IB Math SL Portfolio Type 1

General Introduction:

In its simplest terms, a logarithm is an exponent. It is an exponent needed to produce a given number from a specific base. It is written in the form $\log_g h=j$, which denotes $g^h=j$ (g to the power of h equals j).

LOGARITHM BASES

Consider the following sequences. Write the next two terms of each sequence.

$\log_2 8$, $\log_4 8$, $\log_8 8$, $\log_{16} 8$, $\log_{32} 8$, … $\log_{64} 8$, $\log_{128} 8$, …

$\log_3 81$, $\log_9 81$, $\log_{27} 81$, $\log_{81} 81$, … $\log_{243} 81$, $\log_{729} 81$, …

$\log_5 25$, $\log_{25} 25$, $\log_{125} 25$, $\log_{625} 25$, … $\log_{3,125} 25$, $\log_{15,625} 25$, …

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$\log_m m^k$, $\log_{m^\wedge 2} m^k$, $\log_{m^\wedge 3} m^k$, $\log_{m^\wedge 4} m^k$, … $\log_{m^\wedge 5} m^k$, $\log_{m^\wedge 6} m^k$, …

Find an expression for the n^{th} term of each sequence. Write your expressions in the form p/q, where p and q are integers. Justify your answers using technology.

The first sequence can be otherwise written:

$\log_{2^\wedge 1} 2^3$, $\log_{2^\wedge 2} 2^3$, $\log_{2^\wedge 3} 2^3$, $\log_{2^\wedge 4} 2^3$, $\log_{2^\wedge 5} 2^3$, …

Here, I noticed a pattern: the base of each logarithm is 2^n. Using this knowledge and the concepts of the change of base rule and verifying my theories with a GDC calculator, I developed the expression shown here by evaluating each term.

(Each term to be evaluated can be checked by using a GDC calculator. For example, $\log_2 8$ *reads "the exponent of 2 that yields 8". As these are relatively small numbers, mental math can be used to evaluate the statement, which simplifies to 3. To verify this response, one could type 2^3 in their calculator and press "enter" or "solve". One could also use the "solver" function on a GDC calculator to check their math by simply pressing the "math" key, then pressing "solve". The original statement can be written $2^x=8$, but in order to input this into the solver function, it needs to be set to zero. So, one could input $2^x - 8=0$ and press solve to check their answer).*

The first sequence looked like this:

$\log_{2^1} 2^3 = 3/1$
$\log_{2^2} 2^3 = 3/2$
$\log_{2^3} 2^3 = 3/3$
$\log_{2^4} 2^3 = 3/4$
$\log_{2^5} 2^3 = 3/5$
:
:
$\log_{2^n} 2^3 = 3/n$

The nth term of this sequence can be expressed as 3/n.

I completed the same steps to discover the general expressions of each of the following sequences.

$\log_3 81$, $\log_9 81$, $\log_{27} 81$, $\log_{81} 81$, ... can be written $\log_{3^1} 3^4$, $\log_{3^2} 3^4$, $\log_{3^3} 3^4$, $\log_{3^4} 3^4$, ...

$\log_{3^1} 3^4 = 4/1$
$\log_{3^2} 3^4 = 4/2$
$\log_{3^3} 3^4 = 4/3$
$\log_{3^4} 3^4 = 4/4$
:
:
$\log_{3^n} 3^4 = 4/n$

The nth term of the second sequence can be expressed as 4/n.

$\log_5 25$, $\log_{25} 25$, $\log_{125} 25$, $\log_{625} 25$, ... can be written $\log_{5^1} 25$, $\log_{5^2} 25$, $\log_{5^3} 25$, $\log_{5^4} 25$, ...

$\log_{5^1} 5^2 = 2/1$
$\log_{5^2} 5^2 = 2/2$
$\log_{5^3} 5^2 = 2/3$
$\log_{5^4} 5^2 = 2/4$
:
:
$\log_{5^n} 5^2 = 2/n$

The nth term of the third sequence can expressed as 2/n.

The same rules apply for the fourth and final sequence.

$\log_{m^1} m^k$, $\log_{m^2} m^k$, $\log_{m^3} m^k$, $\log_{m^4} m^k$, ...

$\log_{m^1} m^k = k/1$
$\log_{m^2} m^k = k/2$
$\log_{m^3} m^k = k/3$
$\log_{m^4} m^k = k/4$
:
:
$\log_{m^n} m^k = k/n$

The n^{th} term of the fourth sequence can expressed as k/n.

Now, calculate the following, giving your answers in the form p/q, where p and q are both integers.

