Introduction

In this task, we are going to show how any two vectors are at right angles to each other by using patterns with vectors.

This will be achieved by firstly plotting two vectors and discussing any similar ities and difference between them. Then two randomly selected points will be chosen and a line drawn through them. Different ways of representing this vector will be explained. Finally, the general form of a vector equation will be used to determine and prove how two vectors are perpendicular to each other.

The use of the Geogebra computer software will be used to graph each vector although the methods used will be explained.

Vectors are used to display the magnitude and direction for a path of an object. Examples include velocity, acceleration, force, displayment, momentum and weight. A vector can be written in 3 different forms:

Velocity vector: $\vec{r} = \binom{x}{y} = \binom{a}{b} + t\binom{c}{d}$

Parametric equation: x= a+ct y=b+dt

Cartesian equation: $t = \frac{x-a}{c} = \frac{y-b}{d}$

Vector can be drawn with an initial point (a, b) and then a direction vector $t\binom{c}{d}$. The line that goes through the set of points has an arrow at the end to show its direction.

Two vectors are at right angles (perpendicular) to each other when they satisfy the equation $\vec{a} \bullet \vec{b} = 0$

Results/ Analysis

By plotting the vector equation: $\vec{r} = \binom{x}{y} = \binom{5}{-3} + t\binom{1}{3}$,

After plotting this vector equation, the first step is to find various points in time. That is to substitute values for (t) to find the coordinates of the object.

t = 0, $\vec{r} = \binom{x}{y} = \binom{5}{-3} + 0\binom{1}{3}$,

$\vec{r} = \binom{x}{y} = \binom{5}{-3}$,

t = 1, $\vec{r} = \binom{x}{y} = \binom{5}{-3} + 1\binom{1}{3}$,

$\vec{r} = \binom{x}{y} = \binom{6}{0}$,

t = 2, $\vec{r} = \binom{x}{y} = \binom{5}{-3} + 2\binom{1}{3}$,

$\vec{r} = \binom{x}{y} = \binom{7}{3}$, t = 3, $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$,

t = 4, $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + 4\begin{pmatrix} 1 \\ 3 \end{pmatrix}$,

$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 9 \end{pmatrix}$

From these points for t$\in R$ it is now possible to plot them on a set of Cartesian axes. Thi s was done on the Geogbra computer program. It follows the idea of simply points on the x, y axes when based at the origin.

A graph can be created by plotting those vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ with the equation $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \end{pmatrix}$. For $t \in real\ number$.

