PART I: The Data

[illegible]

The shortest day of the year coincides with the December solstice which occurred on Sunday, December 21st in the year of 2008 at 9:04 PM. At this time, the length of the day (the amount of sunlight) totally at exactly 9h 25m 41s from 7:52 AM when the sun rose to 5:18 PM when the sun set.

[illegible]

The longest day of the year always coincides with the June solstice which occurred on Saturday, June 21st in the year of 2008 at 8:59 AM. The length of the day totaled at precisely 14h 54m 33s, from 5:11 AM when the sun rose, to 8:06 PM when the sun set.

The difference as we can see between the two extremes of the year is 5 hours 28 minutes and 52 seconds, which is quite a difference.

year	Equinox Mar		Solstice June		Equinox Sept		Solstice Dec	
	day	time	day	time	day	time	day	time

2007	21	09:07	22	03:06	23	18:51	22	15:08
2008	20	14:48	21	08:59	23	12:44	21	09:04
2009	20	08:44	21	14:45	23	06:18	22	02:47
2010	21	02:32	21	20:28	23	12:09	22	08:38

Analysis: Here is a graph adapted from Wikipedia that marks the two solstices of the year that occur in June (Summer) and December (winter) as well as the two equinoxes that occur in March (Spring) and September (Autumn). The times are adjusted to Pyongyang time at which this phenomenon occurs. The interesting thing here is that the solstices in June and December coincide with the longest and shortest days of the year respectively.

Acco[illegible]

Interestingly, as I have found above, if we use the difference (5h28m52s) between the two extremes, we can find the median. The median sunlight as I have calculated is 12h10m7s, which is found by dividing the difference by 2 and adding it to the amount of sunlight during the winter solstice and checked the answer by comparing it by subtracting it from the amount of sunlight in the summer solstice.

Summer: 14h 54m 33s

Winter: 9h 25m 41s

$$14h54m33s - 9h25m41s = 5h28m52s$$

$$\therefore \frac{5h28m52s}{2} = 2h44m26s$$

$$Longest: 14h54m33s - 2h44m26s = 12h10m7s$$

$$Shortest: 9h25m41s + 2h44m26s = 12h10m7s$$

Thus the median was 12hrs 10 minutes and 7 seconds.

The date was easier to find. Because the longest day was on June 21st, and the shortest on December 21st, the median was obviously in the middle. Between June and December, the very middle month is September and the very middle date between June 21st and December 21st is none other than September 21st, 2008. Thus the median date according to calculation would occur on September 21st, 2008. However, it is not just one date that has a median amount of sunlight. On the other side between

December and June is March, which is when the median would be reached, exactly 6 months from September 21st. This date I found was March 21st, 2008.

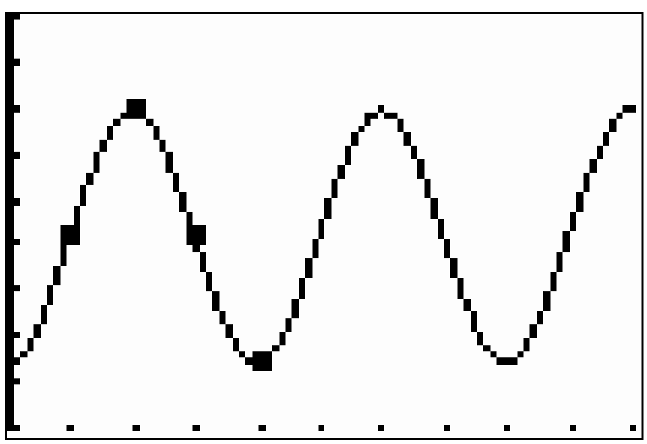
Does this correspond to the date given? Does this correspond with what you would expect? According to the data, on what date did the spring equinox occur?

