Example Math Portfolio SL

Infinite Summation – Type 1:

We being this analysis by investigating series of the type $t_n = \frac{(x \ln a)^n}{n!}$, where t_n represents the nth term in the series, x and a are two parameters that may be varied and n! = n X (n -1) X (n -2) X … X 3 X 2 X 1.

Initially we will consider the summation, S_n where S_n means to sum the first n terms of the series. We will find the S_n for the first 11 summations of this series, that is those terms that obey $0 \leq n \leq 10$.

Using Microsoft Excel we can easily obtain these values which are shown in the table below.

N	Sn
0	1
1	1.693147
2	1.933374
3	1.988878
4	1.998496
5	1.999829
6	1.999983
7	1.999999
8	2
9	2
10	2

To further clarify how these values were obtained a sample calculation is shown below:

$S_2 = t_0 + t_1 + t_2 = 1 + \frac{\ln 2}{1} + \frac{(\ln 2)^2}{2} = 1.693147$, a screenshot highlighting the formula used in Excel to complete the rest is shown in figure 1 below.

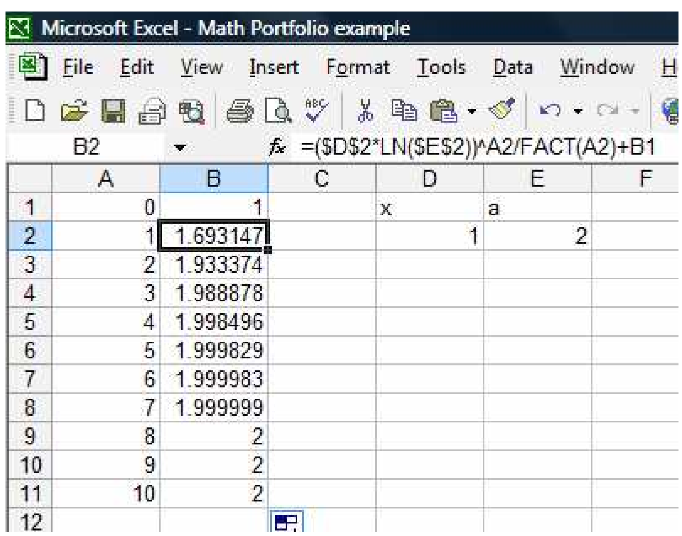
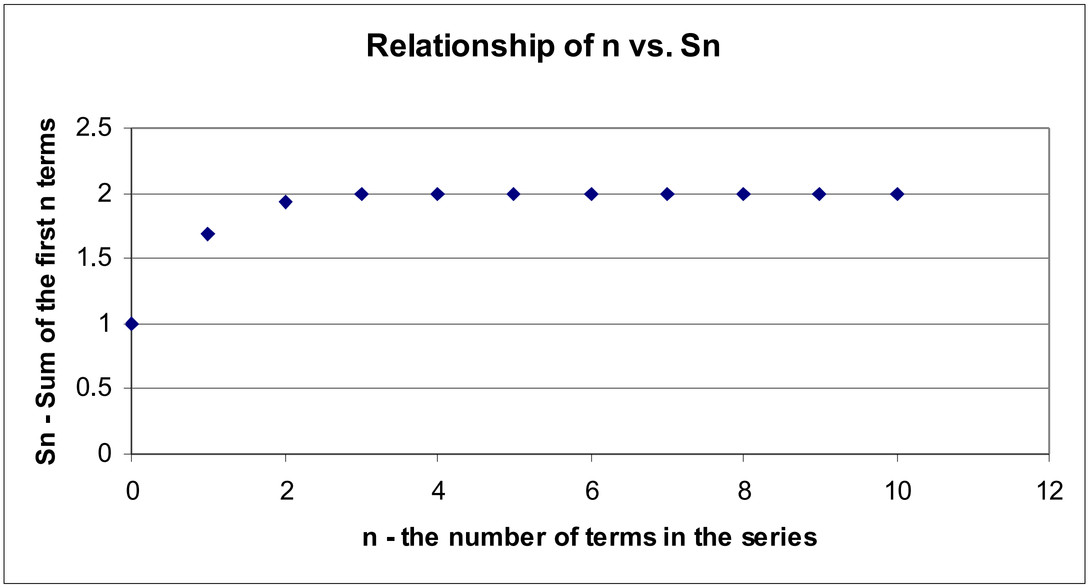


