International Baccalaureate

IB Pre-Calculus

Portfolio

Logarithm Bases

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General Information: Logarithms

A logarithm is an exponent and can be described as the exponent needed to produce a certain number.

> For Example: $2^3=8$, from this you would say that 3 is the logarithm of 8 with base of 2 ($\log_2 8$). 2 is written as a subscript, and 3 is the exponent to which 2 must be raised to produce 8.

The formula or definition that is used in logarithms is: ***$\log_b x=e$, so $b^e=x$.*** So base(b) with exponent(e) produces x. So from the example, 2 is the base(b), 8 is x (the number produced), and the exponent(e) is 3. So, $2^3=8$.

Logarithms in Sequences: Introduction

Since logarithms can be solved ($\log_2 8=3$) to form numbers, this means logarithms are just another way to represent a number; and since numbers can be in sequences, so can logarithms. Given this, consider the following sequences:

1) $\log_2 8$, $\log_4 8$, $\log_8 8$, $\log_{16} 8$, $\log_{32} 8$, …
2) $\log_3 81$, $\log_9 81$, $\log_{27} 81$, $\log_{81} 81$, …
3) $\log_5 25$, $\log_{25} 25$, $\log_{125} 25$, $\log_{625} 25$, …
4) **$\log_m m^k$, $\log_{m^2} m^k$, $\log_{m^3} m^k$, $\log_{m^4} m^k$**

Sequence 4 is the general statement that is used to reflect the previous sequences. So, consider the first sequence; it starts out with a base(m) of 2, the second term has a base(m) of 4, which is equal to 2^2, then the third term has a base(m) of 8, which is equal to 2^3, etc. This can be broken down into a basic sequence just contain the bases: 2, 4, 8, 16, 32, …; this can also be written as 2, 2^2, 2^3, 2^4, 2^5,…; this idea can be applied to each of the sequences represented above.

Given this, the next two or more terms of each sequence can be found. In the first sequence, the next three terms are: $\log_{64} 8$, $\log_{128} 8$, and $\log_{256} 8$. In the second sequence the next four terms are: $\log_{243} 81$, $\log_{729} 81$, $\log_{2187} 81$, and $\log_{6561} 81$. In the third sequence, the next four terms are: $\log_{3125} 25$, $\log_{15625} 25$, $\log_{78125} 25$, and $\log_{390625} 25$. In the fourth sequence, the next four terms are: $\log_m{}^5 m^k$, $\log_m{}^6 m^k$, $\log_m{}^7 m^k$, and $\log_m{}^8 m^k$.

Logarithms in Sequences: Forming Expressions/ Natural Logarithms

In logarithmic sequences, expressions can be found in order to justify how any term in a sequence is found. This basic formula is also used to find the following terms that haven't been found yet. Any term in a sequence is defined as the n^{th} term, because "n" can represent any number. The expression used was in the form of p/q, where p,q εZ. "p" is equal to lnb, and "q" is equal to lna from the expression $\log_a b$=ln a/ln b, where ln stands for natural logarithm.

The system of natural logarithms uses the number called "*e*" as its base. *e* is the base used in natural logarithms in calculus. It is called the "natural" base because of certain technical considerations ($\ln_e$=1)

e^x has the simplest derivative. e can be calculated from the following series involving factorials:

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ . \ \ . \ \ .$$

e is an irrational number, whose decimal value is approximately 2.71828182845904.

To indicate the natural logarithm of a number, we use the notation "ln."

ln x means $\log_e x$ (taken from: http://www.themathpage.com/aPreCalc/logarithms.htm#proof).

After understanding natural logarithms, it makes it easier to understand the expression used when dealing with the sequences mentioned above. The expression used to find the n^{th} term (t_n) of the first sequence is $t_n = \log_{2^n} 8$ = ln 8/ ln 2^n. The expression used to find the n^{th} term ($t_{n)}$ of the second sequence is $t_n = \log_{3^n} 81$= ln 81/ ln 3^n.The expression used to find the n^{th} term (t_n) of the third sequence is $t_n = \log_{5^n} 25$ = ln 25/ln 5^n. The expression used to find the n^{th} term ($t_{n)}$ of the fourth sequence is $t_n = \log_{m^n} m^k$ = ln m^k/ln m^n. The answers can be easily justified by using the highly technological technology in a *Texas Instruments graphing calculator.* Considering $\log_2 8=3$, when plugging in ln 8/ ln2, the calculator provides the answer of 3, which is equivalent to the correct answer mentioned before.