Very interestingly, my results almost corresponded. The median sunlight would be given on the autumn equinox, which in Pyongyang would occur on September 23rd for 2008. This was exceptionally close to what I had predicted, but I was off by two days. This small error, my thoughts are, would be accounted for by time difference and the way the earth moves. The second median date was on the spring equinox, which occurs in March. In 2008 it occurred on March 20th at 14:48. This was also very close to the date I predicted. But since 2008 was a leap year and February had an additional day, this would even out my calculations. Despite a relatively small error in both calculations, the dates corresponded with my calculations.

PART II: The Sine and Cosine Functions

To first graph the results into a sine curve, it was necessary to convert these result values in time to decimal numbers to be calculated in a graph. Thus considering that a minute is 60 seconds and a 60 minutes is an hour, I converted the maximum sunlight (14h54m33s) into the decimal 14.909 and the minimum sunlight (9h25m41s) into the decimal 9.428. The median sunlight (12h10m7s) was then converted into 12.169.

The Sine Function:

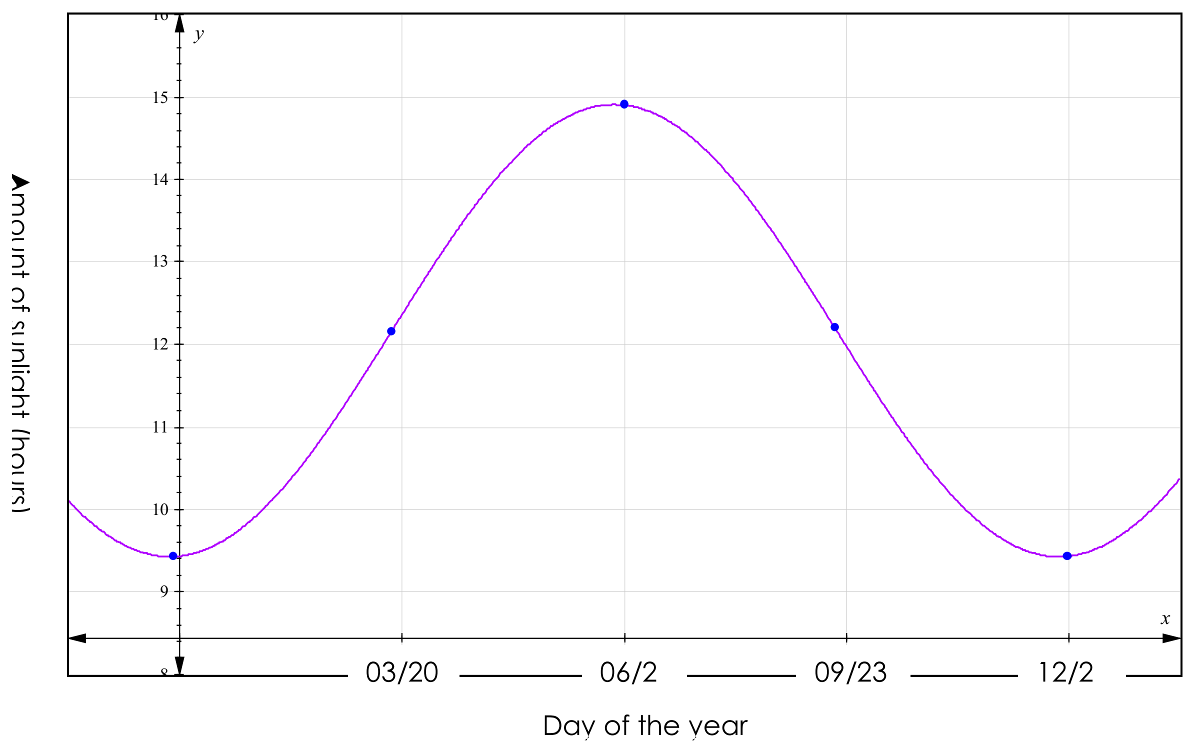


This is a screenshot of my TI-84 Calculator where I used the list function to put in the values of the highest and lowest amounts of sunlight. To set the sine equation onto the values I got from graphing the list, I graphed the function:

