MATHS PORTOFOLIO SL TYPE 1: MATRIX POWER

CANDIDATES NAME: PANJI WICAKSONO

IB NOVEMBER 2008

INTRODUCTION:

Matrices are an array of numeric or algebraic quantities subject to mathematical operations that can be multiplied, subtracted or added. The numbers can be arranged in a rectangular array of numbers set out in rows and columns. The numbers that are inside the matrices are called entries. Matrices are useful to keep track of the coefficients of linear transformations and to record data that depend on multiple parameters. In a more complex use, it can be helpful in encrypting numerical data and also in computer graphics.

In this portfolio, the task that was given was to deduce a general formula or pattern of the given matrices and also to determine the limitation of the general formula that has been found. This task will be done using the help of a GDC calculator (GDC-TI 83 plus) and the knowledge that have been attained during the mathematic SL course.

QUESTION 1:

Consider the matrix

$$M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Calculate $\mathbf{M}^n$ for n = 2, 3, 4, 5, 10, 20, 50

Describe in words any pattern you observe.

Use this pattern to find a general expression for the matrix $\mathbf{M}^n$ in terms of n

Counting for n = 2 manually, it gives:

$$M^2 = M.M = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

For n = 3

$$M^3 = M^2.M = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

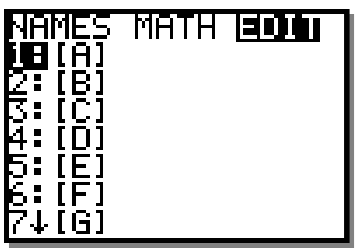
For n = 4

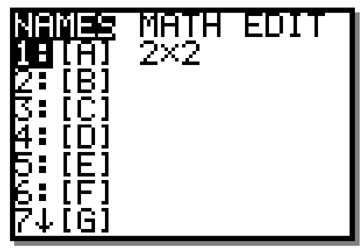
$$M^4 = M^3.M = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$$

For n = 5

$$M^5 = M^4.M = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

For the larger values of n, the GDC was used to help ease the task by first pressing **MATRIX** menu **EDIT** option, and then chose matrix **1: [A]** and defined the correct order of the matrix (dimensions), 2 x 2. Then I entered the matrix entries of $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and save it to memory 1 slot. Then the matrix was used for calculations by pressing MATRIX menu, NAMES submenu and raised it to the power I was calculating.





Using the GDC to calculate n = 10, 20, 50 the result was:

n = 10

$$M^{10} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{10} = \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix} = 1024\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

n = 20

$$M^{20} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{20} = \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix} = 1048576\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

n = 50

$$M^{50} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{50} \approx \begin{pmatrix} 1.126\times10^{15} & 0 \\ 0 & 1.126\times10^{15} \end{pmatrix} = 1.126\times10^{15}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

By calculating the values that were given in question number 1, I noticed a pattern that occurs from one value of n to the other. The 2 inside the matrix are always raised to the value of n in **Mⁿ**. The result gathered was that in entries ‘a’ and ‘d’ which are number greater than one, follows this pattern:

$$M^n = \begin{pmatrix} 2^n & 0 \\ 0 & 2^n \end{pmatrix}$$

To show whether my assumption is right or wrong let us try a couple more value calculating it manually and then comparing it using the graphics calculator.

Let us use values for n such as 25, 30, 40, and 45.

For ➽ = 25

$$M^{25} = M^{20} \cdot M^5 = \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix} \cdot \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

$$= \begin{pmatrix} 33554432 & 0 \\ 0 & 33554432 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{25} = \begin{pmatrix} 2^{25} & 0 \\ 0 & 2^{25} \end{pmatrix} \Rightarrow 2^{25} = 33554432$$

The result above matches my calculation when I used the GDC to find the value of M^{25}.

For ➽ =

$$M^{30} = M^{10} \cdot M^{10} \cdot M^{10} = \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix} \cdot \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix} \cdot \begin{pmatrix} 1024 & 0 \\ 0 & 1024 \end{pmatrix}$$

30

$$= \begin{pmatrix} 1073741824 & 0 \\ 0 & 1073741824 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{30} = \begin{pmatrix} 2^{30} & 0 \\ 0 & 2^{30} \end{pmatrix} \Rightarrow 2^{30} = 1073741824$$

Same goes to this value of n when inserted inside the general formula that was deduced earlier. The value matches the result when I used the GDC.