$\log_4 64$,	$\log_8 64$,	$\log_{32} 64$	3/*1*, 2/*1*, 6/*5*
$\log_7 49$,	$\log_{49} 49$,	$\log_{343} 49$	2/*1*, 2/2, 2/3
$\log_{1/5} 125$,	$\log_{1/125} 125$,	$\log_{1/625} 125$	*3/*-1, *3/-3*, *3/-4*
$\log_8 512$,	$\log_2 512$,	$\log_{16} 512$	*3/*1, *9/*1, *9/4*

Describe how to obtain the third answer in each row from the first two answers.

First, I took the lowest prime base possible of each.

$\log_{2^2} 64$,	$\log_{2^3} 64$,	$\log_{2^5} 64$
$\log_{7^1} 49$,	$\log_{7^2} 49$,	$\log_{7^3} 49$
$\log_{(1/5)^1} 125$,	$\log_{(1/5)^3} 125$,	$\log_{(1/5)^4} 125$
$\log_{2^3} 512$,	$\log_{2^1} 512$,	$\log_{2^4} 512$

In doing this, I noticed a pattern : the exponent of the base of the answer in the third column is simply the sum of the exponents of the bases in the first two columns when each base is in its lowest prime form. ($expbase_1$ + $expbase_2$ = $expbase_3$)

$\log_{2^2} 64$,	$\log_{2^3} 64$,	$\log_{2^5} 64$	(2+3=5)
$\log_{7^1} 49$,	$\log_{7^2} 49$,	$\log_{7^3} 49$	(1+2=3)
$\log_{(1/5)^1} 125$,	$\log_{(1/5)^3} 125$,	$\log_{(1/5)^4} 125$	(1+3=4)
$\log_{2^3} 512$,	$\log_{2^1} 512$,	$\log_{2^4} 512$	(3+1=4)

Create two more examples that fit the pattern above.

$\log_4 256$,	$\log_{16} 256$,	$\log_{64} 256$	
(4)	*(2)*	*(4/3)*	
$\log_{2^{\wedge}2} 256$	$\log_{2^{\wedge}4} 256$	$\log_{2^{\wedge}6} 256$	*(2+4=6)*

$\log_9 729$,	$\log_3 729$,	$\log_{27} 729$	
(3)	*(6)*	*(2)*	
$\log_{3^{\wedge}2} 729$	$\log_{3^{\wedge}1} 729$	$\log_{3^{\wedge}3} 729$	*(2+1=3)*

Let $\log_a x = c$ and $\log_b x = d$. Find the general statement that expresses $\log_{ab} x$, in terms of c and d.

First,

$\log_a x=c$ *can otherwise be written* $x= a^c$, and $\log_b x=d$ *can otherwise be written* $x= b^d$

Next, a and b can be isolated.

$$x^{1/c}=a \qquad x^{1/d}=b$$

They can then be multiplied, as the goal is to find $\log_{ab} x$.

$$x^{1/c+1/d}=ab$$

The variable x can now be isolated.

$$x=(ab)^{1/(1/c+1/d)}$$

The irrational fraction in the exponent can be eradicated my multiplying by 1, which can be written as cd/cd.

$$x=(ab)^{cd/(c+d)}$$

This expression can be rewritten now, as

$$\log_{ab} x= cd/(c+d)$$

Thus, the general statement for $\log_{ab} x$ in terms of c and d is $\log_{ab} x= cd/(c+d)$.

Test the validity of your general statement using other values of *a, b, and x.*

All right, suppose a= 3, b=9, and x=81. *The expression still holds true.*

$\log_a x=c$	$\log_b x=d$
$\log_3 81=4$	$\log_9 81=2$
c=4	d=2

$\log_{ab} x= cd/(c+d)$
$\log_{27} 81= 8/6= 4/3$
or
$27^{4/3}=81$

Or suppose a=8, b=4, and x= 64. *The expression is consistent.*

$\log_a x=c$	$\log_b x=d$
$\log_8 64=4$	$\log_4 64=2$
c=4	d=3

$\log_{ab} x= cd/(c+d)$
$\log_{32} 64= 6/5$
or
$32^{6/5}=64$

Discuss the scope and/or limitations of *a, b* and *x.*

The variables a *and* b *must be integers greater than zero and not equal to one. If a or b were equal to one, then c and d would be rendered undefined.*
(For example, if a=1 and x=3, *the given statement* $\log_a x=c$ *would be undefined for* c *because there is no exponent that can be raised to make one equal three. One to any power always equals one.)*
This is the same for b. *If it were equal to one,* d *would be undefined.*
They cannot be negative, as this does not work with the equations.
The variable c *cannot equal negative* d, *as this would yield a zero in the denominator of the equation, which would make it undefined.*

Explain how you arrived at your general statement.

I explained it step by step above, but generally I just used my knowledge of logarithm and exponent rules to mold and simplify the information I was given. For example, I knew that $\log_a x=c$ *was the same as writing* $x= a^c$, *a point I explained in the brief introduction, which helped me not only with this portion but the portfolio in its entirety.*