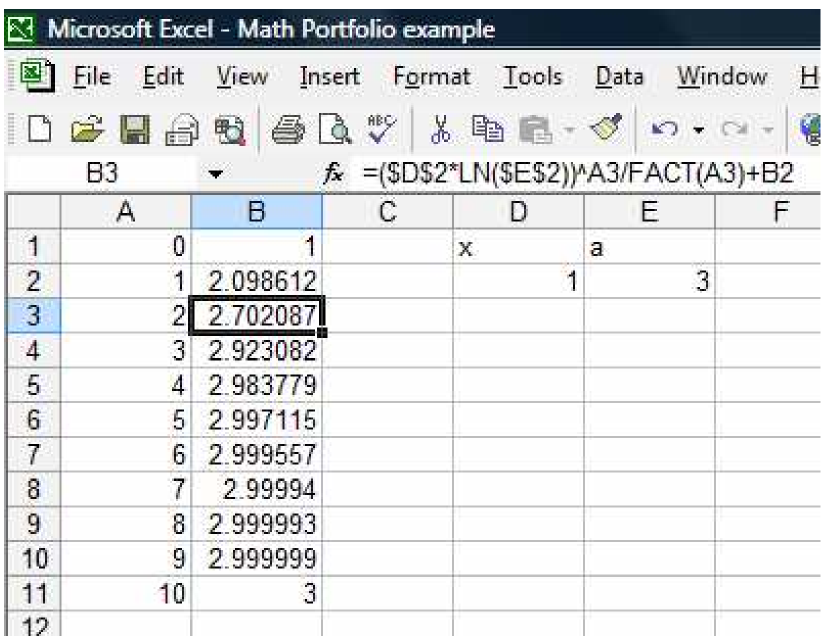
Figure 1: Showing how Excel was used to calculate Sn

If we now take these values and create a graph of n vs. Sn we obtain the result shown in Figure 2



From this graph it is clearly seen that the summation Sn converges on the value 2. It can be said then that as n approaches infinity the summation converges on the value 2.

If we complete this same exercise, but this time change the value of parameter a to equal 3 we obtain the following results for Sn shown as a screenshot for Excel.


Microsoft Excel - Math Portfolio example

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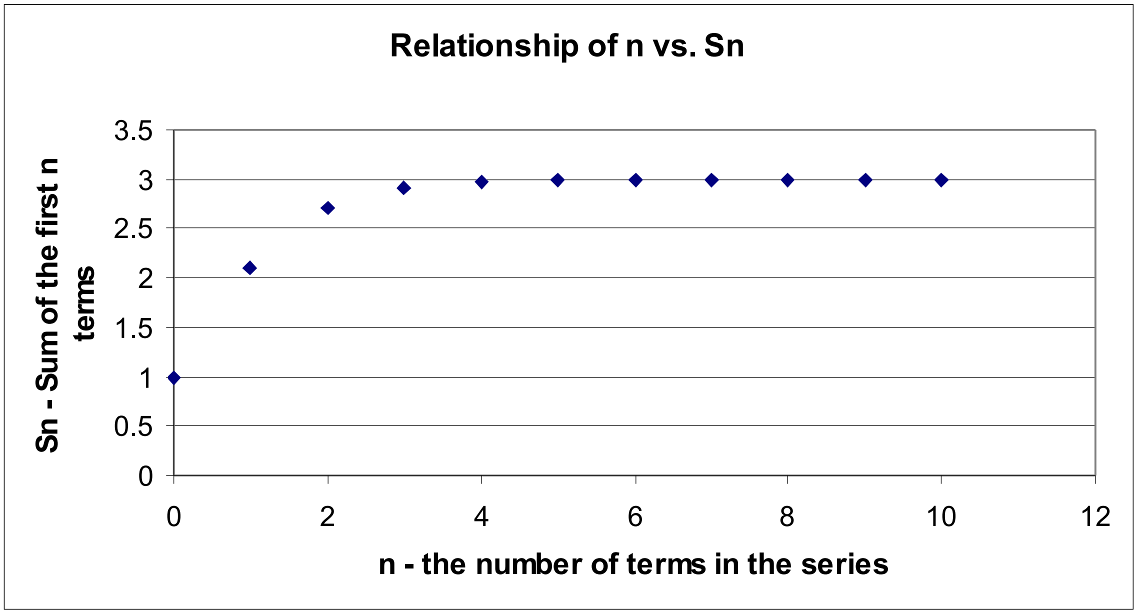
B3 =(D2*LN(E2))^A3/FACT(A3)+B2