For n = 40

$$M^{40} = M^{20} \cdot M^{20} = \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix} \cdot \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix}$$

$$= \begin{pmatrix} 1.0995E12 & 0 \\ 0 & 1.0995E12 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{40} = \begin{pmatrix} 2^{40} & 0 \\ 0 & 2^{40} \end{pmatrix} \Rightarrow 2^{40} = 1.0995E12$$

For n = 45

$$M^{45} = M^{20} \cdot M^{25} = \begin{pmatrix} 1048576 & 0 \\ 0 & 1048576 \end{pmatrix} \cdot \begin{pmatrix} 33554432 & 0 \\ 0 & 33554432 \end{pmatrix}$$

$$= \begin{pmatrix} 3.518437209E13 & 0 \\ 0 & 3.518437209E13 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{45} = \begin{pmatrix} 2^{45} & 0 \\ 0 & 2^{45} \end{pmatrix} \Rightarrow 2^{45} = 3.518437209E13$$

So for all values of n calculated using the general expression in the first question matches to the result when n is calculated using GDC.

Therefore the general expression of $\boldsymbol{M^n} = \begin{pmatrix} 2^n & 0 \\ 0 & 2^n \end{pmatrix}$ is valid.

QUESTION 2

Consider the matrices $P = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ and S $= \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$.

$P^2 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^2 = \begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix} = 2\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$; $S^2 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^2 = \begin{pmatrix} 20 & 16 \\ 16 & 20 \end{pmatrix} = 2\begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}$

Calculate P^n and S^n for other values of n and describe any pattern(s) you observe.

P^n and S^n was calculated using the GDC for values such as n = 3, 4, 5, 6, 7, 8, 9, 10

I started using the GDC by inserting matrices P and S to the memory slot. For P the matrix slot that was used was **[B]** and for S I used matrix slot **[C]**. I specifically entered those matrices in a 2 x 2 dimension to suit the purpose. After storing the matrix in the memory I accessed it using the **MATRIX**, **NAMES** submenu and raise it to the value of n that I was calculating.

The results I got by calculating P in the GDC was

$$P^3 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^3 = \begin{pmatrix} 36 & 28 \\ 28 & 36 \end{pmatrix} \qquad P^7 = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^7 = \begin{pmatrix} 8256 & 8128 \\ 8128 & 8256 \end{pmatrix}$$

$$= 4\begin{pmatrix} 9 & 7 \\ 7 & 9 \end{pmatrix} \qquad = 64\begin{pmatrix} 129 & 127 \\ 127 & 129 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^4 = \begin{pmatrix} 136 & 120 \\ 120 & 136 \end{pmatrix} \qquad = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^8 = \begin{pmatrix} 32896 & 32640 \\ 32640 & 32896 \end{pmatrix}$$

$$= 8\begin{pmatrix} 17 & 15 \\ 15 & 17 \end{pmatrix} \qquad = 128\begin{pmatrix} 257 & 255 \\ 255 & 257 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^5 = \begin{pmatrix} 528 & 496 \\ 496 & 528 \end{pmatrix} \qquad = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^9 = \begin{pmatrix} 131328 & 130816 \\ 130816 & 131328 \end{pmatrix}$$

$$= 16\begin{pmatrix} 33 & 31 \\ 31 & 33 \end{pmatrix} \qquad = 256\begin{pmatrix} 513 & 511 \\ 511 & 513 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^{10} = \begin{pmatrix} 2080 & 2016 \\ 2016 & 2080 \end{pmatrix} \qquad = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^{10} = \begin{pmatrix} 524800 & 523776 \\ 523776 & 524800 \end{pmatrix}$$

$$= 32\begin{pmatrix} 65 & 63 \\ 63 & 65 \end{pmatrix} \qquad = 512\begin{pmatrix} 1025 & 1023 \\ 1023 & 1025 \end{pmatrix}$$

For the second part I will calculate S^n using n as 3, 4, 5, 6, 7, 8, 9, and 10.

$$S^3 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^3 = \begin{pmatrix} 112 & 104 \\ 104 & 112 \end{pmatrix} \qquad S^4 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^4 = \begin{pmatrix} 656 & 640 \\ 640 & 656 \end{pmatrix}$$

$$= 4\begin{pmatrix} 28 & 26 \\ 26 & 28 \end{pmatrix} \qquad = 8\begin{pmatrix} 82 & 80 \\ 80 & 82 \end{pmatrix}$$

$$S^5 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^5 = \begin{pmatrix} 3904 & 3872 \\ 3872 & 3904 \end{pmatrix} \qquad S^6 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^6 = \begin{pmatrix} 23360 & 23296 \\ 23296 & 23360 \end{pmatrix}$$