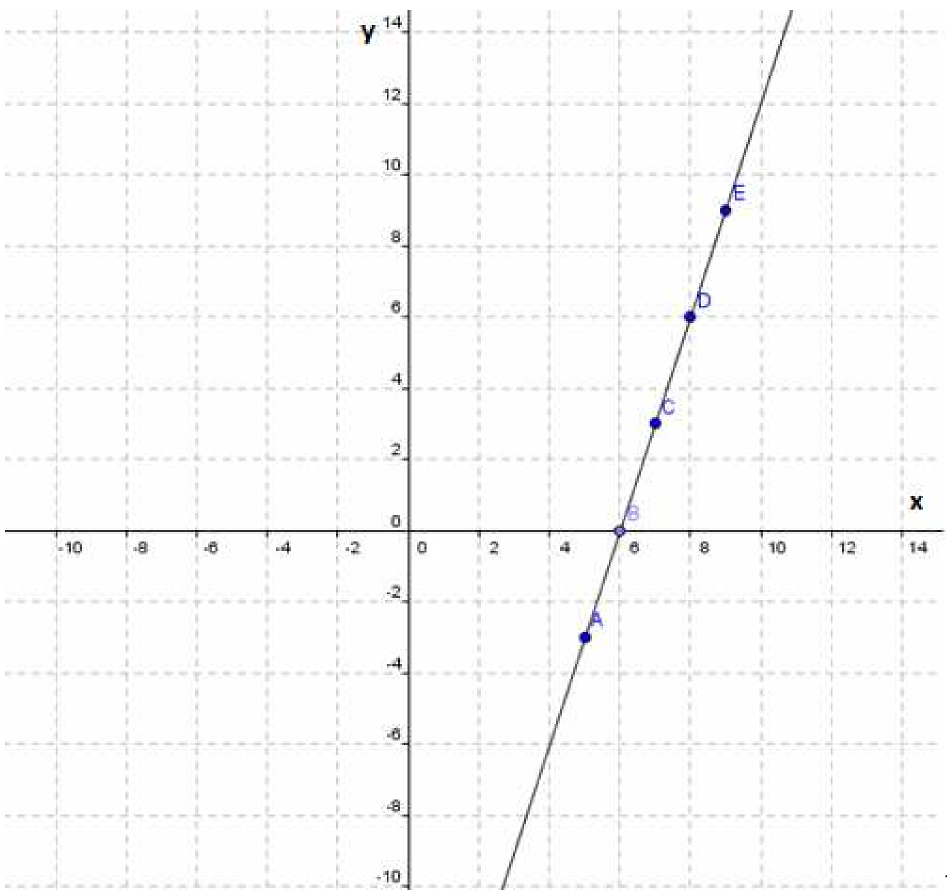


figure 1.

It can be seen from the graph that the 5 points that were plotted are linear and move in the same direction. The initial point being (5, -3) and each successive point moving in the direction of 1 across and 3 up according to the direction vector $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

To determine the parametric equation for this vector, we split the vector equation into the x and y components.

$\therefore x(t) = 5 + t$ and $y(t) = -3 + 3t$

By knowing the value of t, this can be substituted into the 2 parametric equation and the coordinate for (x, y) can be found.

Now, we are going to plot another vector equation: $\vec{r} = \binom{x}{y} = \binom{-2}{6} + t\binom{4}{-3}$ for comparing with the first equation.

After plotting this vector equation, the first step is to find various points in time. That is to substitute values for (t) to find the coordinates of the object.

t = 0, $\vec{r} = \binom{x}{y} = \binom{-2}{6} + 0\binom{4}{-3}$,

$\vec{r} = \binom{x}{y} = \binom{-2}{6}$,

t = 1, $\vec{r} = \binom{x}{y} = \binom{-2}{6} + 1\binom{4}{-3}$,

$\vec{r} = \binom{x}{y} = \binom{2}{3}$,

t = 2, $\vec{r} = \binom{x}{y} = \binom{-2}{6} + 2\binom{4}{-3}$,

$\vec{r} = \binom{x}{y} = \binom{6}{0}$,

t = 3, $\vec{r} = \binom{x}{y} = \binom{-2}{6} + 3\binom{4}{-3}$,

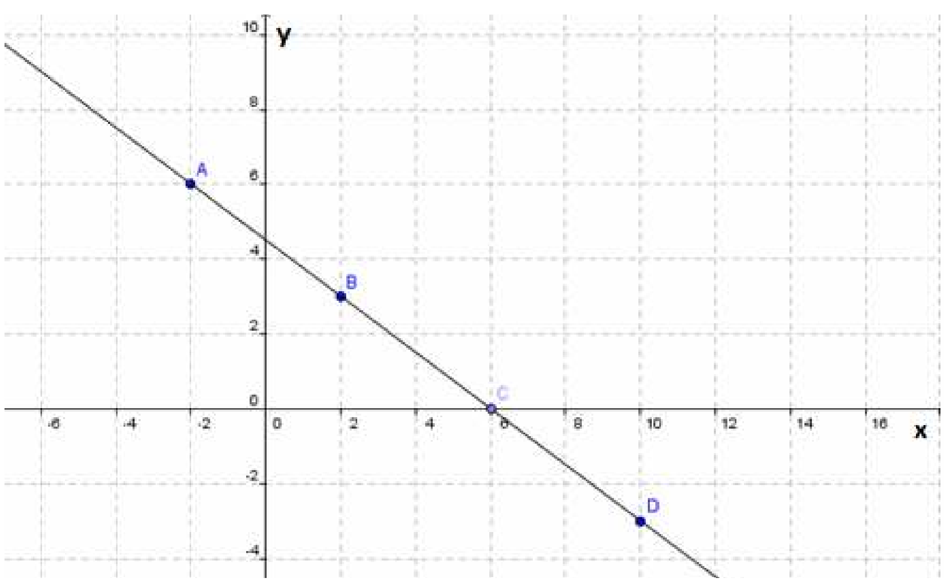
$\vec{r} = \binom{x}{y} = \binom{10}{-3}$,

