$$

$$T_3 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} + \frac{\left(0.56 \ln 3\right)^3}{3!} = 1.843282$$

$$T_4 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} + \frac{\left(0.56 \ln 3\right)^3}{3!} + \frac{\left(0.56 \ln 3\right)^4}{4!} = 1.849251$$

$$T_5 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} + \frac{\left(0.56 \ln 3\right)^3}{3!} + \frac{\left(0.56 \ln 3\right)^4}{4!} + \frac{\left(0.56 \ln 3\right)^5}{5!} = 1.849986$$

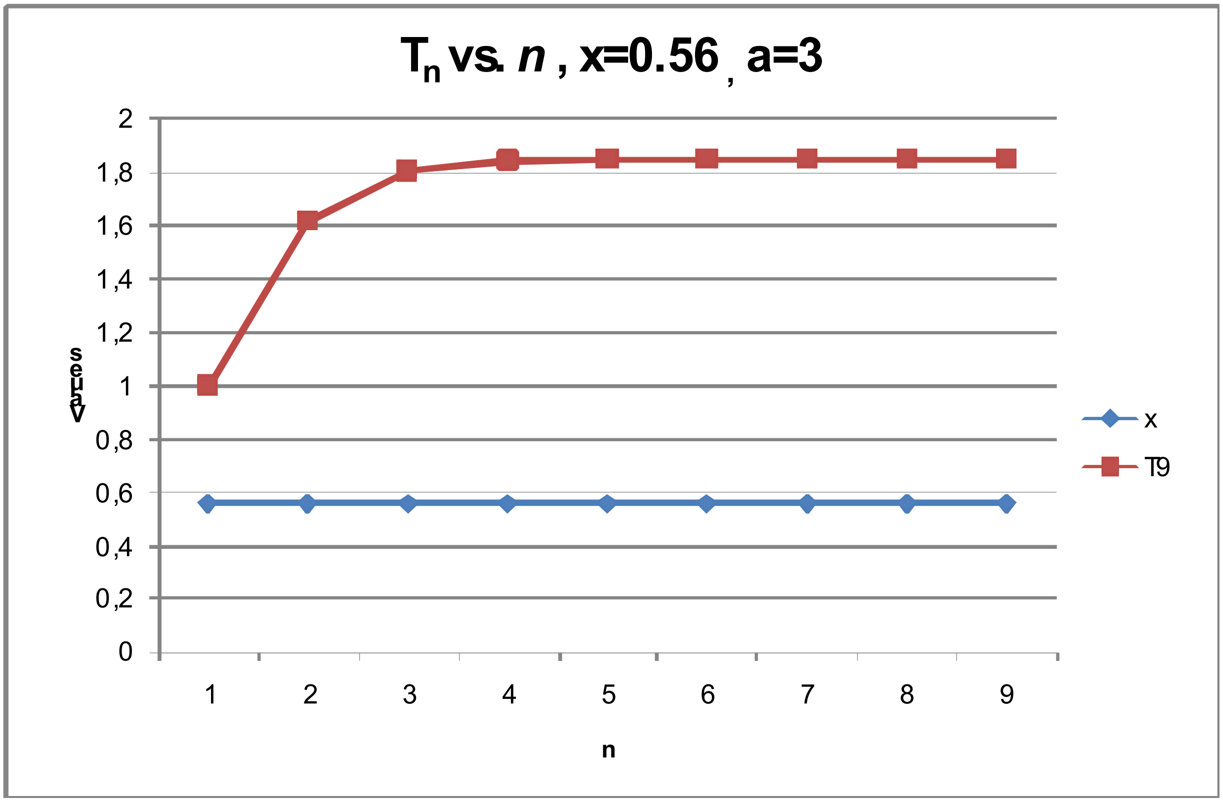
$$T_6 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} + \frac{\left(0.56 \ln 3\right)^3}{3!} + \frac{\left(0.56 \ln 3\right)^4}{4!} + \frac{\left(0.56 \ln 3\right)^5}{5!} + \frac{\left(0.56 \ln 3\right)^6}{6!} = 1.850061$$

$$T_7 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} + \frac{\left(0.56 \ln 3\right)^3}{3!} + \frac{\left(0.56 \ln 3\right)^4}{4!} + \frac{\left(0.56 \ln 3\right)^5}{5!} + \frac{\left(0.56 \ln 3\right)^6}{6!} + \frac{\left(0.56 \ln 3\right)^7}{7!} = 1.850068$$

