

Introduction

In this task, we are required to investigate the mathematical patterns within systems of linear equations. We need to concept of matrices and algebraic equations in this task.

Body

Part A

Consider this 2×2 system of linear equations $\begin{cases} x + 2y = 3 \\ 2x - y = -4 \end{cases}$

- Let the equations be $ax + by = c$,

For the first equation ($x + 2y = 3$): $a = 1, b = 2, c = 3$.

For the second equation ($2x - y = -4$): $a = 2, b = -1, c = -4$.

- **I examined the constants of two equations, I identified there is the pattern of an arithmetic sequence. For the first equation, the constants (1, 2, 3) starts with 1 and has a common difference of 1; for the second equation, the constants (2, -1, -4) starts with 2 and has a common difference of -3.**

- Then I solved the system algebraically:

$$\begin{cases} x + 2y = 3 & - (1) \\ 2x - y = -4 & - (2) \end{cases}$$

$$(1): x + 2y = 3$$

$$2y = 3 - x$$

$$y = \frac{3-x}{2} \quad - (3)$$

$$(2): 2x - y = -4$$

$$-y = -4 - 2x$$

$$y = 4 + 2x \quad - (4)$$

$$\frac{3-x}{2} = 4 + 2x$$

$$3 - x = 8 + 4x$$

$$-5x = 5$$

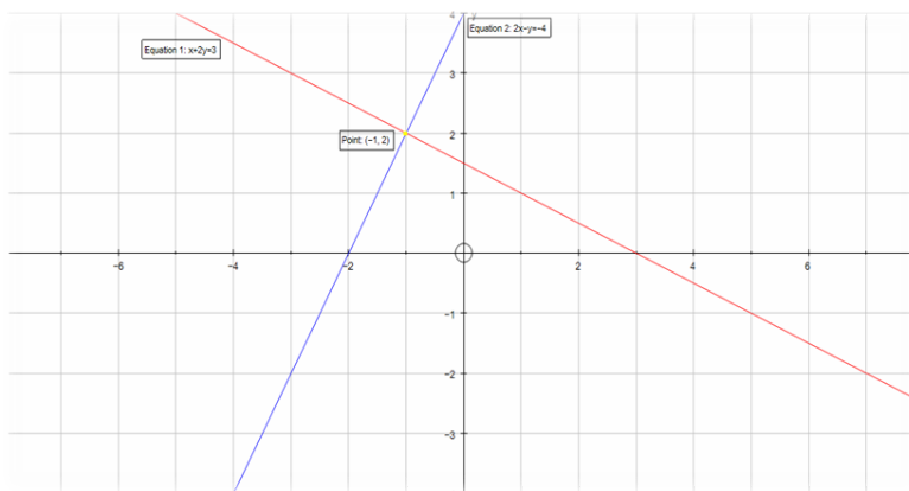
$$x = -1$$

Substituting $x = -1$ into equation (4): $y = 4 + 2(-1)$

$$\therefore y = 2$$

The solution is $x = -1, y = 2$.

- I use autograph to draw two lines on the same set of axes. To check the solution



The solution of this 2×2 system of linear equations is unique.

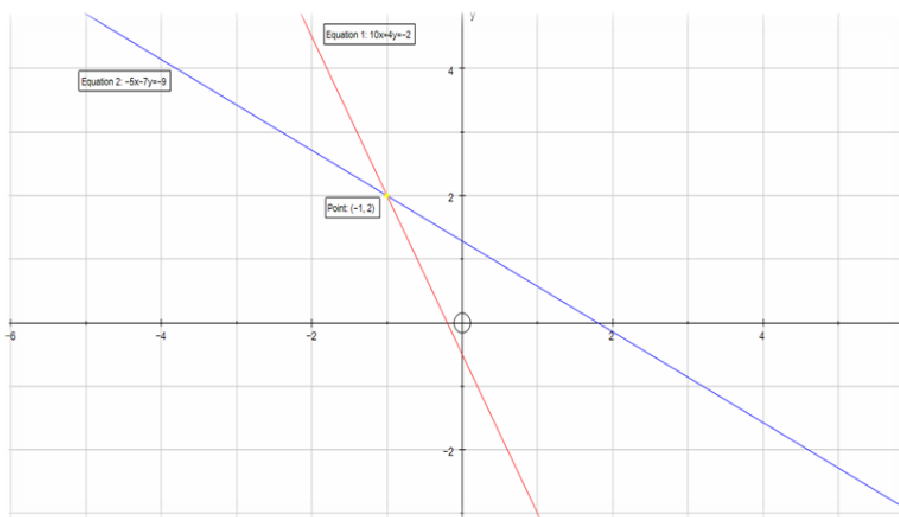
$$1. \begin{cases} 10x + 4y = -2 & - (1) \\ -5x - 7y = -9 & - (2) \end{cases}$$

$$(2) \times 2: -10x - 14y = -18 \quad - (3)$$

$$(1) + (3): -10y = -20$$

$$y = 2$$

Substituting $y = 2$ into equation (1): $x = (-2 - 8) \div 10 = -1$



The solution is $x = -1, y = 2$.

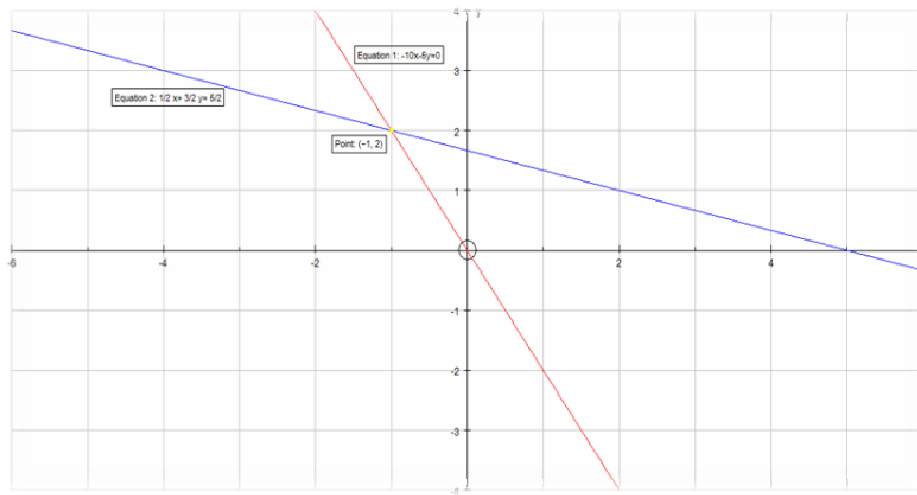
$$2. \begin{cases} -10x - 5y = 0 & - (1) \\ \frac{1}{2}x + \frac{3}{2}y = \frac{5}{2} & - (2) \end{cases}$$

$$(2) \times 20: 10x + 30y = 50 \quad - (3)$$

$$(1) + (3): \quad 25y = 50$$

$$y = 2$$

Substituting $y = 2$ into equation (1): $x = (0 + 10) \div -10 = -1$



The solution is $x = -1, y = 2$.