t = 4, $\vec{r} = \binom{x}{y} = \binom{-2}{6} + 4\binom{4}{-3}$,

$\vec{r} = \binom{x}{y} = \binom{14}{-6}$

Again from these points for t $\in R$ it is now possible to plot them on a set of Cartesian axes. This was done on the Geogbra computer program. It follows the idea of simply points on the x, y axes when based at the origin.

Another graph can be created by plotting those vectors $\binom{x}{y}$ with the equation $\vec{r} = \binom{x}{y} = \binom{-2}{6} + t\binom{4}{-3}$. For $t \in real\ number$.



By comparing those two equations, we are able to see there is a pattern that although the value of (t) changes, the vectors $\vec{r}$ are collinear. This means they lie on a same straight line with the same slope.

In the first equation, the slope of it is lying on the positive side, but the second slope is lying on the negative side. It is considering to their vectors direction 1. $\begin{pmatrix}1\\3\end{pmatrix}$ and 2. $\begin{pmatrix}4\\-3\end{pmatrix}$

In this time, we chose the point $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}10\\-3\end{pmatrix}$ from the vector equation: $\vec{r} = \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2\\6\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix}$. Where r = 1, r = 3. After that, we draw the line **L** that those two point can pass through it.

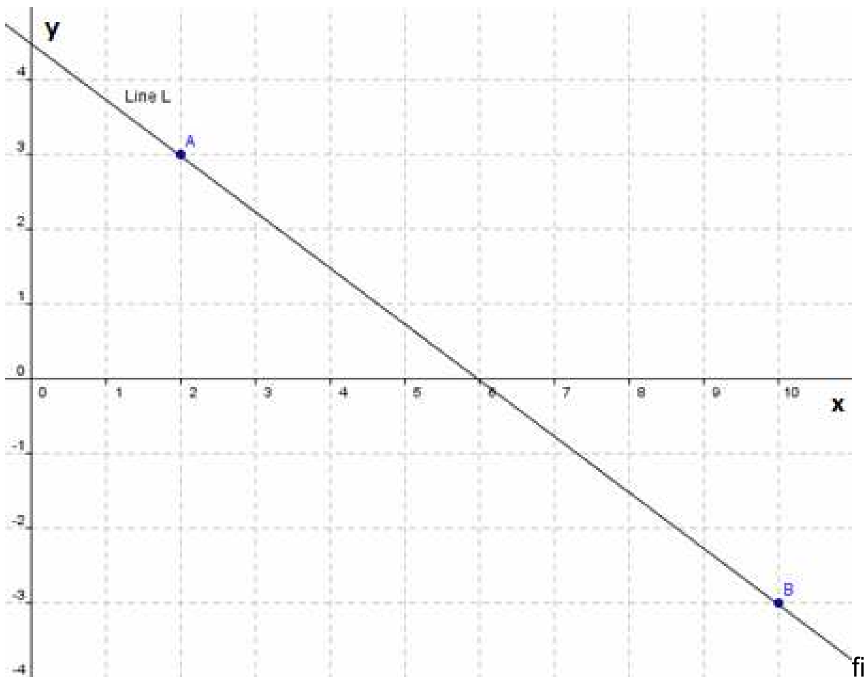


figure 3

The slope of these two point is same as the slope of the equation $\vec{r} = \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2\\6\end{pmatrix} + t\begin{pmatrix}4\\-3\end{pmatrix}$.

