

IB Math SL Type II Portfolio

January 14, 2010

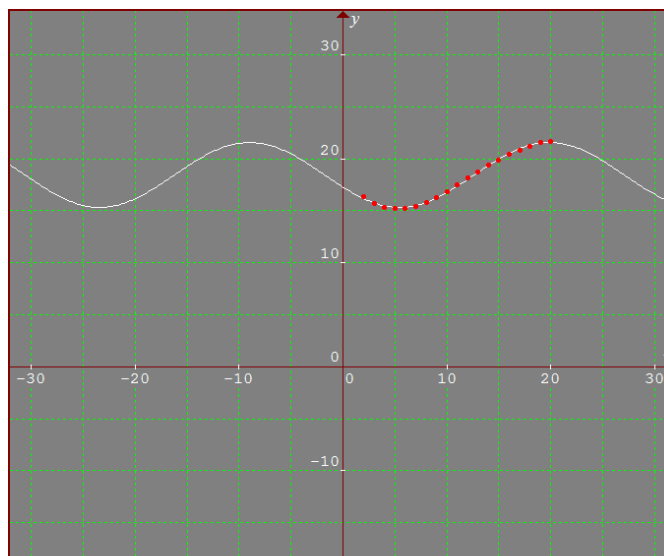
In this portfolio, we will look at the median BMI indexes for females between the ages of two and twenty in the US in the year 2000; and from this data, a model function will be determined. Body mass index (BMI) is a measure of a person's body fat, and is calculated by this formula:

$$\text{BMI} = \frac{\text{mass (kg)}}{\text{height}^2(\text{m}^2)}$$

Looking at the data table we can see that there are two variables: one being the age (in years), and the other being the BMI. The independent variable is the age and the dependent variable is the BMI, since the BMI's of the females depend on what age they are. The parameters are the weight and height of the females, which both directly affect the BMI. After using software to graph the data points, I believe the graph's behavior is best modeled by the cosine function, since it has a wave-like shape and is periodic. Next, I will create an equation that fits the graph by examining the parts that make up a cosine function. First, let's take a look at the general cosine function: $f(x) = a \cdot \cos(bx + c) + d$. To find the amplitude (a), I will simply take the y-maximum (21.65) and subtract the y-minimum (15.20) and then divide by 2. This gives me 3.22 for the amplitude. Then to find the period I will have to see the length of one cycle, which is when the function goes from the maximum to the minimum and back to the maximum. By looking at the graph, I determine that one cycle is 30. I then take 2π and divide by 30 to get the period, since there are 2π radians in one cycle. This gives me 0.21 for the period. Then to find the vertical shift I will simply add the amplitude to the y-minimum:

$3.22 + 15.20 = 18.42$. Finally, to find the horizontal shift (c) I must see how much I need to shift the graph so that it is aligned with the model function. I can do this by entering the model function I have created, leaving out the "c" value, into a graphing calculator.

Then I can compare it to the scatter plot and determine how much I have to shift the function, by looking at the "x" values of the y-minimums of both curves. Since the "x" value of the y-minimum on the scatter plot is 5 and the "x" value of the y-minimum on the model function is 14.96, I can subtract them to get 9.95 as the value of the horizontal shift. Since I



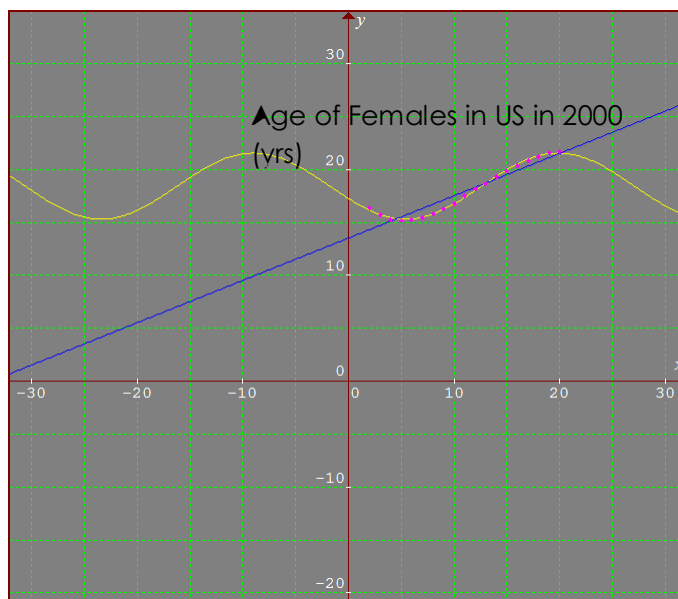
BMI

have to shift the function to the right, I must make the "c" value negative. Now I can substitute all these variables into the general cosine function to get my model function for the data.