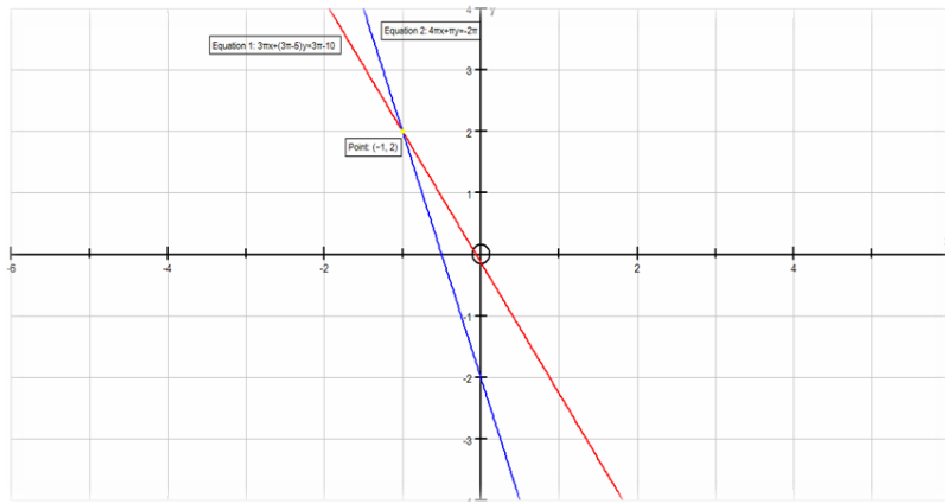
$$3. \begin{cases} 3\pi x + (3\pi - 5)y = 3\pi - 10 & - (1) \\ 4\pi x + \pi y = -2\pi & - (2) \end{cases}$$

$$(2) \times \frac{3}{4}: \quad 3\pi x + \frac{3}{4}\pi y = -\frac{3}{2}\pi \quad - (3)$$

$$(1) - (3): \quad \left(\frac{9}{4}\pi - 5\right)y = \frac{9}{2}\pi - 10$$

$$y = 2$$

Substituting $y = 2$ into equation (2): $x = (-2\pi - 2\pi) \div 4\pi = -1$



The solution is $x = -1, y = 2$.

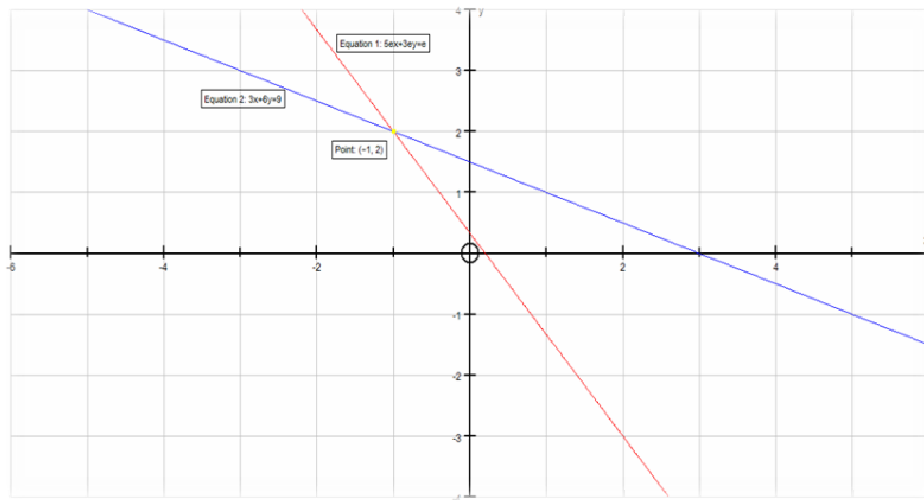
$$4. \begin{cases} 5ex + 3ey = e & - (1) \\ 3x + 6y = 9 & - (2) \end{cases}$$

$$(2) \times \frac{1}{2}e: \quad \frac{3}{2}ex + 3ey = \frac{9}{2}e \quad - (3)$$

$$(3) - (1): \quad -\frac{7}{2}ex = \frac{7}{2}e$$

$$x = -1$$

Substituting $x = -1$ into equation (1): $y = (e + 5e) \div 3e = 2$



The solution is $x = -1$, $y = 2$.

$$5. \begin{cases} \sqrt{2}x + 2\sqrt{2}y = 3\sqrt{2} & - (1) \\ -3\sqrt{5}x - 2\sqrt{5}y = -\sqrt{5} & - (2) \end{cases}$$

$$(1) \times \sqrt{5} : \sqrt{10}x + 2\sqrt{10}y = 3\sqrt{10} \quad - (3)$$

$$(2) \times \sqrt{2} : -3\sqrt{10}x - 2\sqrt{10}y = -\sqrt{10} \quad - (4)$$

$$(4) + (3): \quad -2\sqrt{10}x = 2\sqrt{10}$$

$$x = -1$$

Substituting $x = -1$ into equation (1): $y = (3\sqrt{2} + \sqrt{2}) \div 2\sqrt{2} = 2$

The solution is $x = -1$, $y = 2$.

From the five 2×2 system of linear equations I have investigated, all of them have a unique solution of $x = -1, y = 2$.

Therefore, my conjecture is: for any 2×2 system of linear equations which have the constants in the order of arithmetic sequences. They must have a unique solution of $x = -1, y = 2$.

