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January 20, 2011
Lancaster 3rd

Infinite Summation Internal Assessment

The idea of this internal assessment is to investigate the effect changing the value of *x* and *a* have on the graph of the general sequence given. In able to observe and display the following graph and tables, Microsoft Excel took action during this investigation. It is also vital to take note of the scatter plot lines with connected dots of the values of S_n at the different values of n is used for visual assistance to help understand the affect x and a. The notation of S_n is the sum of the terms and n is the number of the term in the sequence. The sequence that will be examine to determine the effect of different values of *x* and *a* is

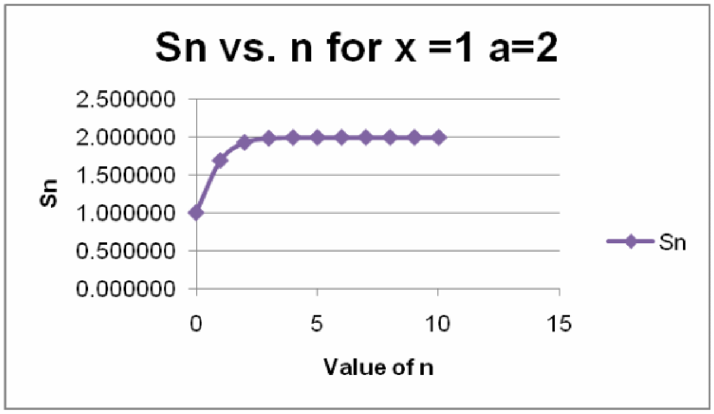
$$t_0=1,\ t_1=\frac{x\ln(a)}{1},\ t_2=\frac{x\ln(a)^2}{2*1},\ t_3=\frac{x\ln(a)^3}{3*2*1}\ \ldots,\ t_n=\frac{x\ln(a)^n}{n!}\ \ldots$$

The first variation of the sequence that will be observed is, $1, \frac{(1)\ln(2)}{1}, \frac{(1)\ln(2)^2}{2\cdot 1}, \frac{(1)\ln(2)^3}{3\cdot 2\cdot 1}$ with *x* = 1 and *a* = 2.

$S_n =$

n	Sn
0	1.000000
1	1.693147
2	1.933374
3	1.988878
4	1.998496
5	1.999829
6	1.999983
7	1.999999
8	2.000000
9	2.000000
10	2.000000

The table shows the value of S_n, which is sum of the terms in the sequence, which gradually increase to 2, but never passes 2 regardless of the value of *n* increasing.



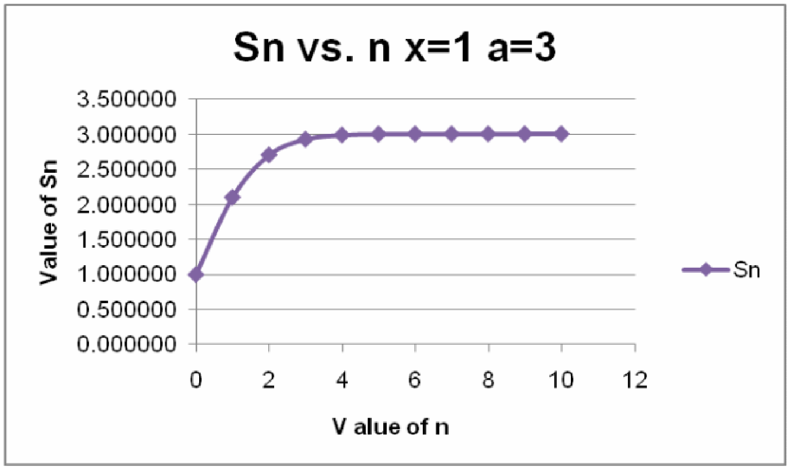
The graph above displays x=1 a=2 in the sequence and how its sum affects this line. $0 \le n \le 10$ which is the x-axis, shows that S_n never passing through 2 because 2 is the horizontal asymptote, thus even if the value of n increases, S_n would still be smaller than 2.

The next sequence will be a similar concept of the previous one, but the change in values of x=1 and a=3 which is $1, \frac{(1)\ln(3)}{1}, \frac{(1)\ln(3)^2}{2*1}, \frac{(1)\ln(3)^3}{3*2*1}\ldots$

$S_n =$

n	Sn
0	1.000000
1	2.098612
2	2.702087
3	2.923082
4	2.983779
5	2.997115
6	2.999557
7	2.999940
8	2.999993
9	2.999999
10	3.000000

This table show the value of S_n, which is the sum of the sequence and never exceeds 3. This may be similar to the previous sequence but it varies as the S_n increase faster throughout the same value of n as S_n reaches to 3.