$$T_8 = 1 + \frac{\left(0.56 \ln 3\right)^1}{1!} + \frac{\left(0.56 \ln 3\right)^2}{2!} + \frac{\left(0.56 \ln 3\right)^3}{3!} + \frac{\left(0.56 \ln 3\right)^4}{4!} + \frac{\left(0.56 \ln 3\right)^5}{5!} + \frac{\left(0.56 \ln 3\right)^6}{6!} + \frac{\left(0.56 \ln 3\right)^7}{7!} +$$

$$+ \frac{\left(0.56 \ln 3\right)^8}{8!} = 1.850068$$

$T_8$




This graph is similar to the case $a = 2$ where $x = 0.56$ and it shows only the bigger value difference between the constant line of $x$ and the line of summation values.

For $x = \sqrt{2}$

$$t_0 = 1$$

$$t_1 = \frac{\left(\sqrt{2}\ln 3\right)^{1}}{1!} = 1.553672$$

$$t_2 = \frac{\left(\sqrt{2}\ln 3\right)^{2}}{2!} = 1.206948$$

$$t_3 = \frac{\left(\sqrt{2}\ln 3\right)^{3}}{3!} = 0.625067$$

$$t_4 = \frac{\left(\sqrt{2}\ln 3\right)^{4}}{4!} = 0.242787$$

$$t_5 = \frac{\left(\sqrt{2}\ln 3\right)^{5}}{5!} = 0.075442$$

$$t_6 = \frac{\left(\sqrt{2}\ln 3\right)^{6}}{6!} = 0.019535$$

$$t_7 = \frac{\left(\sqrt{2}\ln 3\right)^7}{7!} = 0.004335$$

$$t_8 = \frac{\left(\sqrt{2}\ln 3\right)^8}{8!} = 0.000842$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} = 2.553672$$

$$T_2 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} = 3.760621$$

$$T_3 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 3\right)^3}{3!} = 4.385689$$

$$T_4 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 3\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 3\right)^4}{4!} = 4.628476$$

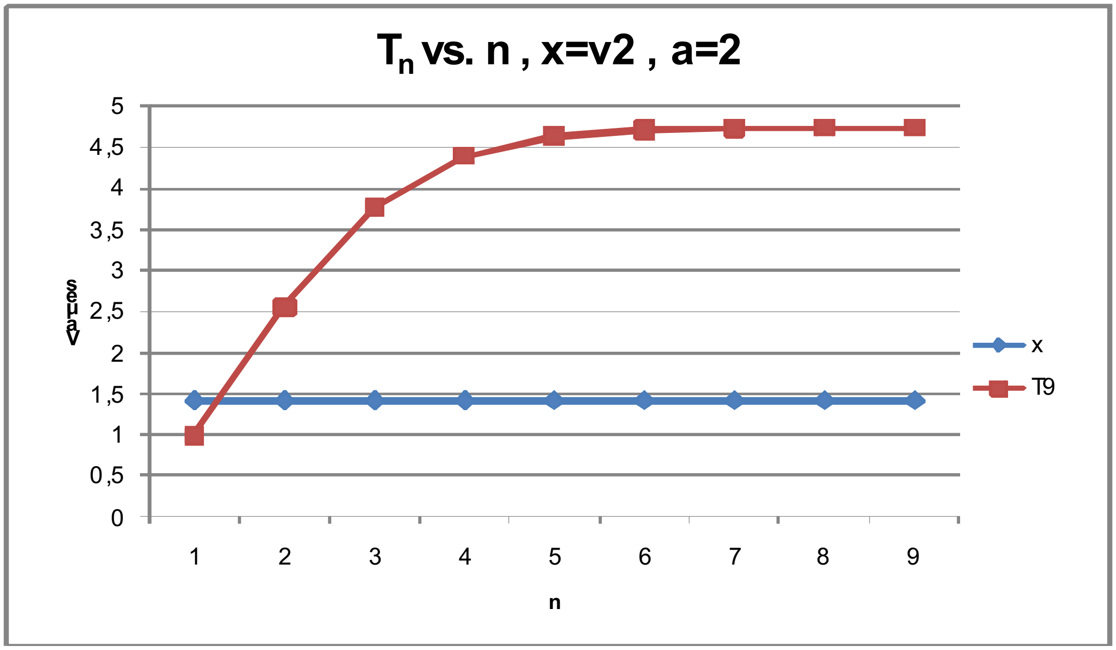
$$T_5 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 3\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 3\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 3\right)^5}{5!} = 4.703919$$