And I came out with these general equations:

$$\begin{cases} ax + (a + c)y = a + 2c & - (1) \\ bx + (b + d)y = b + 2d & - (2) \end{cases}$$

$$(1) \times \frac{b+d}{a+c} : \frac{ab+ad}{a+c}x + (b+d)y = \frac{ab+ad+2bc+2cd}{a+c} \quad - (3)$$

$$(3) - (2): \left(\frac{ab+ad}{a+c} - b \right)x = \frac{ab+ad+2bc+2cd}{a+c} - b - 2d$$

$$\frac{ab+ad-ab-bc}{a+c}x = \frac{ab+ad+2bc+2cd-ab-bc-2ad-2cd}{a+c}$$

$$\frac{ad-bc}{a+c}x = \frac{bc-ad}{a+c}$$

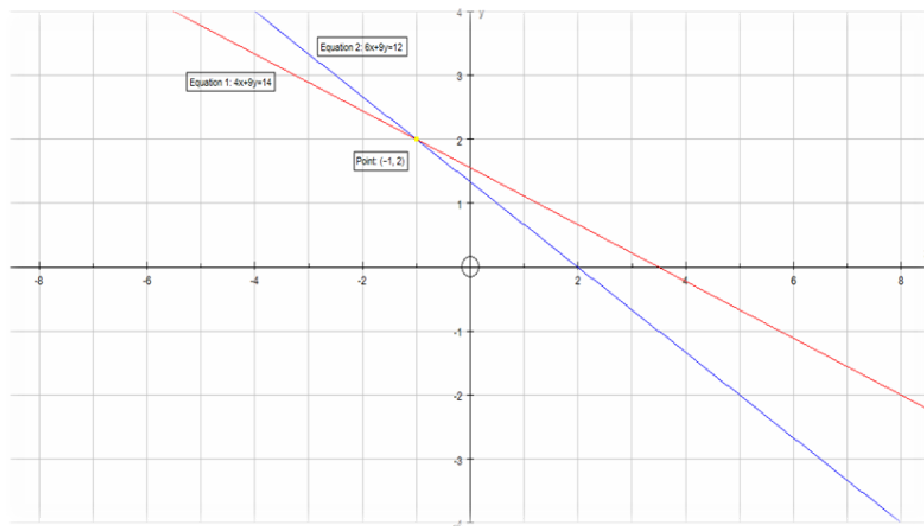
$$x = -1$$

Substituting $x = -1$ into equation (1): $y = (a + 2c + a) \div (a + c) = 2$

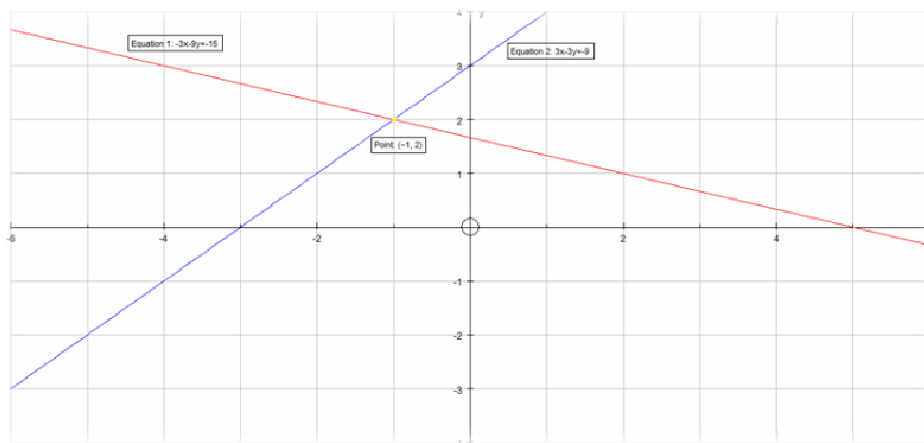
This system has a unique solution of $x = -1, y = 2$.

To test the validity of the conjecture,

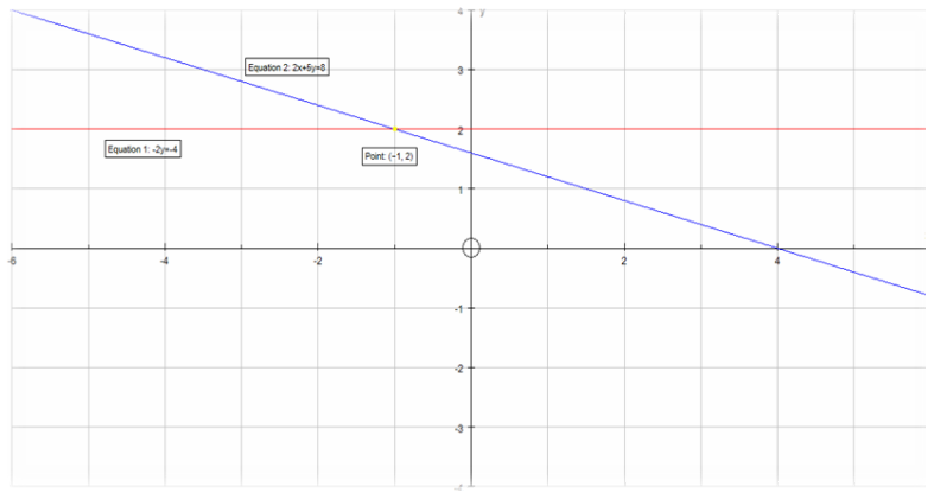
1.
$$\begin{cases} 4x + 9y = 14 \\ 6x + 9y = 12 \end{cases}$$



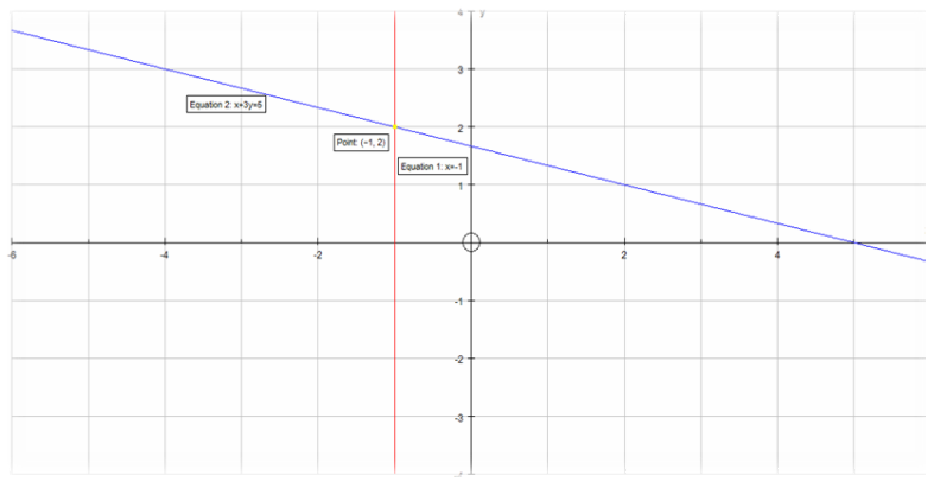
2.
$$\begin{cases} -3x - 9y = -15 \\ 3x - 3y = -9 \end{cases}$$