This graph displays x=1 and a=3, $0 \le n \le 10$ on the value of S_n. 3, which is the horizontal asymptote.

After the value of the x has been kept as 1 from the previous two graphs and tables, it is necessary to change the value of a and observe the affect on the sum of the sequence. Thus, same format of the sequence will be used with changing the value of a

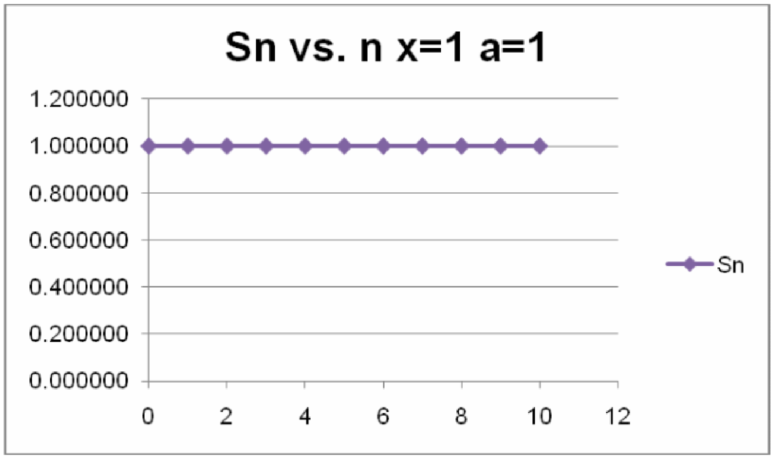
$$1, \frac{(1)\ln(a)}{1}, \frac{(1)\ln(a)^2}{2*1}, \frac{(1)\ln(a)^3}{3*2*1} \ldots$$

x =1 and a =1

$S_n =$

n	Sn
0	1.000000
1	1.000000
2	1.000000
3	1.000000
4	1.000000
5	1.000000
6	1.000000
7	1.000000
8	1.000000
9	1.000000
10	1.000000

This table shows the same result of S_n throughout the increasing value of n which is 1. Therefore, with the table that is consistent, it can be assumed that the line of S_n will be a straight line, with no changing values.



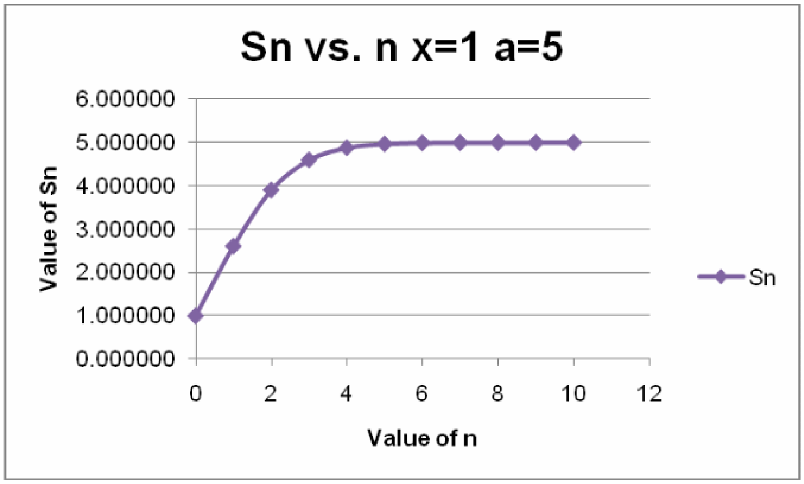
This graph displays the value of x =1 and a =1 and $0 \leq n \leq 10$. Since the value of a is 1 there is no change in the graph because the graph starts at 1 on the y-axis and the horizontal asymptote is also 1, therefore the line is straight.

This is another example of the same previous sequence where x =1 and a =5

$S_n =$

n	Sn
0	1.000000
1	2.609438
2	3.904583
3	4.599402
4	4.878969
5	4.968958
6	4.993096
7	4.998646
8	4.999763
9	4.999962
10	4.999995

This table shows the increasing value of S_n with the value of n increasing till it reached to 10. S_n approaches 5, but never actually reaches 5, which is discovered that 5 is a horizontal asymptote.