$$T_6 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 3\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 3\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 3\right)^5}{5!} + \frac{\left(\sqrt{2}\ln 3\right)^6}{6!} = 4.723454$$

$$T_7 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 3\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 3\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 3\right)^5}{5!} + \frac{\left(\sqrt{2}\ln 3\right)^6}{6!} + \frac{\left(\sqrt{2}\ln 3\right)^7}{7!} = 4.727790$$

$$T_8 = 1 + \frac{\left(\sqrt{2}\ln 3\right)^1}{1!} + \frac{\left(\sqrt{2}\ln 3\right)^2}{2!} + \frac{\left(\sqrt{2}\ln 3\right)^3}{3!} + \frac{\left(\sqrt{2}\ln 3\right)^4}{4!} + \frac{\left(\sqrt{2}\ln 3\right)^5}{5!} + \frac{\left(\sqrt{2}\ln 3\right)^6}{6!} + \frac{\left(\sqrt{2}\ln 3\right)^7}{7!} + \frac{\left(\sqrt{2}\ln 3\right)^8}{8!} =$$

$$= 4.728632$$




$T_8$

No significant change is presented in this graph.

For $x = \pi$

$t_0 = 1$

$$t_1 = \frac{\left(\pi \ln 3\right)^1}{1!} = 3.451392$$

$$t_2 = \frac{\left(\pi \ln 3\right)^2}{2!} = 5.956054$$

$$t_3 = \frac{\left(\pi \ln 3\right)^3}{3!} = 6.852226$$

$$t_4 = \frac{\left(\pi \ln 3\right)^4}{4!} = 5.912430$$

$$t_5 = \frac{\left(\pi \ln 3\right)^5}{5!} = 4.081223$$

$$t_6 = \frac{\left(\pi \ln 3\right)^6}{6!} = 2.347650$$

$$t_7 = \frac{(\pi \ln 3)^7}{7!} = 1.157523$$

$$t_8 = \frac{(\pi \ln 3)^8}{8!} = 0.499383$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{(\pi \ln 3)^1}{1!} = 4.451392$$

$$T_2 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} = 10.407446$$

$$T_3 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} + \frac{(\pi \ln 3)^3}{3!} = 17.259673$$

$$T_4 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} + \frac{(\pi \ln 3)^3}{3!} + \frac{(\pi \ln 3)^4}{4!} = 23.172104$$