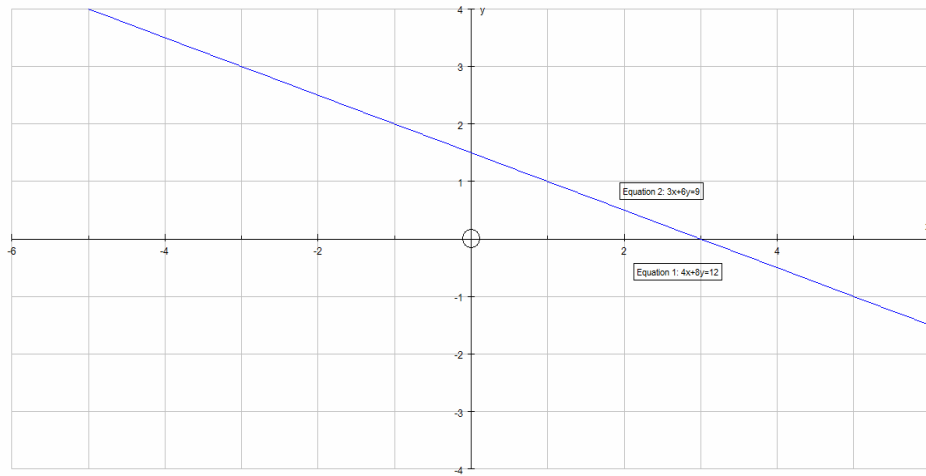
3.
$$\begin{cases} -2y = -4 \\ 2x + 5y = 8 \end{cases}$$



4.
$$\begin{cases} x = -1 \\ x + 3y = 5 \end{cases}$$



5.
$$\begin{cases} 4x + 8y = 12 \\ 3x + 6y = 9 \end{cases}$$



There is limitation to this system: The ratio between the constants can't be the same for both of the equations. If the ratios are same, two lines are overlapped, and the solutions become infinite.

Now look at 3×3 system,

$$1. \begin{cases} x + 3y + 5z = 7 \\ 7x + 8y + 9z = 10 \\ -4x - 8y - 12z = -16 \end{cases}$$

By using GDC,

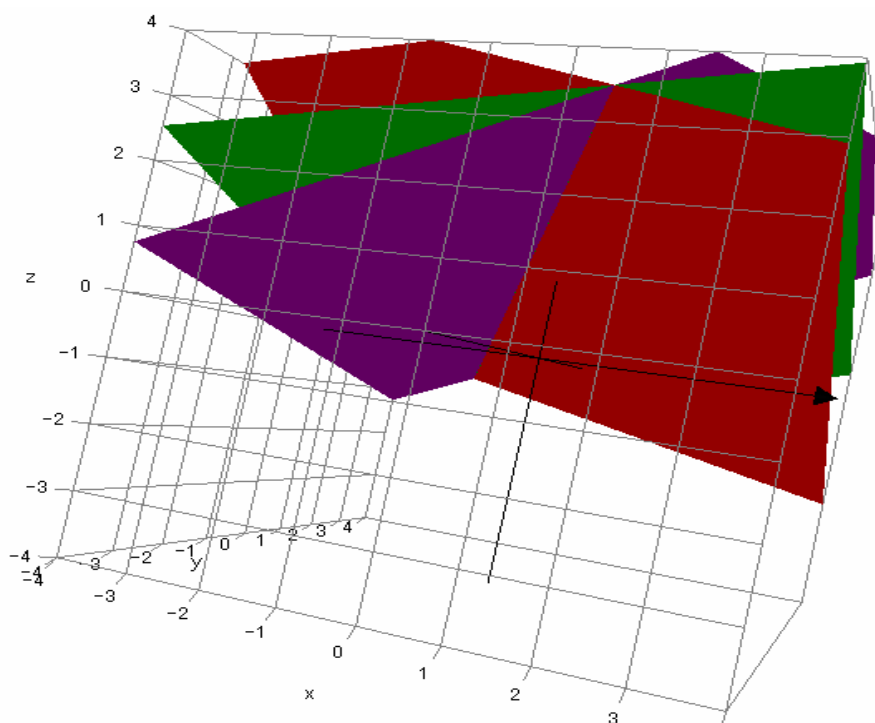
SYSTEM MATRIX (3x4)	SOLUTION SET	RREF (3x4)
$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 7 & 8 & 9 & 10 \\ -4 & -8 & -12 & -16 \end{bmatrix}$	$\begin{aligned} x_1 &= -2 + 1x_3 \\ x_2 &= 3 - 2x_3 \\ x_3 &= x_3 \end{aligned}$	$\begin{bmatrix} 1 & 0 & -.9... & -.1... \\ 0 & 1 & 1.9... & 2.9... \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(3/4)=-16

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSN STD RREF MAIN BACK SYSN STD RREF

From this, it clearly shows there are infinite solutions. And the solution of this system is $x = -2 + m$, $y = 3 - 2m$ and $z = m$

Using 3-D sketch of autograph, we can see that three planes intercept each other at one line



Therefore, $x + y + z = -2 + m + 3 - 2m + m = 1$.

$$2. \begin{cases} -3x - 7y - 11z = -15 \\ 9x + 8y + 7z = 6 \\ x + 5y + 9z = 13 \end{cases}$$

SYSTEM MATRIX (3x4)	SOLUTION SET	RREF (3x4)
$\begin{bmatrix} -3 & -7 & -11 & -15 \\ 9 & 8 & 7 & 6 \\ 1 & 5 & 9 & 13 \end{bmatrix}$	$\begin{cases} x_1 = -2 + x_3 \\ x_2 = 3 - 2x_3 \\ x_3 = x_3 \end{cases}$	$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$(1,1) = -3$$

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSM STO RREF MAIN BACK SYSM STO RREF

It shows the solution of this system is $x = -2 + m$, $y = 3 - 2m$ and $z = m$.

$$3. \begin{cases} \frac{1}{2}x + \frac{3}{2}y + \frac{5}{2}z = \frac{7}{2} \\ 7x + 6y + 5z = 4 \\ \frac{4}{3}x + y + \frac{2}{3}z = \frac{1}{3} \end{cases}$$

