IB Math SL1

23 February 2009

The Practical Uses of Newton's Law of Cooling

Introduction:

Newton's Law of Cooling measures "the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings)" ("Newton's" par.3).

$u(t) = T + (u_0 - T)e^{kt}$ $k < 0$

T : the temperature of the surrounding medium
u_0: the initial temperature of the heated object
t : the length of time in minutes
k : a negative constant
u : represents the temperature at time t

It can practically be used in many places. In this paper, I will deal with the problem of fast food restaurant, McNewton by using the Newton's Law of Cooling; the restaurant needs a container to cool down the temperature of coffee from 170 ºF of brewing temperature to 120~140ºF which is the range of drinkable coffee temperature and it also wants to remain the temperature between 120ºF to 140 ºF for a reasonable time. The Newton's Law of Cooling enables McNewton to compare the time of each container which takes to cool down coffee; therefore, it enables to make a reasonable choice.

The Problem and its Solving Processes:

1. Use Newton's Law of Cooling to find the constant *k* of the formula for each container.

(a) The CentiKeeper Company has a container that will reduce the temperature of a liquid from 200ºF to 100ºF in 90 minutes by maintaining a constant temperature of 70ºF.

Solution: substitutes the numbers at the right place of original equation.

$100=70+(200-70)e^{90k}$

$100-70=130e^{90k}$

$\frac{3}{13}=e^{90k}$

$\ln\frac{3}{13}=\ln e^{90k}$

$\frac{\ln\frac{3}{13}}{90}=k$

$\therefore k \approx -0.016$

(b) The TempControl Company has a container that will reduce the temperature of a liquid from 200ºF to 110ºF in 60 minutes by maintaining a constant temperature of 60ºF.

$110=60+(200-60)e^{60k}$

$\frac{5}{14}=e^{60k}$

$\frac{\ln\frac{5}{14}}{60}=k$

$\therefore k \approx -0.017$

(c) The Hot'n'Cold, Inc., has a container that will reduce the temperature of a liquid from 210ºF to 90ºF in 30 minutes by maintaining a constant temperature of 50ºF.

$90=50+(210-50)e^{30k}$

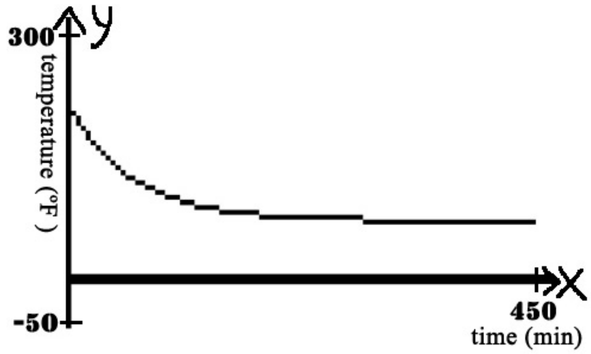
$\frac{1}{4}=e^{30k}$

$\frac{\ln\frac{1}{4}}{30}=k$

$\therefore k \approx -0.046$

2. Use TI Connect to graph and print each graph.

(a) The container of CentiKeeper Company

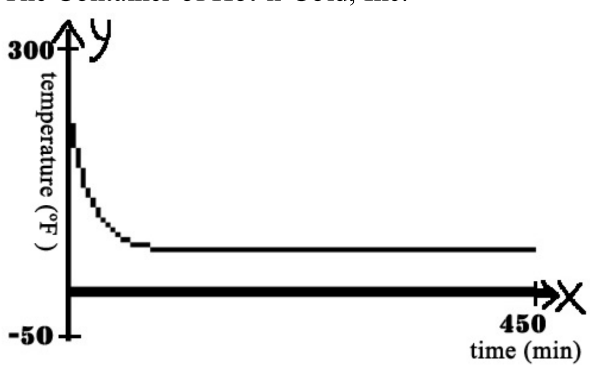


Graph 1: The Change of Temperature According to the Equation of
$y = 70+(200-70)e^{-0.016x}$

(b) The Container of TempControl Company

Graph 2: The Change of Temperature According to the Equation of
$y = 60+(200-60)e^{-0.017x}$

(c) The Container of Hot'n'Cold, Inc.