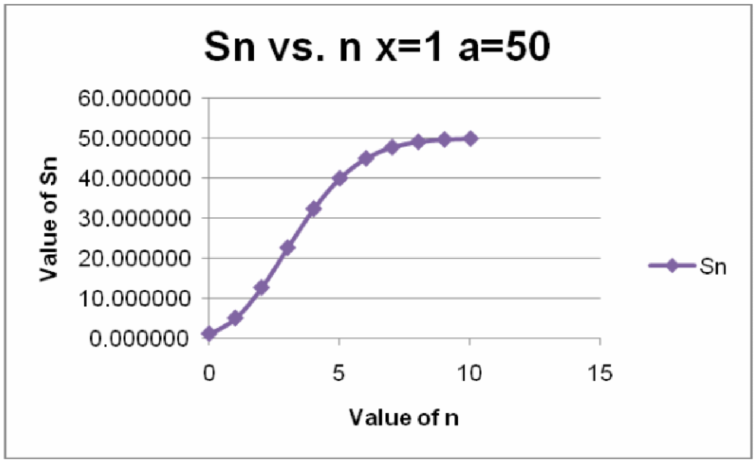
This graph displays the line where x =1 and a =5 and $0 \leq n \leq 10$, the value of S_n approaches the horizontal asymptote, which is 5. This creates a longer and steeper line compare to the previous graph since it reaches towards the horizontal asymptote of 5 in the same values of n .

Now other values with be investigated to ensure the idea about a acting as the horizontal asymptote of the sum of the infinite sequence and the disparity of the graph due to the large value of *a*, x =1 and a =50

$S_n =$

n	Sn
0	1.000000
1	4.912023
2	12.563985
3	22.542202
4	32.300956
5	39.936250
6	44.914491
7	47.696632
8	49.057108
9	49.648464
10	49.879805

This table shows the increasing values of S_n that approach to 50, but never actually reaches 50. As the table is shown, it is visible that at each value of n the value of S_n increases at larger numbers than the other tables before, it is due to the sum of the sequence is reaching a high value of asymptote in the same amount of time.



This graph demonstrates the values of x =1 and a =50 and $0 \le n \le 10$. Also S_n approaches the horizontal asymptote of 50, which is the value of a . Compare to the graphs before, this graph has a different form of a line due to larger value of *a*.

From the investigation of the different values of a with the value of x being 1 in this general sequence,

$$t_0=1,\ t_1=\frac{x\ln(a)}{1},\ t_2=\frac{x\ln(a)^2}{2*1},\ t_3=\frac{x\ln(a)^3}{3*2*1}\ \ldots,\ t_n=\frac{x\ln(a)^n}{n!}.$$

It can be stated that the value of x is 1 while the horizontal asymptote is determined by the value of *a* in the sum of this sequence as it approaches infinite. Since x remained as 1, the only changing variable in this whole sequence was a and the value of a was the number where the horizontal asymptote was present on the y-axis.

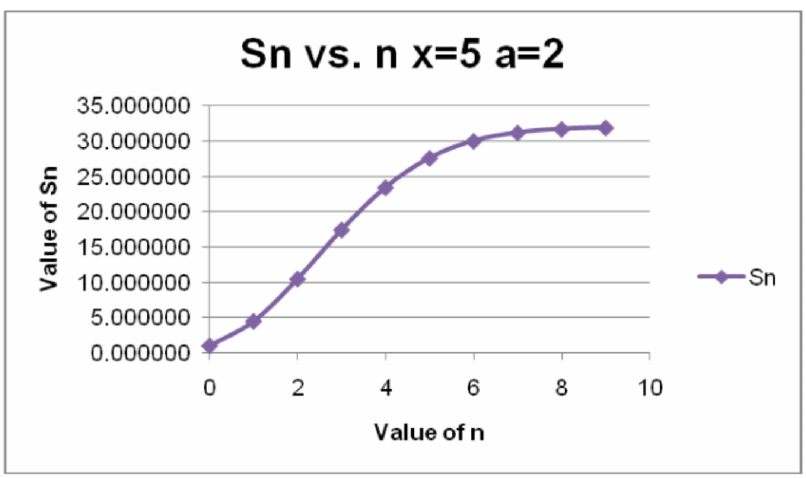
Now the sum of the infinite sequence t_n must be determined by using a different formula unlike the previous one,

$$t_0 = 1,\ t_1 = \frac{(x\ln a)}{1},\ t_2 = \frac{(x\ln a)^2}{2*1},\ t_3 = \frac{(x\ln a)^3}{3*2*1} \ldots$$

This is an example of $T_n(a,x)$, where T_9 (2, 5) which means a = 2 and x = 5

a	x	n	t	Sn
2	5	0	1	1
		1	3.465736	4.46573590
		2	6.005663	10.47139858
		3	6.938014	17.40941216
		4	6.011331	23.42074285
		5	4.166737	27.58747977
		6	2.406802	29.99428140
		7	1.19162	31.18590123
		8	0.51623	31.70213118
		9	0.198791	31.90092192

As the table is shown, the value of a is defined as 2 instead of 1 looking back at the section before, and the value of x being 5 with the increasing value of *n* until 9. The value of S_n continues to increase, but never exceeds 32.