$$= 16\begin{pmatrix} 244 & 242 \\ 242 & 244 \end{pmatrix} \qquad = 32\begin{pmatrix} 730 & 728 \\ 728 & 730 \end{pmatrix}$$

$$S^7 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^7 = \begin{pmatrix} 140032 & 139904 \\ 139904 & 140032 \end{pmatrix} \qquad S^8 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^8 = \begin{pmatrix} 839936 & 839680 \\ 839680 & 839936 \end{pmatrix}$$

$$= 64\begin{pmatrix} 2188 & 2186 \\ 2186 & 2188 \end{pmatrix} \qquad = 128\begin{pmatrix} 6562 & 6560 \\ 6560 & 6562 \end{pmatrix}$$

$$S^9 = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^9 = \begin{pmatrix} 5039104 & 5038592 \\ 5038592 & 5039104 \end{pmatrix} \qquad S^{10} = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}^{10} = \begin{pmatrix} 30233600 & 30232576 \\ 30232576 & 30233600 \end{pmatrix}$$

$$= 256\begin{pmatrix} 19684 & 19682 \\ 19682 & 19684 \end{pmatrix} \qquad = 512\begin{pmatrix} 59050 & 59048 \\ 59048 & 59050 \end{pmatrix}$$

By calculating the different values of n for P^n and S^n a specific pattern could be observed, which is:

$$X^n = 2^{(n-1)} \cdot \begin{pmatrix} R^n + 1 & R^n - 1 \\ R^n - 1 & R^n + 1 \end{pmatrix}$$

X in this general expression corresponds to the matrix P or S, and R inside the matrix is 2 when P is used, and it is 3 when S is used. I will now try to recheck whether the general formula by using the general formula that I observed and comparing it to the results calculated using the GDC.

$$P^5 = 2^{(5-1)} \cdot \begin{pmatrix} 2^5 + 1 & 2^5 - 1 \\ 2^5 - 1 & 2^5 + 1 \end{pmatrix} = 2^4\begin{pmatrix} 33 & 31 \\ 31 & 33 \end{pmatrix} = 16\begin{pmatrix} 33 & 31 \\ 31 & 33 \end{pmatrix}$$

$$S^5 = 2^{(5-1)} \cdot \begin{pmatrix} 3^5 + 1 & 3^5 - 1 \\ 3^5 - 1 & 3^5 + 1 \end{pmatrix} = 2^4\begin{pmatrix} 244 & 242 \\ 242 & 244 \end{pmatrix} = 16\begin{pmatrix} 244 & 242 \\ 242 & 244 \end{pmatrix}$$

The general formula gave the same answer as to the result gathered using the GDC. That means the general formula is acceptable.

QUESTION 3

Now consider the matrices of the form $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$.

Steps one and two contain examples of these matrices for k = 1, 2 and 3.

Consider other values of k, and describe any pattern(s) you observe.

Generalize these results in terms of k and n.

The matrices that were given in question 1, 2, and 3 were derived from the form $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$. Example, if K was given to be 2, then the result would be $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, which was the question in number one. It is the same in number 2 where K for the matrix P is 2 and in S is 3 the result in correspondingly are $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ and $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$. So it could be said that the matrices that was given in the previous questions follow the matrix form of $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$. So when K is 4, 5, 6, and 7 we get:

$k=4=\begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$ $\quad k=5=\begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}$ $\quad k=6=\begin{pmatrix} 7 & 5 \\ 5 & 7 \end{pmatrix}$ $\quad k=7=\begin{pmatrix} 8 & 6 \\ 6 & 8 \end{pmatrix}$

I am able to analyse that the difference within the entries are always 2 (i.e. 3-1 = 2, 8-6 = 2). Thus if the difference is the same, the will be the same as the previous:

$$X^n = 2^{(n-1)} \cdot \begin{pmatrix} R^n+1 & R^n-1 \\ R^n-1 & R^n+1 \end{pmatrix}$$

To make sure that my statement is true, I will try and use an example:

Take K as 5 for example. By using the $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$ matrix I would get the value of K= 5 as $\begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}$. Lets call this matrix of K=5 as Z.

By calculating manually: $Z^6 = \begin{pmatrix} 5^6+1 & 5^6-1 \\ 5^6-1 & 5^6+1 \end{pmatrix} = 32\begin{pmatrix} 15626 & 15624 \\ 15624 & 15626 \end{pmatrix}$.