$$f(x) = 12.169 + 2.741\sin\frac{\pi}{2}(x - \frac{\pi}{2})$$

Where 12.169 was the median temperature right in between the two extremes, and the 2.741 was the amplitude where the variations were between the extremes and the median. Moreover, to make the sine graph fit with the results, I had to make the "b" part of the function (corresponding to the period) as $\frac{\pi}{2}$, because then the period would be exactly 4, which is what I want, because I put in four values into the input (day of the year), thus making a full rotation of the amount of sunlight in four steps. Lastly, to correspond with the right place, I put $\pi / 2$ as the horizontal displacement to put it in line with the graph results.

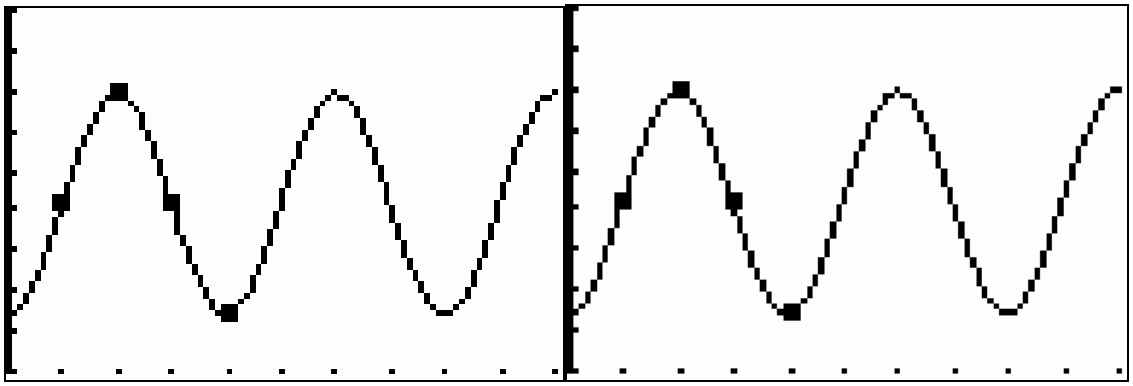
If we were to make a more precise graph/picture:



For this one, I used the program GraphingPackage that came with our digital textbook. I put in the exact same equation I used in the calculator, and even marked the points that were the extremes and the medians.

The Cosine Function:

$$f(x) = 12.169 + 2.741\cos\frac{\pi}{2}(x - \pi)$$



COSINE **SINE**

The figure on the left is the cosine screenshot of the equation, which we can see is identical to the sine function on the right, which was shown before in section I. Except the only variation is that the equation of the cosine function is

$f(x) = 12.169 + 2.741\cos\frac{\pi}{2}(x - \pi)$, whereas the sine function is

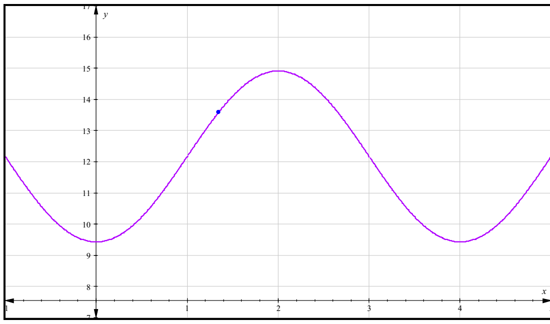
$f(x) = 12.169 + 2.741\sin\frac{\pi}{2}(x - \frac{\pi}{2})$. The difference between the two is the horizontal shift shown by the constants π for cosine and $\pi/2$ for sine. Because of the shape of these graphs, cosine requires a larger shift to enter its position while sine requires less.

To find the amount of sunlight on April 27th, I first needed to convert this date in terms of a decimal in order put it into a x input. Since December 21 was equal to 4 and March 20 equal to 1, I had to find out what April 27th was. If one year between December 21 2007 to December 21 2008 was divided exactly in 4, it would equal 366 days (2008 is a leap year) divided by 4.