Graph 3: The Change of Temperature According to the Equation of
$y = 50+(210-50)e^{-0.416x}$

3. How long does it take each container to lower the coffee temperature from 170 ºF to 140 ºF?

(a) $140=70+(170-70)e^{-0.016t}$

$\frac{7}{10}= e^{-0.016t}$

$\frac{\ln\frac{7}{10}}{-0.016}= t$

$\therefore t \approx 22.292$ minutes

(b) $140=60+(170-60)e^{-0.017t}$

$\frac{8}{11}=e^{-0.017t}$

$\frac{\ln\frac{8}{11}}{-0.017}=t$

$\square t \approx 18.733$ minutes

(c) $140=50+(170-50)e^{-0.046t}$

$\frac{3}{4}=e^{-0.046t}$

$\frac{\ln\frac{3}{4}}{-0.046}=t$

$\therefore t \approx 6.254$ minutes

4. How long will the coffee temperature remain between 120 ºF and 140 ºF?

Solution: In other word, we can find how much each container takes to cool down from 140 ºF to 120 ºF.

(a) $120=70+(140-70)e^{-0.016t}$

$\frac{5}{7}= e^{-0.016t}$

$\frac{\ln\frac{5}{7}}{-0.016}= t$

$\therefore t \approx 21.026$ minutes

(b) $120=60+(140-60)e^{-0.017t}$

$\frac{3}{4}=e^{-0.017t}$

$\frac{\ln\frac{3}{4}}{-0.017}=t$

$\therefore t \approx 16.922$ minutes

(c) $120=50+(140-50)e^{-0.046t}$

$\frac{7}{9}=e^{-0.046t}$

$\frac{\ln\frac{7}{9}}{-0.046}=t$

$\therefore t \approx 5.463$ minutes

5. On the basis of this information, which company should get the contact with McNewton's? What are your reasons?

The most reasonable choice would be to choose a container which cools down the coffee quickly between 170 ºF to 140 ºF, the drinkable temperature, and remains the temperature of coffee slowly between 140 ºF to 120 ºF. The container of the Hot'n'Cold, Inc., cools down the quickest; however, the coffee cools down too fast that it can remain at drinkable temperate range, between 140 ºF to 120 ºF, only for 5.463 minutes. On the other hand, the container of the CentiKeeper Company cools down from 170 ºF to 140ºF too slowly although it remains the drinkable temperature for the longest time, 21.026 minutes. So most moderate and reasonable choice is to buy TempControl Company's container which takes 18.733 minutes to cool down from 170 ºF to 140 ºF and remains the temperature of coffee for 16.922 minutes between 140 ºF to 120 ºF.

Another consideration of choosing container would be the number of customers. Since McNewton is a fast food restaurant, a lot of customers may come for a short period time. McNewton's job is to serve as many customers as possible at the shortest time period; they cannot make their customers wait so long. Then, Hot'n'Cold, Inc.'s container would be the best choice for McNewton. It has to consider both the efficiency and the number of customers when they choose its new equipment.

6. Define "Capital cost" and "operating cost." How might they affect your choice?

Compared to capital cost is the one-time cost which is used to start the business such as buying land, buildings and equipment, operating cost is which is used periodically such as paying rental fees, salaries for employees and electricity. Therefore, to make a rational choice, both the capital cost and the operating cost should be considered with the efficiency of the container; although the capital cost is very reasonable, if the operating cost is too expensive, McNewton is likely to buy another container that costs reasonably.

Conclusion:

The Newton's Law of Cooling can be practically used in the real world as it is exemplified above—to decide which container McNewton should buy. Considering just the time takes to cool down from brewing temperature (170 ºF) to drinkable temperature (140 ºF) and time that remains from drinkable temperature range (120~140 ºF), the container from the TempControl Company would be the reasonable choice since it takes 22.292 minutes to cool down and remains at the drinkable temperature for 16.922 minutes. However, if the McNewton has a lot of customers to serve, the container of Hot'n'Cold, Inc. which takes 6.254 minutes to cool down and remains at drinkable temperature for 5.463 minutes would fit the most. Since the problem of McNewton does not specify the capital cost, the price of device in this case, and the operating cost, it is hard to state which container would be better. But when there is more information, McNewton has to consider every factors when it buys the container.

It is notable to recognize that the Newton's Law of Cooling can be used in every situation where it is needed to find the temperature to cooling substances, the time and the rate of cooling. The Newton's Law of Cooling equation is really useful and practical equation not in just theory but in the real world.

Works Cited

"Newton's Law of Cooling." The UBC Calculus Online. The University of British Columbia Department of Mathematics. 21 Feb. 2009 <http://www.ugrad.math.ubc.ca/coursedoc/math100/notes/diffeqs/cool.html>.