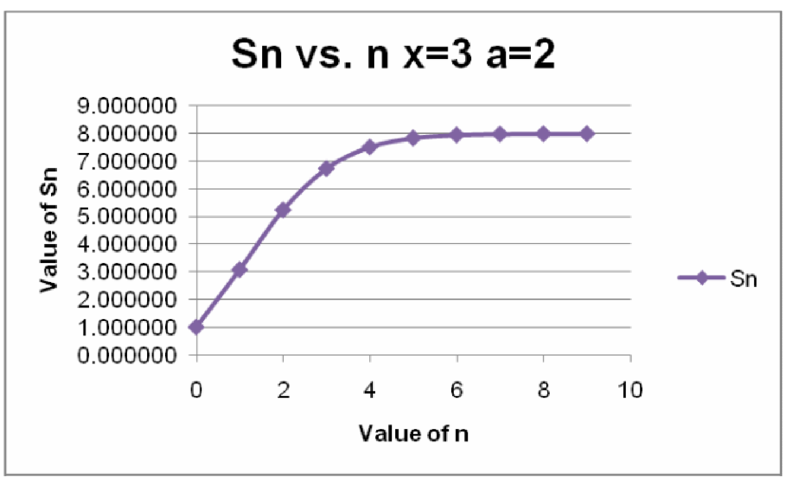


This graph displays the large increasing values of S_n in the beginning that approaches a horizontal asymptote at 32. The value of n is $0 \leq n \leq 9$ and there is exponential growth until the value of *n* is 7, which is when the rate of increase of S_n begins to slow down and it never actually reaches 32.

Another example where *a*=2 and *x*=3, T_9 (2, 3)

a	x	n	t	Sn
2	3	0	1.000000	1.000000
		1	2.079442	3.079442
		2	2.162039	5.241480
		3	1.498611	6.740091
		4	0.779068	7.519159
		5	0.324005	7.843165
		6	0.112292	7.955457
		7	0.033358	7.988814
		8	0.008671	7.997485
		9	0.002003	7.999488

This table shows the value of a being 2 again while the value of x is now 3 and the value of n still increases until 9. Examining the table, S_n increases quickly until the value of n is about 4, where the value of S_n continues to stay in the range of 7 never reaching 8, since the horizontal asymptote is 8.

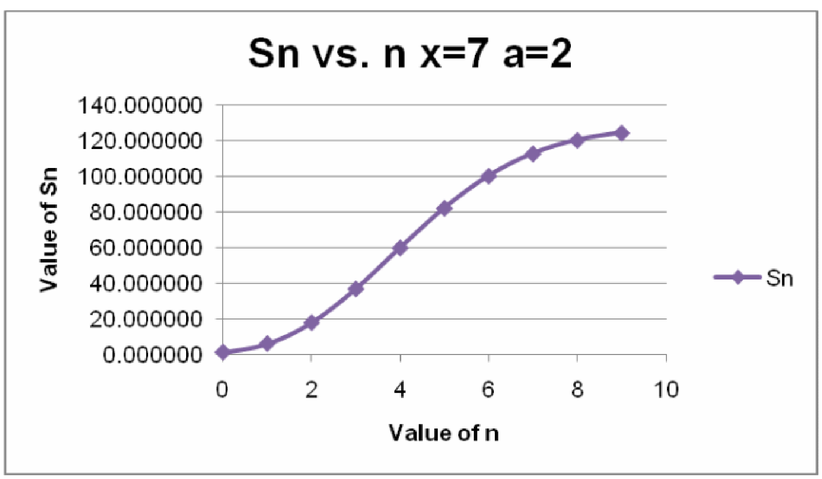


This graph $0 \leq n \leq 9$ displays a quick increase of S_n and later on levels out when *n* is 7, but never exceeds 8 because 8 is the horizontal asymptote. At the later values of n (8 and 9) it is visible that the line being shown as a straight line proving that 8 is the horizontal asymptote and S_n will never exceed.