(Refer to figure 2)

We assume that the vector $\vec{r}$ is $\overrightarrow{AB}$ so

$\overrightarrow{AB}$ = t**d {distance = time × speed}**

Now by drawing the diagram

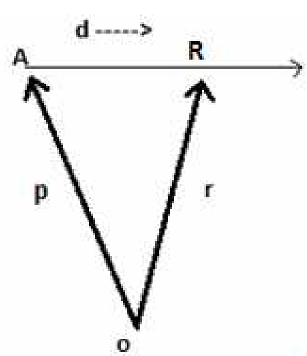


Figure 4 $r = \overrightarrow{OA} + \overrightarrow{AR}$

$$\therefore \boldsymbol{r} = \boldsymbol{p} + t\boldsymbol{d} \qquad t \geq 0$$

We are trying to find out that all the equation of ***L*** can explain by the general formula above. Therefore, we can use another two point $\binom{x}{y} = \binom{6}{0}$ and $\binom{x}{y} = \binom{8}{6}$ from the vector equation: $\vec{r} = \binom{x}{y} = \binom{5}{-3} + t\binom{1}{3}$. Where r = 1, r = 3. Another graph can be dr awn below.

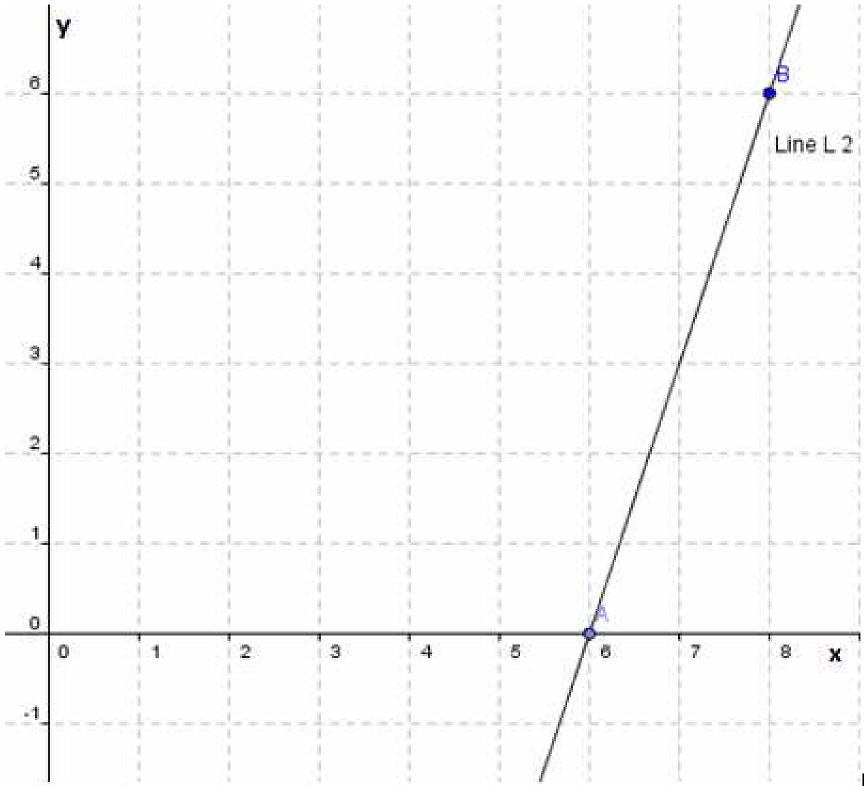


Figure 5

Also, the slope of these two point is same as the slope of the equation $\vec{r} = \binom{x}{y} = \binom{5}{-3} + t\binom{1}{3}$. (Refer to figure 1)

Again, by using the diagram

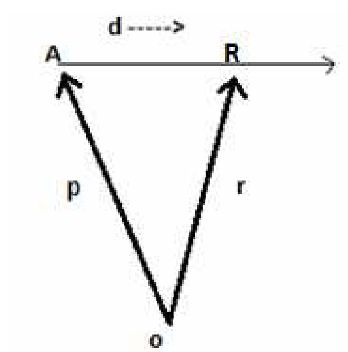


Figure 6

We can find out that the formula $\boldsymbol{r} = \boldsymbol{p} + t\boldsymbol{d}$ $t \geq 0$ is also fix on this two point $\binom{x}{y} = \binom{6}{0}$ and $\binom{x}{y} = \binom{8}{6}$ from the vector equation: $\vec{r} = \binom{x}{y} = \binom{5}{-3} + t\binom{1}{3}$.