SYSTEM MATRIX (3x4)	SOLUTION SET	RREF (3x4)
$\begin{bmatrix} .5 & 1.5 & 2.5 & 3.5 \\ 7 & 6 & 5 & 4 \\ 1.33... & 1 & .66... & .33... \end{bmatrix}$	$\begin{cases} x_1 = -2 + x_3 \\ x_2 = 3 - 2x_3 \\ x_3 = x_3 \end{cases}$	$\begin{bmatrix} 1 & 0 & -.9... & -.1... \\ 0 & 1 & 1.9... & 2.9... \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$(1,1) = .5$$

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSM STO RREF MAIN BACK SYSM STO RREF

Again it shows the solution of this system is $x = -2 + m$, $y = 3 - 2m$

and $z = m$.

$$4. \begin{cases} 0x + y + 2z = 3 \\ 2x + 0y - 2z = -4 \\ 6x + 3y + 0z = -3 \end{cases}$$

SYSTEM MATRIX (3x4)	SOLUTION SET	RREF (3x4)
$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 0 & -2 & -4 \\ 6 & 3 & 0 & -3 \end{bmatrix}$	$\begin{cases} x_1 = -2 + x_3 \\ x_2 = 3 - 2x_3 \\ x_3 = x_3 \end{cases}$	$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$(1,1)=0$$

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSM STD RREF MAIN BACK SYSM STD RREF

Again it shows the solution of this system is $x = -2 + m$, $y = 3 - 2m$

and $z = m$.

$$5. \begin{cases} \sqrt{2}x + (\sqrt{2} + 4)y + (\sqrt{2} + 8)z = \sqrt{2} + 12 \\ ex + (e + 3)y + (e + 6)z = e + 9 \\ (7 - \pi)x + (5 - \pi)y + (3 - \pi)z = 1 - \pi \end{cases}$$

SYSTEM MATRIX (3x4)	SOLUTION SET	RREF (3x4)
$\begin{bmatrix} 1.414 & 5.414 & 9.414 & 13.414 \\ 2.718 & 5.718 & 8.718 & 11.718 \\ 3.818 & 1.818 & -1.182 & -2.182 \end{bmatrix}$	$\begin{cases} x_1 = -2 + 1x_3 \\ x_2 = 3 - 2x_3 \\ x_3 = x_3 \end{cases}$	$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$(3,4) = -2.14159265...$$

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSM STD RREF MAIN BACK SYSM STD RREF

From the above solution, my conjecture is: for any 3×3 system of linear equations which have the constants in the order of arithmetic sequences. They must have a infinite solution of $x = -2 + m$, $y = 3 - 2m$ and $z = m$.
And $x + y + z = 1$

Therefore, I came out with a general system:

$$\begin{cases} ax + (a + d)y + (a + 2d)z = a + 3d & - (1) \\ bx + (b + e)y + (b + 2e)z = b + 3e & - (2) \\ cx + (c + f)y + (c + 2f)z = c + 3f & - (3) \end{cases}$$

$$(1) \times \frac{b+e}{a+d} : \frac{ab+ae}{a+d}x + (b+e)y + \frac{ab+ae+2bd+2de}{a+d}z = \frac{ab+ae+3bd+3de}{a+d} \quad - (4)$$

$$(4) - (2): \left(\frac{ab+ae}{a+d} - b \right)x + \left(\frac{ab+ae+2bd+2de}{a+d} - b - 2e \right)z = \frac{ab+ae+3bd+3de}{a+d} - b - 3e$$

$$\frac{ab+ae-ab-bd}{a+d}x + \frac{ab+ae+2bd+2de-ab-bd-2ae-2de}{a+d}z = \frac{ab+ae+3bd+3de-ab-bd-3ae-3de}{a+d}$$

$$(ae - bd)x + (bd - ae)z = 2bd - 2ae$$

$$x = \frac{2(bd-ae)-(bd-ae)z}{ae-bd}$$

$$x = z - 2$$

Substituting $x = z - 2$ into equation (1):

$$az - 2a + (a + d)y + az + 2dz = a + 3d$$

$$(a + d)y + 2(a + d)z = 3(a + d)$$

$$y = 3 - 2z$$

Let $z = m$

The solution of this system is $x = -2 + m$, $y = 3 - 2m$ and $z = m$

To test the validity of the conjecture,

$$\begin{cases} 8x + 6y + 4z = 2 \\ -12x - 9y - 6z = -3 \\ 20x + 15y + 10z = 5 \end{cases}$$

SYSTEM MATRIX (3x4)	SOLUTION SET	RREF (3x4)
$\begin{bmatrix} 8 & 6 & 4 & 2 \\ -12 & -9 & -6 & -3 \\ 20 & 15 & 10 & 5 \end{bmatrix}$	$\begin{cases} x_1 = 1/4 - 3/4 x_2 - 1/2 x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{cases}$	$\begin{bmatrix} 1 & .75 & .5 & .25 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$(1,1)=8$$

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSN STD RREF MAIN BACK SYSN STD RREF

There is limitation to this system: The ratio between the constants can't be the same for the three of the equations.

Now look at 4x4 system,

$$1. \begin{cases} x_1 + 4x_2 + 7x_3 + 10x_4 = 13 \\ 10x_1 + 11x_2 + 12x_3 + 13x_4 = 14 \\ -2x_1 - 4x_2 - 6x_3 - 8x_4 = -10 \\ -3x_1 + 1x_2 + 5x_3 + 9x_4 = 13 \end{cases}$$

SYSTEM MATRIX (4x5)	SOLUTION SET
$\begin{bmatrix} 1 & 4 & 7 & 10 & 13 \\ 10 & 11 & 12 & 13 & 14 \\ -2 & -4 & -6 & -8 & -10 \\ -3 & 1 & 5 & 9 & 13 \end{bmatrix}$	$\begin{cases} x_1 = -3 + x_3 + 2x_4 \\ x_2 = 4 - 2x_3 - 3x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{cases}$

$$(4,5)=13$$

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSN STD RREF

The solution of this system is $x_1 = -3 + m + 2n$, $x_2 = 4 - 2m - 3n$, $x_3 = m$

and $x_4 = n$.

$$2. \begin{cases} 5x_1 + 8x_2 + 11x_3 + 14x_4 = 17 \\ \frac{7}{4}x_1 + \frac{5}{4}x_2 + \frac{3}{4}x_3 + \frac{1}{4}x_4 = -\frac{1}{4} \\ -5x_1 - 3x_2 - x_3 + x_4 = 3 \\ \frac{7}{6}x_1 + x_2 + \frac{5}{6}x_3 + \frac{2}{3}x_4 = \frac{1}{2} \end{cases}$$

SYSTEM MATRIX (4x5)	SOLUTION SET
[5 8 11 14 -	x1 = -3 + x3 + 2x4
[1.75 1.25 .75 .25 -	x2 = 4 - 2x3 - 3x4
[-5 -3 -1 1 -	x3 = x3
[1.1... 1 .83... .66... -	x4 = x4

(1,1)=5

MAIN MODE CLR LOAD SOLVE MAIN MODE SYSM STO RREF

Again it shows the solution of this system to be $x_1 = -3 + m + 2n$,

$x_2 = 4 - 2m - 3n$, $x_3 = m$ and $x_4 = n$.