	A	B	C	D	E	F
1	0	1		x	a	
2	1	2.098612		1	3	
3	2	2.702087				
4	3	2.923082				
5	4	2.983779				
6	5	2.997115				
7	6	2.999557				
8	7	2.99994				
9	8	2.999993				
10	9	2.999999				
11	10	3				
12						

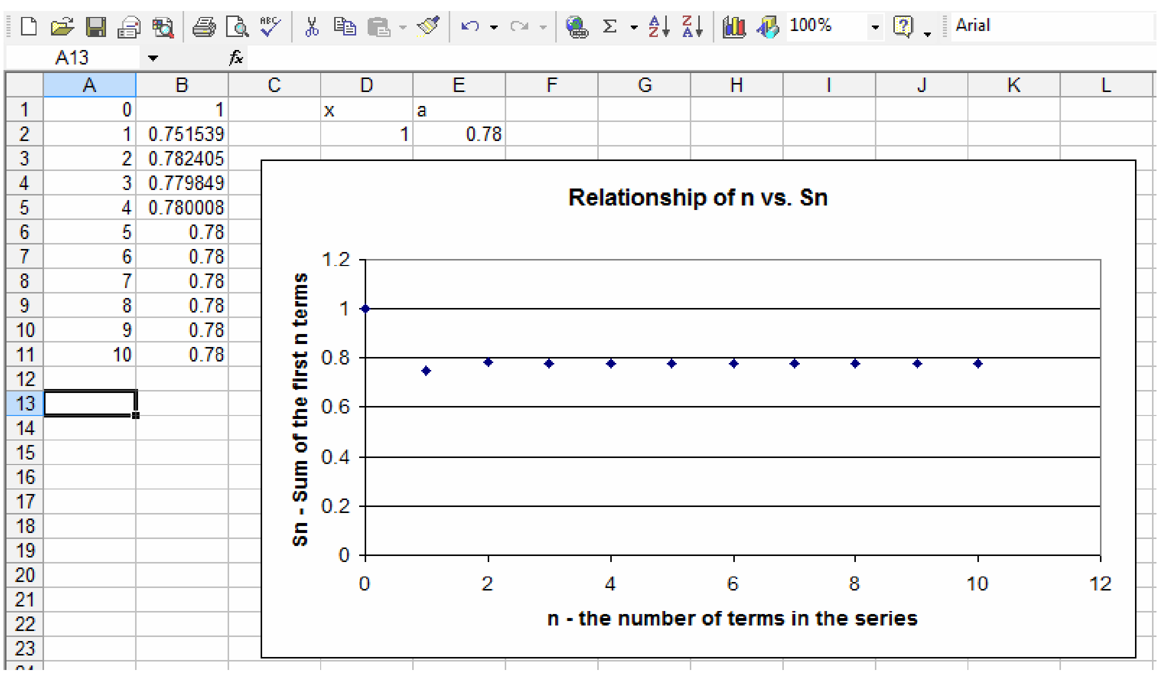
Again if we create a graph of n versus Sn we obtain the results shown in the figure below.



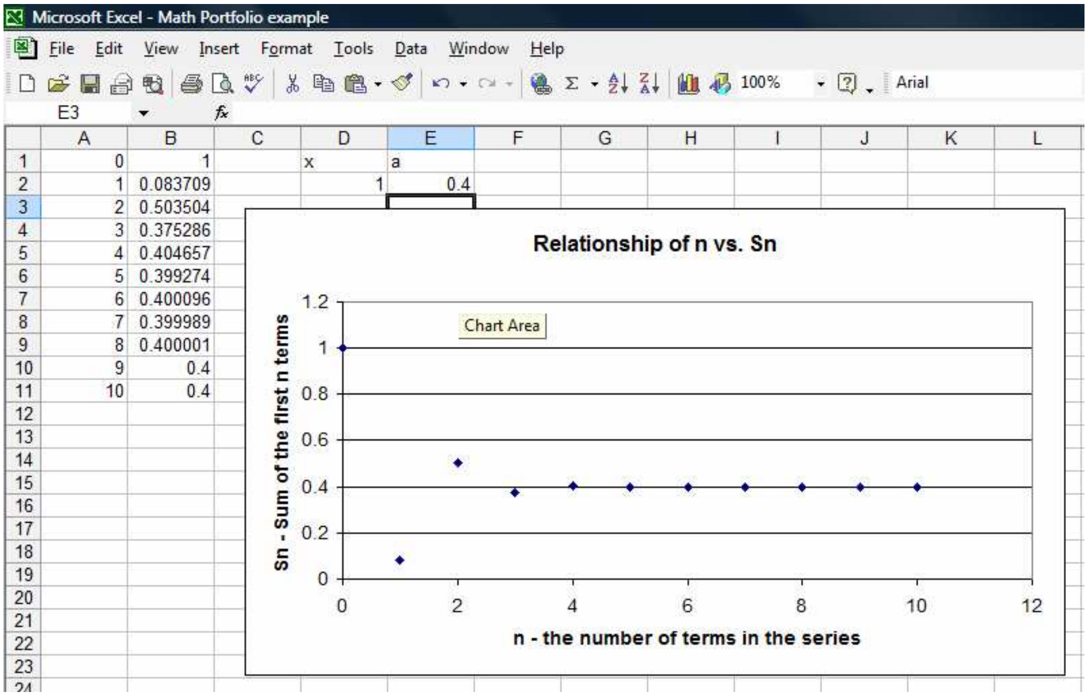
Again we notice that as n approaches infinity the value of Sn converges to 3 which is equal to the parameter a.