By using the GDC: $Z^6 = \begin{pmatrix} 6 & 4 \\ 4 & 6 \end{pmatrix}^6 = \begin{pmatrix} 500032 & 499968 \\ 499968 & 500032 \end{pmatrix} = 32\begin{pmatrix} 15626 & 15624 \\ 15624 & 15626 \end{pmatrix}$

Therefore, we can say that the general formula of

$X^n = 2^{(n-1)} \cdot \begin{pmatrix} R^n+1 & R^n-1 \\ R^n-1 & R^n+1 \end{pmatrix}$ is applicable with the matrix $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$.

Again it is important to make clear what the symbols represent; the X in this equation would represent the symbol of the matrix related (such as P, S or M). The n would be the power in which the matrix is being raised to, and K is going to be the R of Matrix X. To make this concept perfectly clear I shall undertake another example:

Take these values into consideration:

K= 3 ; n = 2

K=3 = $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$ = $\begin{pmatrix} 3+1 & 3-1 \\ 3-1 & 3+1 \end{pmatrix}$ = $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$

Let us name this matrix Y. If Y was raised to **n = 2**, than we would use the equation of $X^n = 2^{(n-1)} \cdot \begin{pmatrix} R^n+1 & R^n-1 \\ R^n-1 & R^n+1 \end{pmatrix}$.

Now remember that K=R thus R would be 3

$$Y^2 = 2^{(2-1)} \cdot \begin{pmatrix} 3^n+1 & 3^n-1 \\ 3^n-1 & 3^n+1 \end{pmatrix} = 2\begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}$$

This result matches with our results that we had counted in question 2 for S^2.

Thus from all this calculation we have gathered the following:

1. That K=R
2. That the general formula $X^n = 2^{(n-1)} \cdot \begin{pmatrix} R^n+1 & R^n-1 \\ R^n-1 & R^n+1 \end{pmatrix}$. Can be used in the matrix $\begin{pmatrix} k+1 & k-1 \\ k-1 & k+1 \end{pmatrix}$.

Thus we can change R in our general formula with K to become

$$\boxed{X^n = 2^{(n-1)} \cdot \begin{pmatrix} k^n+1 & k^n-1 \\ k^n-1 & k^n+1 \end{pmatrix}}$$

QUESTION 4

Use technology to investigate what happens with further values of k and n. State the scope or limitations of k and n.

To find the limitation of this general formula I used the GDC to calculate some values regarding to K and n. The values include whole numbers, integers, fractions and irrational numbers. In this part I tried to compare between my manual calculation using the general formula and comparing it with my calculation using the GDC. In this testing I will keep my K=1 through out the testing to find the limitation for n. The results are as followed:

k = 1
n = 0

Solving Manually:

$$X^0 = 2^{(0-1)} \cdot \begin{pmatrix} 1^0+1 & 1^0-1 \\ 1^0-1 & 1^0+1 \end{pmatrix} = 2^{-1}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Solving Technologically:

$$X^0 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

k = 1
n = 5

Solving Manually:

$$X^0 = 2^{(5-1)} \cdot \begin{pmatrix} 1^5+1 & 1^5-1 \\ 1^5-1 & 1^5+1 \end{pmatrix} = 2^{4}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 16\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

Solving Technologically:

$$X^5 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^5 = \begin{pmatrix} 32 & 0 \\ 0 & 32 \end{pmatrix}$$

k = 1
n = −2

Solving Manually:

$$X^{-2} = 2^{(-2-1)} \cdot \begin{pmatrix} 1^{-2}+1 & 1^{-2}-1 \\ 1^2-1 & 1^{-2}+1 \end{pmatrix} = 2^{(-3)}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.25 \end{pmatrix}$$

Solving Technologically:

$$X^{-2} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{-2} = \text{ERR: DOMAIN}$$

k = 1

$n = \frac{1}{2}$

Solving Manually:

$$X^{\frac{1}{2}} = 2^{(\frac{1}{2}-1)} \cdot \begin{pmatrix} 1^{\frac{1}{2}}+1 & 1^{\frac{1}{2}}-1 \\ 1^{\frac{1}{2}}-1 & 1^{\frac{1}{2}}+1 \end{pmatrix} = 2^{(-\frac{1}{2})}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0.707\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1.414 & 0 \\ 0 & 1.414 \end{pmatrix}$$

Solving Technologically:

$$X^{\frac{1}{2}} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{\frac{1}{2}} = \text{ERR: DOMAIN}$$