From the above solution, my conjecture is: for any 4×4 system of linear

equations which have the constants in the order of arithmetic sequences. They

must have a infinite solution of $x_1 = -3 + m + 2n$, $x_2 = 4 - 2m - 3n$, $x_3 = m$

and $x_4 = n$. And $x_1 + x_2 + x_3 + x_4 = -3 + m + n + 4 - 2m - 3n + m + n = 1$

Now look at 5×5 system,

$$1. \begin{cases} x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 = 11 \\ 9x_1 + 10x_2 + 11x_3 + 12x_4 + 13x_5 = 14 \\ -2x_1 - 4x_2 - 6x_3 - 8x_4 - 10x_5 = -12 \\ -3x_1 + 1x_2 + 5x_3 + 9x_4 + 13x_5 = 17 \\ 7x_1 + 5x_2 + 3x_3 + 1x_4 - 1x_5 = -3 \end{cases}$$

SYSTEM MATRIX (5×6)					SOLUTION SET				
[1]	1	3	5	7	-	x1	=	-4+x3+2x4+3x5	
[9]	9	10	11	12	-	x2	=	5-2x3-3x4-4x5	
[-2]	-2	-4	-6	-8	-	x3	=	x3	
[-3]	-3	1	5	9	-	x4	=	x4	
[7]	7	5	3	1	-	x5	=	x5	
(1,1)=1									
[MAIN]MODE[CLR]LOAD[SOLVE][MAIN]MODE[SYSM] STO [RREF]									

This shows the solution of this system is $x_1 = -4 + m + 2n + 3v$

$x_2 = 5 - 2m - 3n - 4v, x_3 = m, x_4 = n$ and $x_5 = v$.

And $x_1 + x_2 + x_3 + x_4 + x_5 = -4 + t + 2u + 3v + 5 - 2t - 3u - 4v + t + u + v = 1$

$$2. \begin{cases} 2x_1 + 5x_2 + 8x_3 + 11x_4 + 15x_5 = 18 \\ \frac{9}{4}x_1 + \frac{5}{4}x_2 + \frac{1}{4}x_3 - \frac{3}{4}x_4 - \frac{7}{4}x_5 = -\frac{11}{4} \\ -5x_1 - 3x_2 - x_3 + x_4 + 3x_5 = 5 \\ \frac{7}{6}x_1 + x_2 + \frac{5}{6}x_3 + \frac{2}{3}x_4 + \frac{1}{2}x_5 = \frac{1}{3} \\ x_1 + 6x_2 + 11x_3 + 16x_4 + 21x_5 = 26 \end{cases}$$

SYSTEM MATRIX (5×6)					SOLUTION SET				
[2]	2	5	8	11	-	x1	=	-1+x3+2x4	
[2.25]	2.25	1.25	.25	-.75	-	x2	=	1-2x3-3x4	
[-5]	-5	-3	-1	1	-	x3	=	x3	
[1.1...]	1.1...	1	.83...	.66...	-	x4	=	x4	
[1]	1	6	11	16	-	x5	=	1	
(5,1)=1									
[MAIN]MODE[CLR]LOAD[SOLVE][MAIN]MODE[SYSM] STO [RREF]									

Again this shows the solution of this system is $x_1 = -4 + m + 2n + 3v$

$x_2 = 5 - 2m - 3n - 4v, x_3 = m, x_4 = n$ and $x_5 = v$.

From the above solution, my conjecture is: for any 5×5 system of linear equations which have the constants in the order of arithmetic sequences. They

must have infinite solutions of $x_1 = -4 + m + 2n + 3v$

, $x_2 = 5 - 2m - 3n - 4v$, $x_3 = m$, $x_4 = n$ and $x_5 = v$.

And

$x_1 + x_2 + x_3 + x_4 + x_5 = -4 + t + 2u + 3v + 5 - 2t - 3u - 4v + t + u + v = 1$

Now look at the $n \times n$ system of equations,

When $n \geq 2$, there are infinite solutions. The general solution is

$$x_1 = -(n-1) + x_3 + 2x_4 + \dots + (n-2)x_n$$

$$x_2 = n - 2x_3 - 3x_4 - \dots - (n-1)x_n$$

$$x_3 = x_3$$

$$x_4 = x_4$$

...

$$x_n = x_n$$

And $x_1 + x_2 + x_3 + x_4 + \dots + x_n = 1$.

There is limitation to this system: The ratio between the constants can't be the same for the all of the equations.

Part B

Consider the 2×2 system of linear equations $\begin{cases} x + 2y = 4 \\ 5x - y = \frac{1}{5} \end{cases}$

I find that the constants follow an order of geometric sequences,

For the first equation: $x + 2y = 4$, the constants (1, 2, 4) starts with 1 and has a common ratio of 2.

For the second equation: $5x - y = \frac{1}{5}$, the constants (5, -1, $\frac{1}{5}$) starts with 5 and has a common ratio of $-\frac{1}{5}$.

Rewrite the two equations in the form of $y = ax + b$:

First equation: $2y = 4 - x$

$$y = -\frac{1}{2}x + 2$$

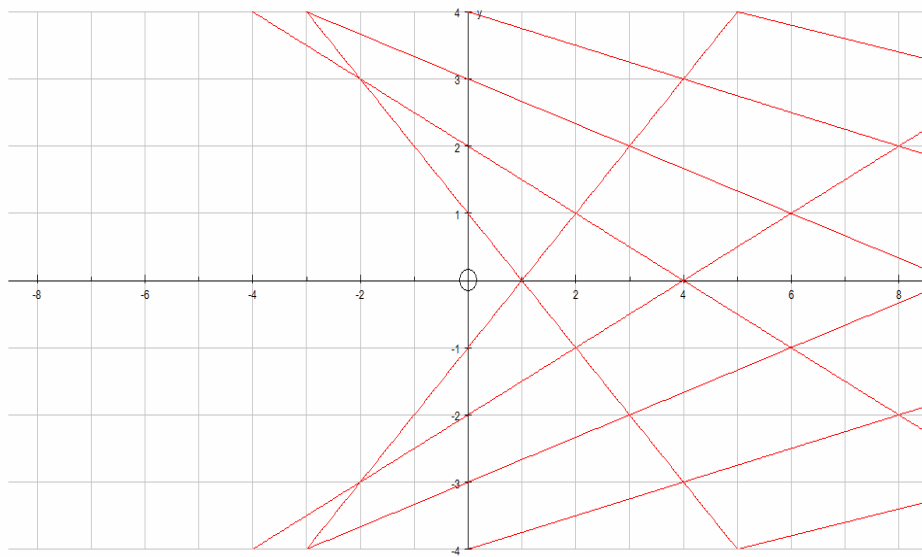
Second equation: $-y = \frac{1}{5} - 5x$

$$y = 5x - \frac{1}{5}$$

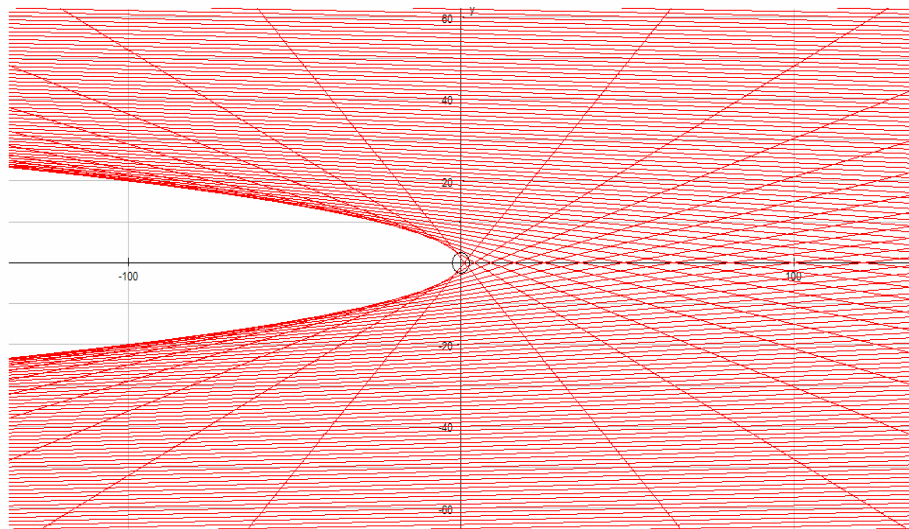
The relationship between constants a and b is $a = -\frac{1}{b}$

By using autograph to draw a line which $y = -\frac{1}{b}x + b$. And vary the constant b from

-100 to 100. So there are 201 lines on the same set of axes.



By zooming out the graph, we would be able to see,



It shows a U-shape region on the left of the graph which no line intersects.

The general 2×2 system which the constants of each equations is a geometric sequence is,

$$\begin{cases} ax + ary = ar^2 \\ bx + bsy = bs^2 \end{cases}$$

a is the common ratio of equation 1 and b is the common ratio of equation 2.

$$\begin{cases} ax + ary = ar^2 & - (1) \\ bx + bsy = bs^2 & - (2) \end{cases}$$

$$(2) \times \frac{ar}{bs} : \frac{ar}{s}x + ary = ars \quad - (3)$$

$$(3) - (1) : \left(\frac{ar}{s} - a \right) x = ars - ar^2$$

$$\frac{a(r-s)}{s} x = ar(s-r)$$

$$x = -rs$$

Substituting $x = -rs$ into equation (1): $-ars + ary = ar^2$

$$y = \frac{ar^2 + ars}{ar}$$

$$y = r + s$$

There is a unique solution of this 2×2 system of equations. The general solution is

$$x = -rs \quad \text{and} \quad y = r + s .$$

To test the validity of the solution,

$$1. \begin{cases} x + 3y = 9 \\ 6x - y = \frac{1}{6} \end{cases}$$

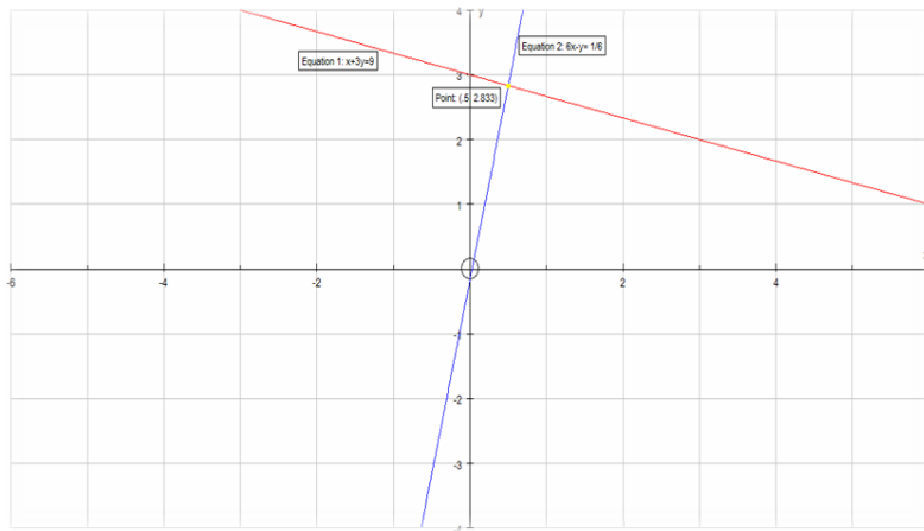
From this system, I observed the common ratio for the first equation is $r = 3$ and

common ratio for the second equation is $s = -\frac{1}{6}$.

The general solution state that $x = -rs$ and $y = r + s$.