We can begin to notice a pattern that when x = 1 the value of Sn converges to the value of a, as n approaches infinity. In order to verify our assumption we will look at some different values of a.

a = 0.78:

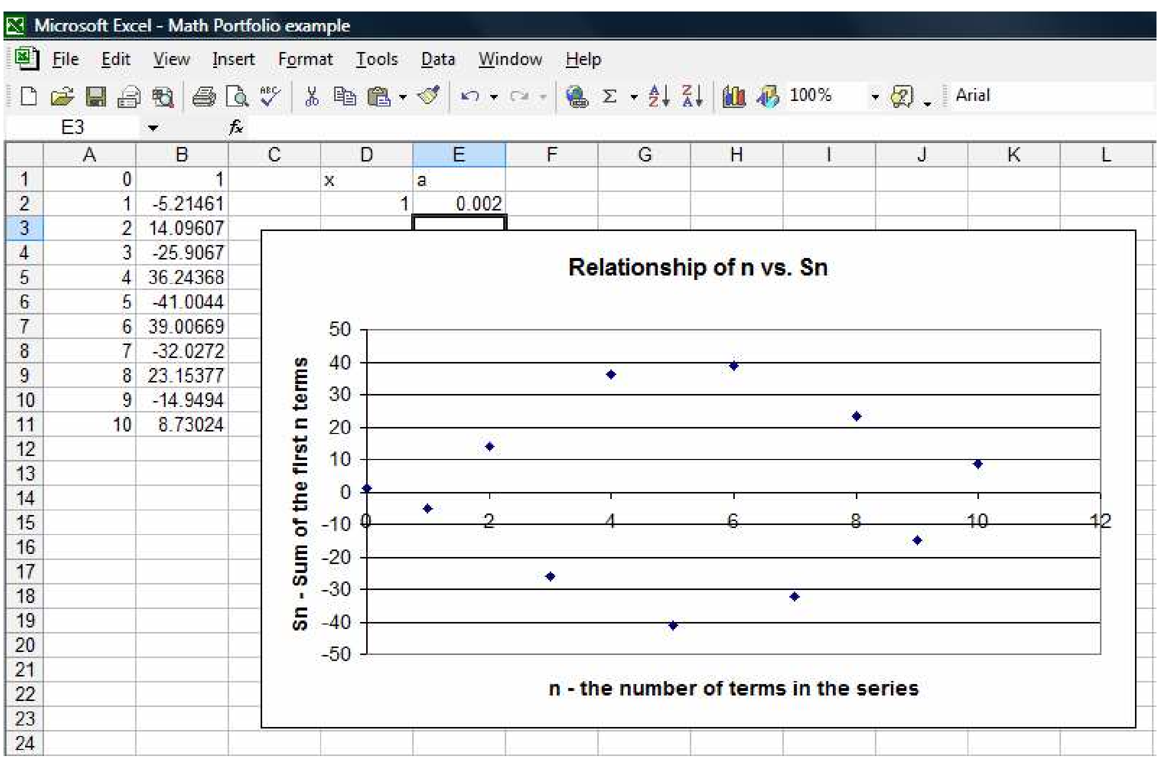


Again we see that as n approaches infinity the series converges to the value of a. It is also interesting to notice that as the value of a decreases the value of Sn seems to converge quicker. To test this hypothesis we will use a = 0.40.