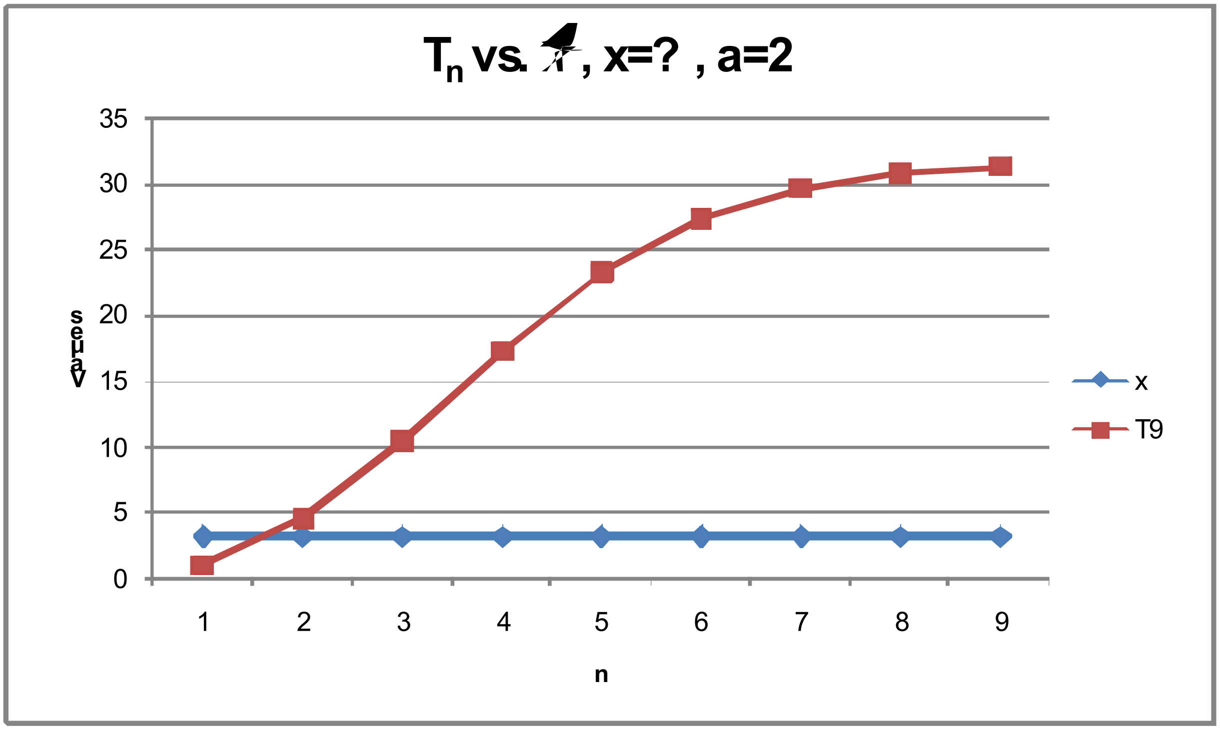
$$T_5 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} + \frac{(\pi \ln 3)^3}{3!} + \frac{(\pi \ln 3)^4}{4!} + \frac{(\pi \ln 3)^5}{5!} = 27.253327$$

$$T_6 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} + \frac{(\pi \ln 3)^3}{3!} + \frac{(\pi \ln 3)^4}{4!} + \frac{(\pi \ln 3)^5}{5!} + \frac{(\pi \ln 3)^6}{6!} = 29.600978$$

$$T_7 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} + \frac{(\pi \ln 3)^3}{3!} + \frac{(\pi \ln 3)^4}{4!} + \frac{(\pi \ln 3)^5}{5!} + \frac{(\pi \ln 3)^6}{6!} + \frac{(\pi \ln 3)^7}{7!} = 30.758501$$

$$T_8 = 1 + \frac{(\pi \ln 3)^1}{1!} + \frac{(\pi \ln 3)^2}{2!} + \frac{(\pi \ln 3)^3}{3!} + \frac{(\pi \ln 3)^4}{4!} + \frac{(\pi \ln 3)^5}{5!} + \frac{(\pi \ln 3)^6}{6!} + \frac{(\pi \ln 3)^7}{7!} + \frac{(\pi \ln 3)^8}{8!} = 31.257884$$

$T_8$




All above presented cases show that the best general statement is $S_n = a^x$, however as much as $n$ approaches infinity there appears a certain difference between the exact values of $S_n$ and $a^x$, which increases with the increasing of $n$.

We can show 3 more cases that present different combinations of $a$ and $x$ values.

As we have tried decimals for $x$, but we haven't tried decimals for $a$ we shall do it:

For $x = 2 \quad a = 4.8$

$t_0 = 1$

$$t_1 = \frac{(2\ln 4.8)^1}{1!} = 3.137231$$

$$t_2 = \frac{(2\ln 4.8)^2}{2!} = 4.921111$$

$$t_3 = \frac{(2\ln 4.8)^3}{3!} = 5.146222$$

$$t_4 = \frac{(2\ln 4.8)^4}{4!} = 4.036223$$

$$t_5 = \frac{(2\ln 4.8)^5}{5!} = 2.532513$$

$$t_6 = \frac{(2\ln 4.8)^6}{6!} = 1.324180$$

$$t_7 = \frac{(2\ln 4.8)^7}{7!} = 0.593465$$

$$t_8 = \frac{(2\ln 4.8)^8}{8!} = 0.232730$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{(2\ln 4.8)^1}{1!} = 4.137231$$

$$T_2 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} = 9.058343$$

$$T_3 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} + \frac{(2\ln 4.8)^3}{3!} = 14.204566$$

$$T_4 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} + \frac{(2\ln 4.8)^3}{3!} + \frac{(2\ln 4.8)^4}{4!} = 18.240790$$

$$T_5 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} + \frac{(2\ln 4.8)^3}{3!} + \frac{(2\ln 4.8)^4}{4!} + \frac{(2\ln 4.8)^5}{5!} = 20.773303$$