\k =1

$n = -\frac{1}{2}$

Solving Manually:

$$X^{-\frac{1}{2}} = 2^{(-\frac{1}{2}-1)} \cdot \begin{pmatrix} 1^{-\frac{1}{2}}+1 & 1^{-\frac{1}{2}}-1 \\ 1^{-\frac{1}{2}}-1 & 1^{-\frac{1}{2}}+1 \end{pmatrix} = 2^{(-\frac{3}{2})}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0.354\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0.708 & 0 \\ 0 & 0.708 \end{pmatrix}$$

Solving Technologically:

$$X^{-\frac{1}{2}} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{-\frac{1}{2}} = \text{ERR: DOMAIN}$$

k = 1

n = 3.75

Solving Manually:

$$X^{3.75} = 2^{(3.75-1)} \cdot \begin{pmatrix} 1^{3.75}+1 & 1^{3.75}-1 \\ 1^{3.75}-1 & 1^{3.75}+1 \end{pmatrix} = 2^{(2.75)}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 6.73\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 13.46 & 0 \\ 0 & 13.46 \end{pmatrix}$$

Solving Technologically:

$$X^{3.75} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{3.75} = \text{ERR: DOMAIN}$$

k = 1

n = - 3.75

Solving Manually:

$$X^{-3.75} = 2^{(-3.75-1)} \cdot \begin{pmatrix} 1^{-3.75}+1 & 1^{-3.75}-1 \\ 1^{-3.75}-1 & 1^{-3.75}+1 \end{pmatrix} = 2^{(-4.75)}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0.0372\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0.0744 & 0 \\ 0 & 0.0744 \end{pmatrix}$$

Solving Technologically:

$$X^{-3.75} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{-3.75} = \text{ERR: DOMAIN}$$

k = 1

$n = \sqrt{3}$

Solving Manually:

$$X^{\sqrt{3}} = 2^{(\sqrt{3}-1)} \cdot \begin{pmatrix} 1^{\sqrt{3}}+1 & 1^{\sqrt{3}}-1 \\ 1^{\sqrt{3}}-1 & 1^{\sqrt{3}}+1 \end{pmatrix} = 2^{(\sqrt{3})}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 0.732\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1.464 & 0 \\ 0 & 1.464 \end{pmatrix}$$

Solving Technologically:

$$X^{\sqrt{3}} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{\sqrt{3}} = \text{ERR: DOMAIN}$$

k = 1
n = π

Solving Manually:

$$X^{\pi} = 2^{(\pi-1)} \cdot \begin{pmatrix} 1^{\pi}+1 & 1^{\pi}-1 \\ 1^{\pi}-1 & 1^{\pi}+1 \end{pmatrix} = 2^{(\pi)} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 8.825 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 17.65 & 0 \\ 0 & 17.65 \end{pmatrix}$$

Solving Technologically:

$$X^{\pi} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{\pi} = \text{ERR: DOMAIN}$$

k = 1
n = 255

Solving Manually:

$$X^{255} = 2^{(255-1)} \begin{pmatrix} 1^{255}+1 & 1^{255}-1 \\ 1^{255}-1 & 1^{255}+1 \end{pmatrix} = 2^{254} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2.894802231E76 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5.789604462E76 & 0 \\ 0 & 5.789604462E76 \end{pmatrix}$$

Solving Technologically:

$$X^{255} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{255} = \begin{pmatrix} 5.789604462E76 & 0 \\ 0 & 5.789604462E76 \end{pmatrix}$$

k = 1
n = 256

Solving Manually:

$$X^{256} = 2^{(255-1)} \begin{pmatrix} 1^{256}+1 & 1^{256}-1 \\ 1^{256}-1 & 1^{256}+1 \end{pmatrix} = 2^{255} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 5.789604462E76 \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1.157920892E77 & 0 \\ 0 & 1.157920892E77 \end{pmatrix}$$

Solving Technologically:

$$X^{255} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}^{256} = \text{ERR: DOMAIN}$$

From the results above, it is evident that some types of numbers are not suitable when being calculated by the GDC. Those numbers include, work for negative whole numbers, fractions, negative fractions, decimals, negative decimals, surds, irrational numbers and numbers greater than 255 as proved using the GDC above.

However, the general formula does work when the value of n is a positive integer, a shown above using the value n = 0, 5 and 255. Thus it can be concluded that the general equation only works when the **value of n is a positive integer.**

On the other hand, any value of K is acceptable and can be used in this general formula as long as the entries inside the matrix has a difference of 2, which it follows the matrix formula of k+1 and k-1. If the matrix has a difference of 2 (regardless of it being negative, positive, fractions, decimals, irrational, surds etc.) the general formula would work.