Here is another example of the sequence taking the same concept where $a = 2$ *and* $x = 7$, $T_9\ (2,7)$

a	x	n	t	Sn
2	7	0	1.000000	1.000000
		1	4.852030	5.852030
		2	11.771099	17.623129
		3	19.037909	36.661038
		4	23.093128	59.754166
		5	22.409711	82.163878
		6	18.122099	100.285977
		7	12.561282	112.847259
		8	7.618465	120.465724
		9	4.107225	124.572949

This table shows the large changes in the value of S_n throughout $0 \leq n \leq 9$, while quickly reaching the horizontal asymptote of 128, but this horizontal asymptote is not visible in the small values of n .



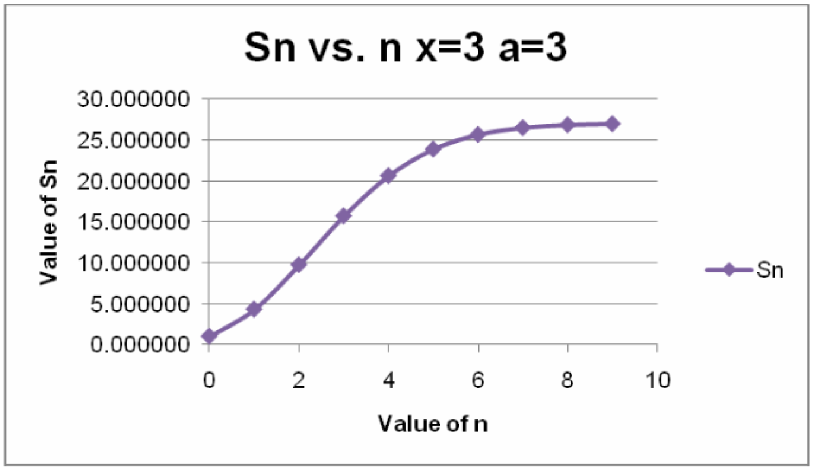
This graph displays $a = 2$ and $x = 7$ and $0 \leq n \leq 9$, which creates the horizontal asymptote at 128 as 2^7 is 128. Unusually, this graph does not display the horizontal asymptote because the end of this line is not yet straight, so it can be assumed that it will straighten more as the value of S_n gets closer to 128. Thus from the three different values of x it is visible that x changes the horizontal asymptote because it is how a is rooted, creating a larger or smaller horizontal asymptote.

Next the same sequence will be used, $t_0 = 1$, $t_1 = \frac{(x\ln a)}{1}$, $t_2 = \frac{(x\ln a)^2}{2*1}$, $t_3 = \frac{(x\ln a)^3}{3*2*1}$, $T_9(3, x)$ with changing values of x to determine the affect x has on the sequence with $a = 3$, which is also changed.

This is the first example where $a = 3$ and $x = 3$, $T_9(3, 5)$

a	x	n	t	Sn
3	3	0	1.000000	1.000000
		1	3.295837	4.295837
		2	5.431270	9.727107
		3	5.966860	15.693968
		4	4.916450	20.610417
		5	3.240763	23.851180
		6	1.780171	25.631351
		7	0.838165	26.469516
		8	0.345307	26.814823
		9	0.126453	26.941276

This table shows the fast increasing values of S_n towards 27 then later on skirling around 25 to 26 when *n* is 6, it can be estimated that it will never exceed 27 because it quickly increases and then the rate of increase slows around 25 with increased values of n .

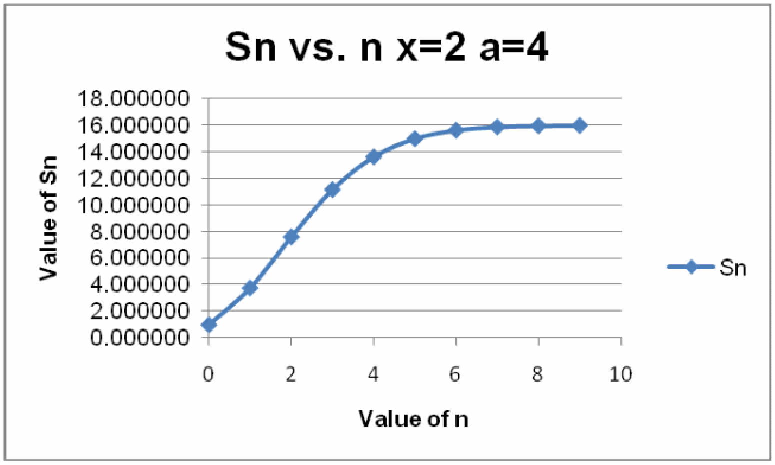


The graph displays the value of a = 3 and x = 3 and $0 \le n \le 9$. The line begins to flatten around 27, which is the horizontal asymptote, and it can be stated that 3^3 is 27. Since the horizontal asymptote is smaller, the exponential growth is shorter and gets straighter at top of the line because there is a smaller rate of increase of S_n as it approaches 27.