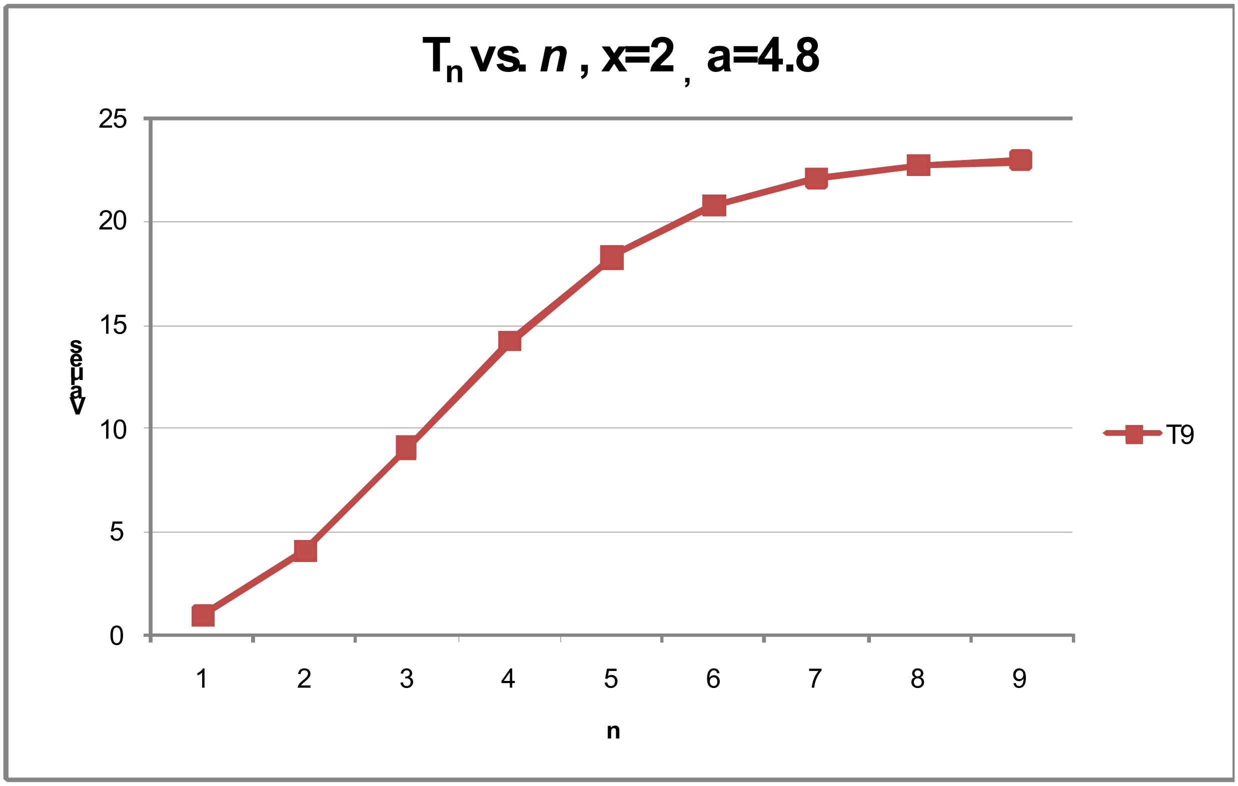
$$T_6 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} + \frac{(2\ln 4.8)^3}{3!} + \frac{(2\ln 4.8)^4}{4!} + \frac{(2\ln 4.8)^5}{5!} + \frac{(2\ln 4.8)^6}{6!} = 22.097484$$

$$T_7 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} + \frac{(2\ln 4.8)^3}{3!} + \frac{(2\ln 4.8)^4}{4!} + \frac{(2\ln 4.8)^5}{5!} + \frac{(2\ln 4.8)^6}{6!} + \frac{(2\ln 4.8)^7}{7!} = 22.690950$$

$$T_8 = 1 + \frac{(2\ln 4.8)^1}{1!} + \frac{(2\ln 4.8)^2}{2!} + \frac{(2\ln 4.8)^3}{3!} + \frac{(2\ln 4.8)^4}{4!} + \frac{(2\ln 4.8)^5}{5!} + \frac{(2\ln 4.8)^6}{6!} + \frac{(2\ln 4.8)^7}{7!} + \frac{(2\ln 4.8)^8}{8!} =$$

$$= 22.923680$$

$T_8$




The values of summation reach the asymptote slower than the previous cases but still it is not a limitation of the common behaviour and statement. The same is mentioned when $a = \pi$.