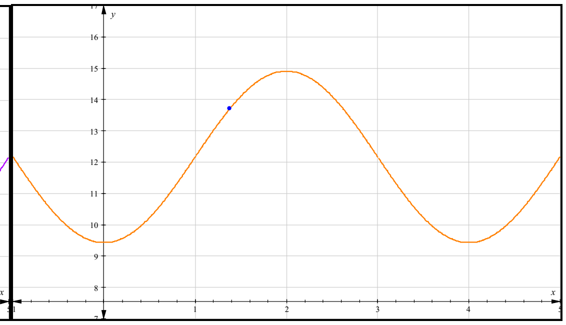
$366 \div 4 = 91.5$

$\frac{127}{91.5} = 1.3779...$ ←The 127comes from the number of days from December 21st to April 27th

If we add this to 1 (the time from December 21st to March 20th), we get 1.378. Now I plugged into the equation to get the actual amount of sunlight on that exact day of April 27th.



SINE



COSINE

Whew… And the result was 13.59 according to the Graphing Package program for both sine and cosine functions . To verify if this was accurate, I went back to the Sunrise and Sunset program.

According to the site, Pyongyang had the following statistics of sunlight on April 27.

Apr 27, 2008	5:44 AM	7:26 PM	13h 41m 39s

However, I needed to compare 13.59 with the time in 13:41:39. So I converted the latter into decimal form.

$$(\frac{39}{60}+41)/60+13=13.694$$

$$\therefore 13.694 \approx 13.59$$

Although the two amounts of time weren't identical, it was pretty clear that it was darn close. The small error probably can be accounted to the fact that the increments for my x input values were too small and created this slight error.

$$13.694-13.59=0.104$$

The percentage error: $\frac{0.104}{13.694}*100\%=0.759\%$

I have to say so myself, less than 1% error is pretty amazing.

Some more dates:

January 27th (my birthday)

$$366 \div 4 = 91.537$$

$\frac{37}{91.5}=0.40437...$ ←37 is the days from December 21st to Jan.27th

Y=9.963 hours Real time: 10:02:41 (10.04) Percentage error: 0.8%

August 15th

$366 \div 4 = 91.537$

$\frac{238}{91.5} = 2.601...$ ←37 is the days from December 21st to Aug 15th

Y=13.78 Real time: 13:41:44 (13.696) Percentage error: 0.6%

June 25th

$366 \div 4 = 91.537$

$\frac{187}{91.5} = 2.043...$ ←37 is the days from December 21st to June 25th

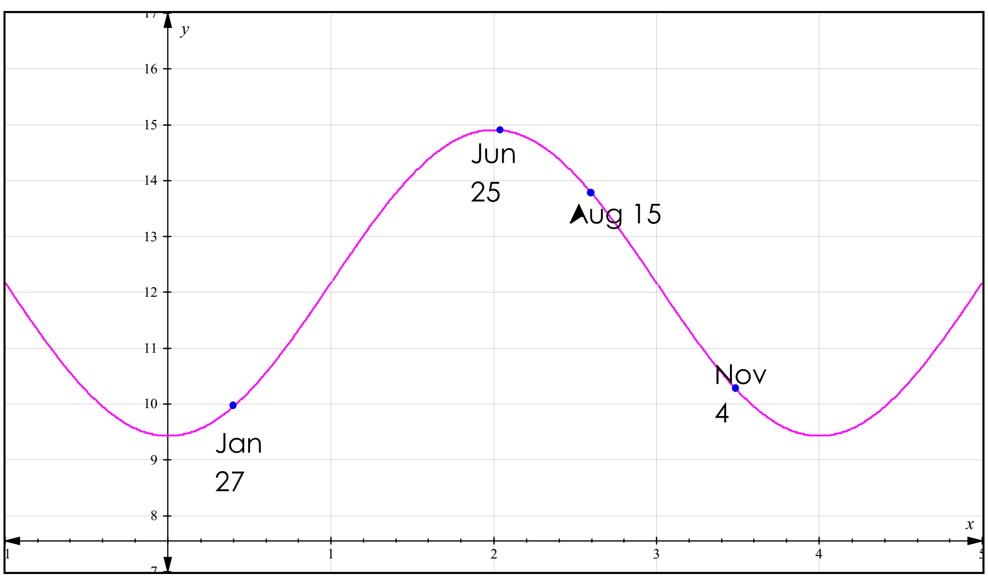
Y=14.9 Real time: 14:54:03 (14.9008) Percentage error: 0.006%