My model function is:

$$f(x) = 3.22 \cdot \cos(0.21x - 9.95) + 18.42$$

As you can clearly see, the first model function fits the data almost perfectly, but it is very unrealistic for predicting BMI's of women, as it just keeps rising and falling; so it is virtually useless for estimating the BMI of a 30-year-old woman in the US. This is fairly obvious when looking at the cosine graph, as you can see that a 30-year-old woman has a lower BMI than a 20-year-old woman. This is essentially impossible unless a woman became shorter and/or lost a lot of weight. It would even



be reasonable if a 30-year-old woman had the same BMI as when she was 20, but it is much more likely that she would have a slightly increased BMI, at the least. I believe that a linear function will do a much more accurate job in predicting the BMI of women in the US, but only for several years after the age of 20; probably just enough to accurately predict the BMI of a 30-year-old woman in the US.

According to my cosine model function, the predicted BMI for a 30-year-old woman in the US is:

$$f(30) = 3.22 \cdot \cos((0.21)(30) - 9.95) + 18.42$$

$$f(30) = 15.61$$

If this function was used to predict the BMI of a 30-year-old woman, you would be implying that a 30-year-old woman has the same BMI as a 10-year-old; which is basically physically impossible.

However, if the linear function was used to predict the age of a 30-year-old woman, I am positive that it will be more accurate and certainly reasonable. The linear function I obtained from technology is:

$$f(x) = 0.4x + 13.52$$

$$f(30) = 0.4(30) + 13.52$$

$$f(30) = 25.52$$

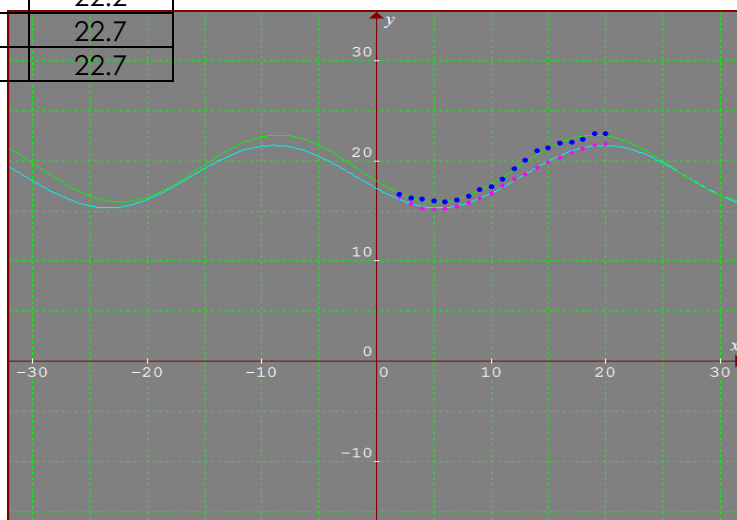
A body mass index of 25.52 for a 30-year-old American female seems fairly reasonable, although maybe a little too high. According to the BMI categories, anything 25 and above is considered overweight, but with the high rate of obesity in the US this seems like a reasonable median BMI for females.

The table below gives the median BMI for females aged 2-20 in London, England for the years 1995-97.

Age (yrs)	BMI
2	16.7
3	16.3
4	16.2
5	16.0
6	15.9
7	16.1
8	16.5
9	17.1
10	17.4
11	18.2
12	19.2
13	20.1
14	21.0
15	21.3
16	21.8
17	21.9
18	22.2
19	22.7
20	22.7

(Source: <http://www.archive.official-documents.co.uk/document/doh/survey97/hst3-4.htm>)

This BMI data for females aged 2-20 in London fits my model to a certain extent, as it also follows the same trend. However, you can see from the graph that the



data points from London are just slightly higher. To fix this, a simple change in amplitude and vertical shift would be sufficient. Well, the biggest limitation on my model is that you can't accurately predict the BMI for all ages of women in the US.



Age of Females in US in 2000
(yrs)