For $x = 2 \quad a = \pi$

$$t_0 = 1$$

$$t_1 = \frac{(2\ln \pi)^1}{1!} = 2.289459$$

$$t_2 = \frac{(2\ln \pi)^2}{2!} = 2.620813$$

$$t_3 = \frac{(2\ln \pi)^3}{3!} = 2.000081$$

$$t_4 = \frac{(2\ln \pi)^4}{4!} = 1.144776$$

$$t_5 = \frac{(2\ln \pi)^5}{5!} = 0.524184$$

$$t_6 = \frac{(2\ln \pi)^6}{6!} = 0.200016$$

$$t_7 = \frac{(2\ln\pi)^7}{7!} = 0.065418$$

$$t_8 = \frac{(2\ln\pi)^8}{8!} = 0.018721$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{(2\ln\pi)^1}{1!} = 3.289459$$

$$T_2 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} = 5.910272$$

$$T_3 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} + \frac{(2\ln\pi)^3}{3!} = 7.910354$$

$$T_4 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} + \frac{(2\ln\pi)^3}{3!} + \frac{(2\ln\pi)^4}{4!} = 9.055131$$

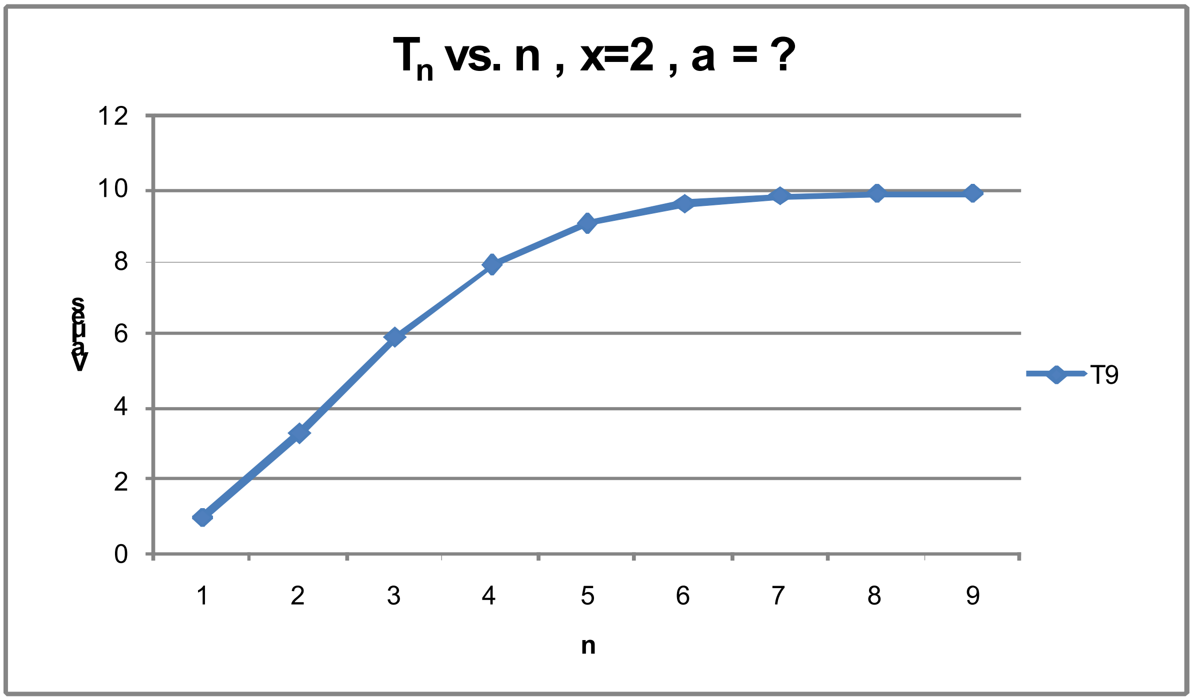
$$T_5 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} + \frac{(2\ln\pi)^3}{3!} + \frac{(2\ln\pi)^4}{4!} + \frac{(2\ln\pi)^5}{5!} = 9.579315$$