November 4th

$366 \div 4 = 91.537$

$\frac{319}{91.5} = 3.486...$ ←37 is the days from December 21st to Nov.4th

Y=10.27 Real time: 10:25:32 (10.426) Percentage error: 1.5%

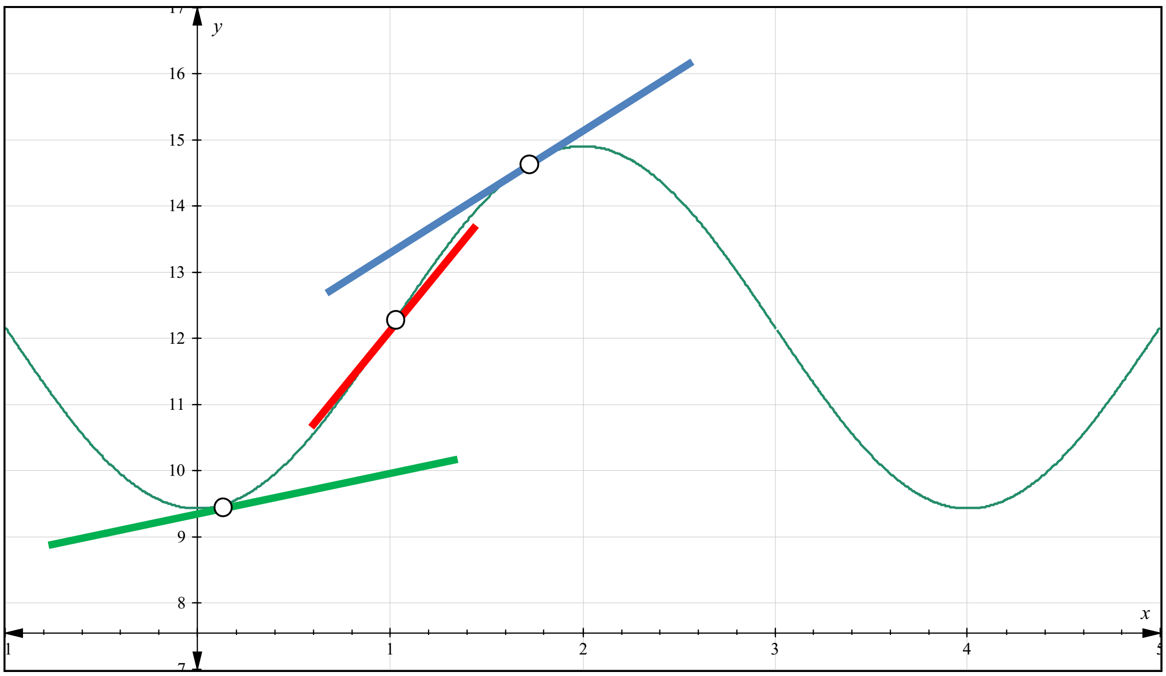


Analysis: Ultimately, this reveals the almost amazing accuracy of the sine graph in terms of finding the amount of sunlight on a certain date of the year. The difference was miniscule, almost even dead on precise in a case like August 15th, where my graph found the result with a 0.006% percentage error. The greatest difference in contrast was November 4th with 1.5%.