Here it again converges to the value of a, however it does not directly go there instead it oscillates above and below it.

a = 0.002



Here it seems to not converge at all but rather oscillates widely back and forth from positive to negative values. However, if we include a higher value of n (i.e. n = 30) the value of Sn does indeed converge to a.

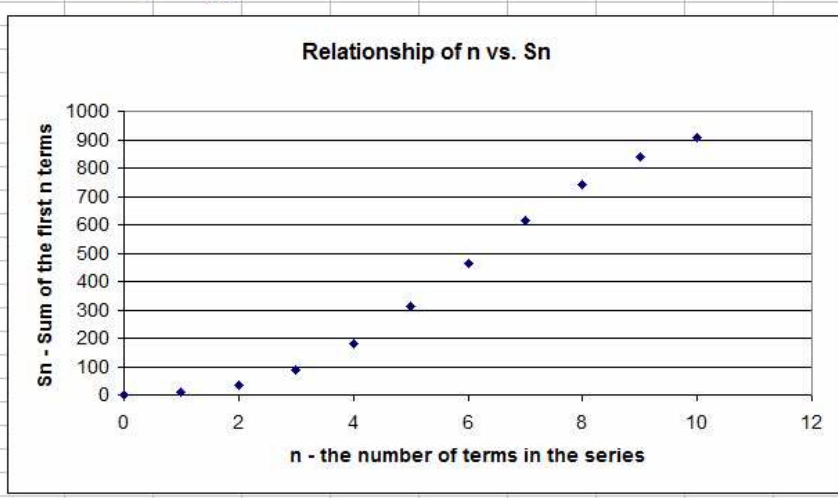
So clearly as the value of a decreases the value of Sn does converge on it, however it does not do so in a smooth systematic way, but rather chaotically.

Now we will try large values of a:

a = 1000

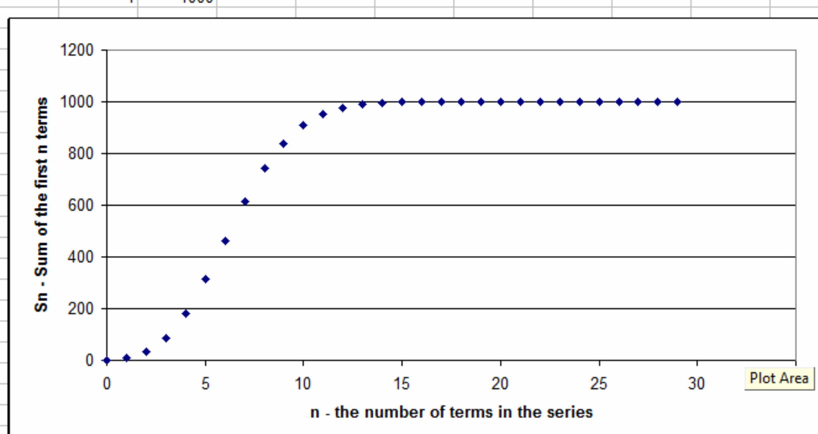
Microsoft Excel - Math Portfolio example

	A	B	C	D	E
1	0	1		x	a
2	1	7.907755		1	1000
3	2	31.7663			
4	3	86.70262			
5	4	181.5743			
6	5	312.6443			
7	6	463.5443			
8	7	612.4558			
9	8	741.0362			
10	9	839.7254			
11	10	907.8974			



Now it does not converge to the value of a, after 10 steps; however if the value of n is increased the value of Sn does indeed converge on 1000 as shown. It also appears to do so in a smooth fashion.

	A	B	C	D	E
1	0	1		x	a
2	1	7.907755		1	1000
3	2	31.7663			
4	3	86.70262			
5	4	181.5743			
6	5	312.6443			
7	6	463.5443			
8	7	612.4558			
9	8	741.0362			
10	9	839.7254			
11	10	907.8974			
12	11	950.708			
13	12	975.3517			
14	13	988.4465			
15	14	994.9077			
16	15	997.8831			
17	16	999.1677			
18	17	999.6897			
19	18	999.8901			
20	19	999.9629			
21	20	999.988			
22	21	999.9963			
23	22	999.9989			
24	23	999.9997			
25	24	999.9999			
26	25	1000			
27	26	1000			
28	27	1000			
29	28	1000			
30	29	1000			



We can clearly observe a general trend for Sn in which assuming n is large enough (i.e n approaches infinity) then the value of Sn converges upon the value of a.