Therefore, $x = -3 \times \left(-\frac{1}{6}\right) = 0.5$

$$y = 3 + \left(-\frac{1}{6}\right) \approx 2.833$$



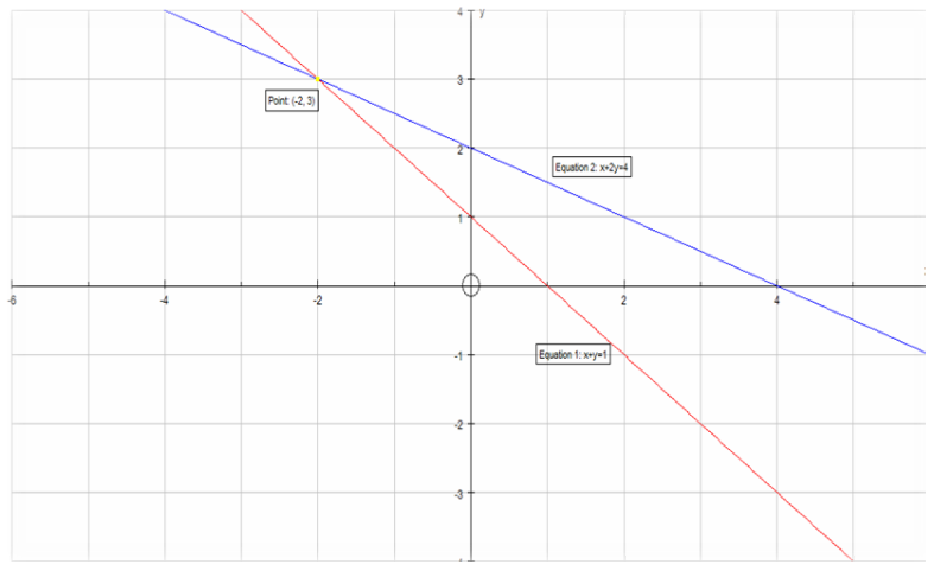
I used autograph to find the intersection point of the lines is at (0.5, 2.833).

$$2. \begin{cases} x + y = 1 \\ x + 2y = 4 \end{cases}$$

For this system, the common ratio of the first equation is $r = 1$ and common ratio of the second equation is $s = 2$.

$$x = -1 \times 2 = -2$$

$$y = 1 + 2 = 3$$



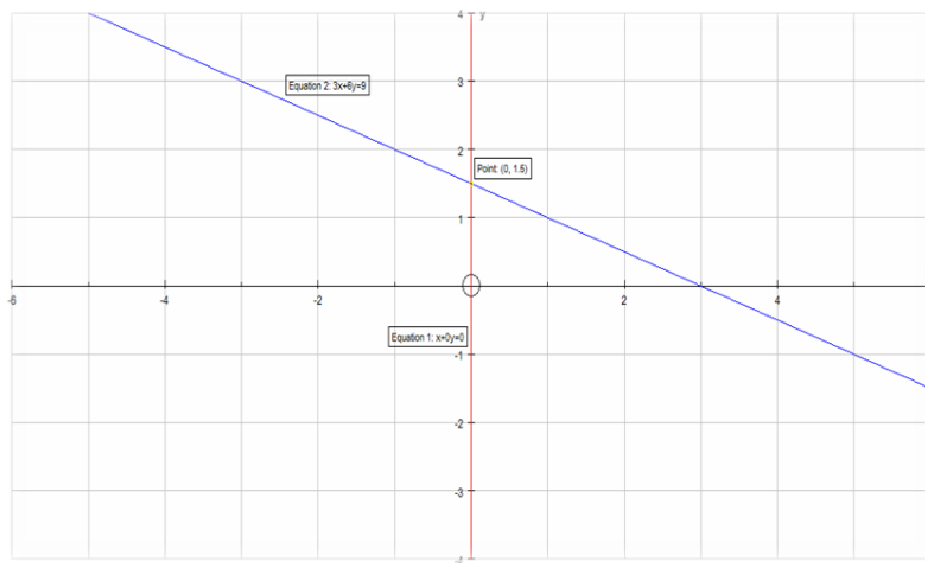
I used autograph to find the intersection point of the lines is at $(-2, 3)$.

$$3. \begin{cases} x + 0y = 0 \\ x + 3y = 6 \end{cases}$$

For this system the common ratio of the first equation is $r = 0$ and common ratio of the second equation is $s = 3$.

$$x = -0 \times 3 = 0$$

$$y = 0 + 3 = 3$$



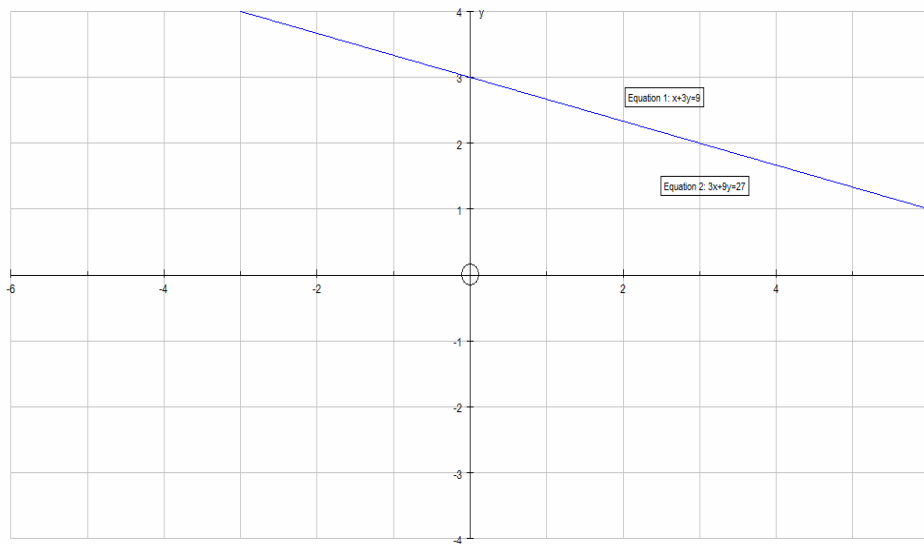
There is limitation to this system: The common ratio of the two equations can't be 0.

$$4. \begin{cases} x + 3y = 9 \\ 3x + 9y = 27 \end{cases}$$

For this system, the common ratio of the first equation is $r = 3$ and common ratio of the second equation is $s = 3$.

$$x = -3 \times 3 = -9$$

$$y = 3 + 3 = 6$$



I used autograph, I observed that there are infinite solutions when two lines overlapped.

There is another limitation to this system: The common ratio of the two equations can't be the same.

Conclusion

Part A

For any 2×2 system of linear equations which have the constants in the order of arithmetic sequences. They must have a unique solution of $x = -1, y = 2$. However the limitation to this system is the ratio between the constants can't be the same for both of the equations.

For any $n \times n$ system ($n > 2$) of linear equations which have the constants in the order of arithmetic sequences. They must have infinite solutions. However the limitation to this system is the ratio between the constants can't be the same for the three of the equations. However the limitation to this system is the ratio between the constants can't be the same for the all of the equations.

Part B

The 2×2 system of linear equations which have the constants in the order of geometric sequences. They must have a unique value. However, the limitations would be: The common ratio of the two equations can't be the same or equal to 0.