Logarithms in Sequences: Calculating Logarithms

As mentioned earlier, logarithms can be solved and produce number answers, such as $\log_2 8=3$. Using the expressions from above this logarithm can also be solved like this: ln 8/ ln 2=3. Given this, consider the following logarithmic sequences:

1) $\log_4 64$, $\log_8 64$, $\log_{32} 64$
2) $\log_7 49$, $\log_{49} 49$, $\log_{343} 49$
3) $\log_{1/5} 125$, $\log_{1/125} 125$, $\log_{1/625} 125$
4) $\log_8 512$, $\log_2 512$, $\log_{16} 512$

Each of these logarithms can be calculated by using the formula $\log_a b = \ln a / \ln b$. In row 1, the calculated answers are as follows: ln 64/ ln 4, ln 64/ ln 8, ln 64/ ln 32. In row 2, the calculated answers are as follows: ln 49/ ln 7, ln 49/ ln 49, ln 49/ ln 343. In row 3 the calculated answers are as follows: ln 125/ ln (1/5), ln 125/ ln (1/125), ln 125/ ln (1/625). In row 4 the calculated answers are as follows: ln 512/ ln 8, ln 512/ ln 2, ln 512/ ln 16.

The answer obtained in the third column of each row can be deduced by just that of the first and second. If you didn't notice already, a pattern exists. The answer of the third row can be obtained by keeping the same numerator while multiplying the coefficients of the first and second row's denominators to produce the coefficient of the third row's denominator. So basically, the base of the first log is multiplied by the base of the second logarithm to form the base of the third logarithm, while keeping this in mind the coefficient of the third logarithm stays the same as the other two logarithms.

> Ex: ln 64/ ln 4, ln 64/ ln 8. ln 64 remains in the third logarithm as its numerator; and ln 4 is multiplied by ln 8 = ln (4 * 8) = ln 32. So ln 64/ ln 32.

A few more sequences can be formed like the previous ones:

1) $\log_9 81$= ln 81/ ln 9, $\log_{81} 81$= ln 81/ ln 81, $\log_{729} 81$= ln 81/ ln 729
2) $\log_6 6$= ln 6/ ln 6, $\log_{36} 6$= ln 6/ ln 36, $\log_{216} 6$= ln6/ ln 216

These sequences also follow the same patterns. In the first sequence, 9 from the first term is multiplied by 81 in the second term to form 729 in the third term; and in the second sequence, 6 from the first term is multiplied by 36 from the second term to form 216 in the third term.

Logarithms in Sequences: Finding the General Statement/ Concluding Statements

Consider the following: let $\log_a x$= c and $\log_b x$= d. given this you can find the general statement that expresses $\log_{ab} x$, in terms of c and d, which is cd/c+d. This can only be tested using a calculator, so from the first sequence plug in ln 64/ ln 4, ln 64/ ln 8, and ln 64/ ln 32 all separately. Consecutively you should get 3, 2, and 1.2. Now convert 1.2 into the fraction of 6/5. From this you can notice that 6 is made when multiplying 3 and 2, and 5 is made when adding 3 and 2. Now try another sequence. From sequence 3, plug in ln 125/ ln (1/5), ln 125/ ln (1/125), and ln 125/ ln (1/625) all separately. You should get -3, -1, and -.75 respectively. Now lets try the formula of cd/ c+d. when plugged in you should get, (-3 * -1)/ (-3 + -1), which is reduced to 3/ -4, which is equal to -.75. This proves the statement to be valid. With formulas comes a limitation. However it seems as though this one lacks a limit in terms of a, b and x. It seems this formula of cd/c+d is only useful when trying to find the calculation of a logarithm using its roots. By splitting up a logarithm into two smaller ones, you can find its solution easier by using the formula. This can be helpful when used in certain situations; however it can be useless when unneeded in smaller logarithms. In terms of a, b and x, x must repeat in order to use this expression. To find the general statement of cd/ c+d, was by reverse solution. I found the answer of the third term and tried to find out how this answer could be derived by using the values calculated from the first two terms.

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