Therefore, $\boldsymbol{r} = \boldsymbol{p} + t\boldsymbol{d}$ is the general formula for determining the vector $\vec{r}$ with AB two points.

The line **L** is every all points which extend along a directio n vector, in other words all scalar multiples of a direction vector, which extend from a particular point in space defined by its own direction vector.

r or $\binom{x}{y} = \binom{h}{k} + t\binom{a}{b}$

t is a scalar multiple of arbitrary value, any value in the set of all real numbers. $\binom{a}{b}$ is a direction vector describing the lines direction. It and its multiples are being vector added to the constant $\binom{h}{k}$, which is a vector describing the point $\binom{h}{k}$ which the vector passes through.

If a line *L* passes through a point U (*h*, *k*) in a direction $\boldsymbol{d} = \begin{pmatrix} a \\ b \end{pmatrix}$. Then we can create a vector equation R $\binom{x}{y} = \binom{h}{k} + t\binom{a}{b}$ which is the general position vector for point R on line **L.**

By looking to the diagram,

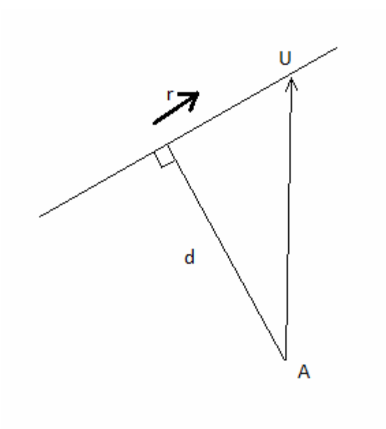


Figure 7

Now, changing the velocity equation R $\binom{x}{y} = \binom{h}{k} + t\binom{a}{b}$ in the form $cx + dy + e = 0$:

$$x = h + at \ \rightarrow \ t = \frac{x-h}{a} \qquad\qquad y = k + bt \ \rightarrow \ t = \frac{y-k}{b}$$

$$\frac{x-h}{a} = \frac{y-k}{b} \ \rightarrow \ b(x-h) = a(y-k) \ \rightarrow \ bx - bh = ay - ak$$

$$\therefore bx - ay + ak - bh = 0$$

$\vec{L} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix} + t\begin{pmatrix} a \\ b \end{pmatrix}$. Line L passed through a point U (h, k) in a direction $\mathrm{d} = \begin{pmatrix} a \\ b \end{pmatrix}$,

Suppose a plane in space has normal vector $\boldsymbol{n} = \begin{pmatrix} -b \\ a \end{pmatrix}$

And that is includes the fixed point $\mathrm{A\ (u1,k1)}$. U(u, k) is any other point in the plane.

Now $\overrightarrow{AU}$ is perpendicular to **n**

$\therefore \mathbf{n} \bullet \overrightarrow{AU} = 0$

$\therefore \begin{pmatrix} -b \\ a \end{pmatrix} \bullet \begin{pmatrix} u1 - u \\ k1 - k \end{pmatrix} = 0$

$\therefore -\mathrm{b(u1 - u) + a(k1 - k) = 0}$

$\therefore -\mathrm{bu1 + ak1 = -bu + ak}$ where the RHS is a constant.

$\mathbf{n} \bullet \overrightarrow{AU} = 0$ could also be written as $\mathbf{n} \bullet (\mathbf{r} - \mathbf{u}) = 0$, which implies $\mathbf{r} \bullet \mathbf{n} = \mathbf{u} \bullet \mathbf{n}$.

If a plane has normal vector $\mathbf{n} = \begin{pmatrix} -b \\ a \end{pmatrix}$ and pass through $\mathrm{(u1, k1)}$

then is has equation -bu + ak = -bu1 + ak1 = d,   where d is a constant. This is called the **Cartesian equation of the plane.**

Now, we are trying to sub $\mathbf{d} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ into $\mathbf{d} = \begin{pmatrix} a \\ b \end{pmatrix}$, $\mathbf{n} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ and then provide the thoery above.

$\mathbf{n} \bullet \overrightarrow{AU} = 0$

$\therefore \begin{pmatrix} -1 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 0$ where t = 1