$$T_6 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} + \frac{(2\ln\pi)^3}{3!} + \frac{(2\ln\pi)^4}{4!} + \frac{(2\ln\pi)^5}{5!} + \frac{(2\ln\pi)^6}{6!} = 9.779332$$

$$T_7 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} + \frac{(2\ln\pi)^3}{3!} + \frac{(2\ln\pi)^4}{4!} + \frac{(2\ln\pi)^5}{5!} + \frac{(2\ln\pi)^6}{6!} + \frac{(2\ln\pi)^7}{7!} = 9.844750$$

$$T_8 = 1 + \frac{(2\ln\pi)^1}{1!} + \frac{(2\ln\pi)^2}{2!} + \frac{(2\ln\pi)^3}{3!} + \frac{(2\ln\pi)^4}{4!} + \frac{(2\ln\pi)^5}{5!} + \frac{(2\ln\pi)^6}{6!} + \frac{(2\ln\pi)^7}{7!} + \frac{(2\ln\pi)^8}{8!} = 9.863472$$

$T_8$




For $x=2 \quad a=\pi$ , the statement is still the same. The last check we can do is with a negative value of $x$, as for $a$ we cannot do that because of the limitation if the $ln$ function.
For $x=-8 \quad a=7$

$$t_0 = 1$$

$$t_1 = \frac{|-8\ln 7|^1}{1!} = -15.567281$$

$$t_2 = \frac{|-8\ln 7|^2}{2!} = 121.170121$$

$$t_3 = \frac{|-8\ln 7|^3}{3!} = -628.76311 \qquad 9$$

$$t_4 = \frac{|-8\ln 7|^4}{4!} = 2447.03307 \qquad 2$$

$$t_5 = \frac{|-8\ln 7|^5}{5!} = -7618.7303 \qquad 84$$

$$t_6 = \frac{|-8\ln 7|^6}{6!} = 19767.1530 \qquad 4$$

$$t_7 = \frac{|-8\ln 7|^7}{7!} = -43960.118 \qquad 53$$

$$t_8 = \frac{|-8\ln 7|^8}{8!} = 85542.407 \qquad 9$$

$$T_0 = 1$$

$$T_1 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} = -14.567281$$

$$T_2 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} = 106.602840$$

$$T_3 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} + \frac{\left(-8\ln 7\right)^3}{3!} = -522.16027 \qquad 9$$

$$T_4 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} + \frac{\left(-8\ln 7\right)^3}{3!} + \frac{\left(-8\ln 7\right)^4}{4!} = 1924.87279 \qquad 3$$