Next other values of x and a will be investigated to guarantee the general statement is valid, $a = 4$ and $x = 2$, T_9 (4, 2)

a	x	n	t	Sn
4	2	0	1.000000	1.000000
		1	2.772589	3.772589
		2	3.843624	7.616213
		3	3.552263	11.168476
		4	2.462241	13.630717
		5	1.365356	14.996073
		6	0.630929	15.627002
		7	0.249901	15.876903
		8	0.086609	15.963512
		9	0.026681	15.990193

The table above shows a fast increasing value of the sum of the sequence until n=6 as it approaches 16 at a slow rate, which 16 is the horizontal asymptote since 4^2 = 16.

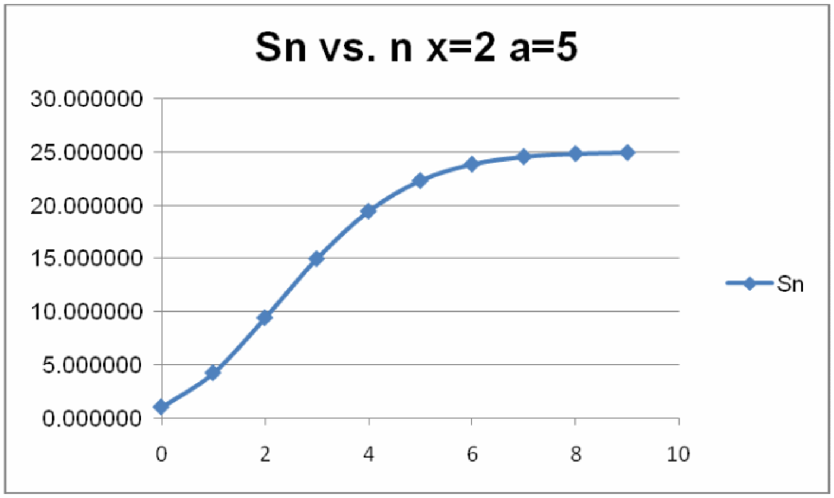


This graph displays the value of $a = 4$ and $x = 2$ and $0 \le n \le 9$. There is exponential growth in the sum of the sequence until the value of n is about 3, where the rate of line of increase begins to slow since it is approaching the horizontal asymptote of 16.

Another example is with $a = 5$ and $x = 2$, T_9 (5, 2) and using the same sequence as before

a	x	n	t	Sn
5	2	0	1.000000	1.000000
		1	3.218876	4.218876
		2	5.180581	9.399457
		3	5.558549	14.958005
		4	4.473070	19.431075
		5	2.879651	22.310726
		6	1.544873	23.855599
		7	0.710394	24.565993
		8	0.285834	24.851826
		9	0.102229	24.954056

This table shows the increasing value of the sum of the sequence approaching 25 as the value of n increases, but looks as it will never exceed 25, due to the fact that 25 is the horizontal asymptote.



This graph displays the value of the sum of this sequence when $a = 5$ and $x = 2$ and $0 \leq n \leq 9.$ Since in this sequence a^x is 5^2 = 25, the horizontal asymptote is then 25, which is why the line has exponential growth until it approaches 25 creating an almost straight line.

After the investigation of the infinite sequence,

$$t_0 = 1,\ t_1 = \frac{(x \ln a)}{1},\ t_2 = \frac{(x \ln a)^2}{2 * 1},\ t_3 = \frac{(x \ln a)^3}{3 * 2 * 1} \ldots$$

Where $T_n(a, x)$, with the changing values of x and a , it is clear that a^x is the horizontal asymptote of the sum of the terms in this sequence as n approaches infinite. The limitations of this general statement could vary since n was only investigated until certain numbers not infinity, therefore it was examination and judgment that created the general statements about the horizontal asymptotes that affect the graph of the sum of the infinite sequence.