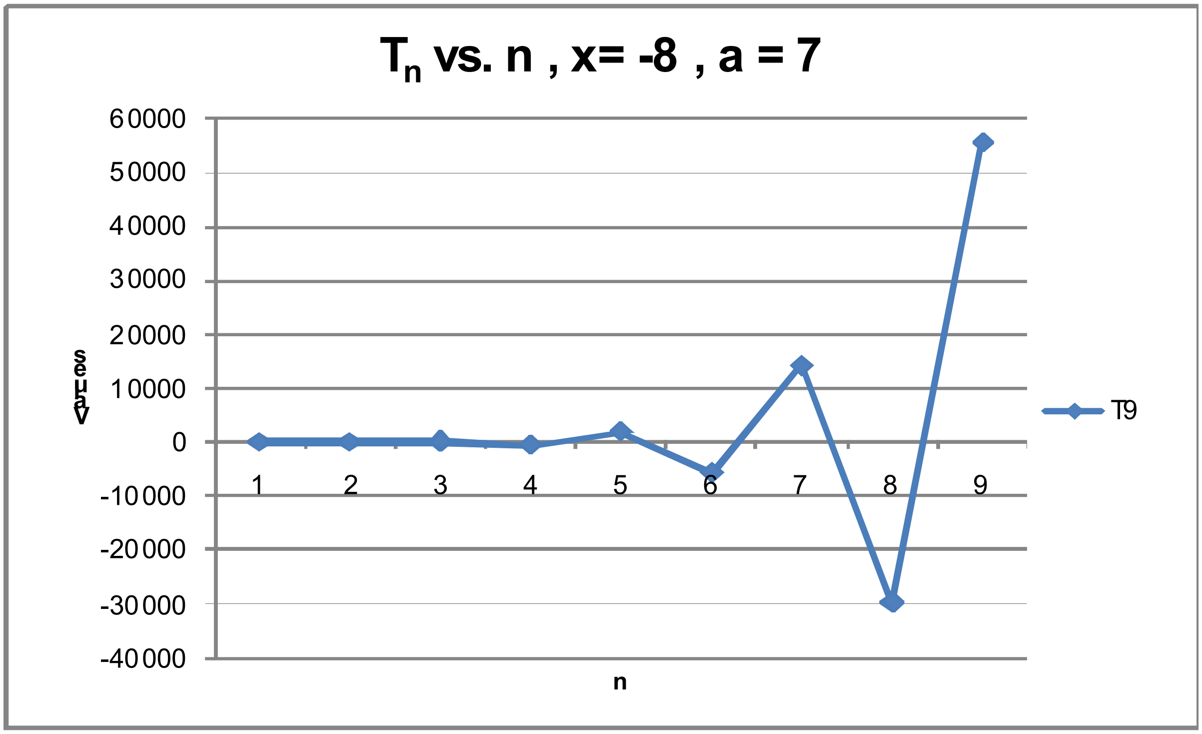
$$T_5 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} + \frac{\left(-8\ln 7\right)^3}{3!} + \frac{\left(-8\ln 7\right)^4}{4!} + \frac{\left(-8\ln 7\right)^5}{5!} = -5693.8575 \qquad 91$$

$$T_6 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} + \frac{\left(-8\ln 7\right)^3}{3!} + \frac{\left(-8\ln 7\right)^4}{4!} + \frac{\left(-8\ln 7\right)^5}{5!} + \frac{\left(-8\ln 7\right)^6}{6!} = 14073.2954 \qquad 4$$

$$T_7 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} + \frac{\left(-8\ln 7\right)^3}{3!} + \frac{\left(-8\ln 7\right)^4}{4!} + \frac{\left(-8\ln 7\right)^5}{5!} + \frac{\left(-8\ln 7\right)^6}{6!} + \frac{\left(-8\ln 7\right)^7}{7!} = -29886.823 \qquad 08$$

$$T_8 = 1 + \frac{\left(-8\ln 7\right)^1}{1!} + \frac{\left(-8\ln 7\right)^2}{2!} + \frac{\left(-8\ln 7\right)^3}{3!} + \frac{\left(-8\ln 7\right)^4}{4!} + \frac{\left(-8\ln 7\right)^5}{5!} + \frac{\left(-8\ln 7\right)^6}{6!} + \frac{\left(-8\ln 7\right)^7}{7!} + \frac{\left(-8\ln 7\right)^8}{8!} = 55655.6$$

$T_8$




Clearly it is the first graph that is distinguishable and doesn't follow the common behavior as well as these are the first values that don't follow the statement $S_n = a^n$. The values as well as the graph seem to oscillate under and above the 0 with increasing amplitude of oscillation as much as $n$ approaches infinity.

As a conclusion we can say that the general statement $S_n = a^n$ fulfils quite well the presented infinite summation, including the first case when we stated that $S_n = a$, which is a particular case when $x = 1$, so it is according to $a^1 = a$.

The statement should consider the following limitations:

$a \neq 0, x \neq 0$ – as this will give us a single value "0" and the graph will be a straight line on the $x$ axis.

$a > 0, x > 0$ – as $a > 0$ is the natural limitation of the $ln$, and $x > 0$ shows an oscillating graph.

Works Cited:

[1] "Taylor's Theorem." Wikipedia*, the Free Encyclopedia*. Web. 09 Dec. 2011. <http://en.wikipedia.org/wiki/Taylor's_theorem>.

[2] "Euler–Maclaurin formula." Wikipedia, the Free Encyclopedia. Web. 09 Dec. 2011.<http://en.wikipedia.org/wiki/Euler%E2%80%93Maclaurin_formula>

[3] "Series (mathematics)." Wikipedia, the Free Encyclopedia. Web. 09 Dec. 2011. <http://en.wikipedia.org/wiki/Series_(mathematics)>.