PART III. Rate of Change

Rate of change is truly interesting, especially in an applicable case like this. In this case I will start off with a direct analysis of the rate of change (found by the derivative of the curve on a certain point) in various different points.

In terms of Calculus, we know that as the derivative or gradient of a certain point leans closer to the horizontal, its derivative value will be less than when it is more erect to the vertical side.



In my self-made example diagram here, we can see three gradients (slopes) of this exact curve. Interestingly, the green gradient, as it leans more than the blue gradient, will have a smaller rate of change. However the red gradient will have the greatest rate of change in the whole graph because it is situated right at the inflection point where the peak of the rate of change value will be. Here the concavity of the graph modifies, and at the same time the graph reaches its greatest change.

Now looking at concrete examples from the actual sunlight amount in Pyongyang, it is possible to determine how this change can fit into the context of the actual project.

Here I have the difference in sunlight time from January 9 to 13.

Jan 9, 2008	7:56 AM	5:32 PM	9h 35m 33s	+ 1m 02s
Jan 10, 2008	7:56 AM	5:32 PM	9h 36m 40s	+ 1m 06s
Jan 11, 2008	7:56 AM	5:33 PM	9h 37m 49s	+ 1m 09s

Between the first two days the difference is 1 minute and 6 seconds, which increases over time as January continues

Jan 12, 2008	7:55 AM	5:34 PM	9h 39m 02s	+ 1m 12s
Jan 13, 2008	7:55 AM	5:36 PM	9h 40m 17s	+ 1m 15s

Thus we can conclude that the same amount indeed does not change every day, rather increases in January. However, by May:

May 15, 2008	5:24 AM	7:43 PM	14h 18m 31s	+ 1m 48s
May 16, 2008	5:23 AM	7:44 PM	14h 20m 18s	+ 1m 46s
May 17, 2008	5:22 AM	7:45 PM	14h 22m 03s	+ 1m 44s
May 18, 2008	5:22 AM	7:45 PM	14h 23m 45s	+ 1m 42s

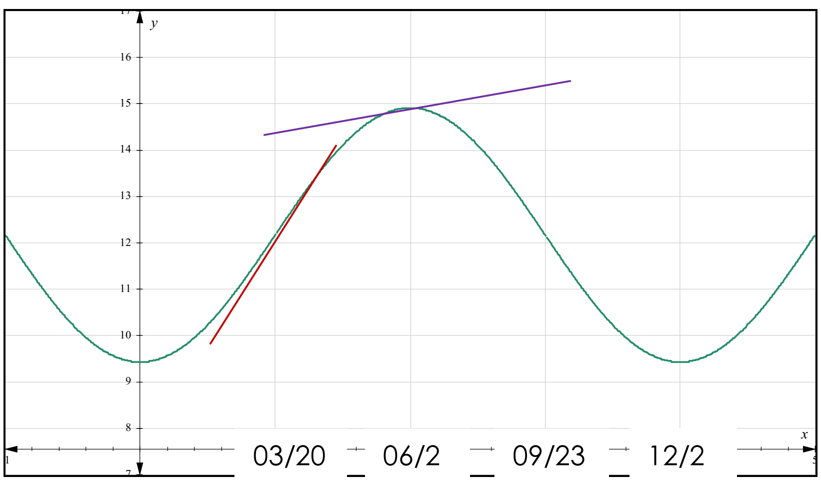
There is a notable decrease as sunlight decreases by seconds.

We can see that somewhere between January and May, there was a peak, because the rate of change is decreasing by May when it was increasing in January. And interestingly, the peak was exactly at the Spring Equinox in March 20th:

Mar 7, 2008	+ 2m 32s
Mar 20, 2008	+ 2m 33s
Mar 28, 2008	+ 2m 32s

For length reasons I skipped a few dates, but it shows that the peak is in March 20!

Why would this make sense? The answer is in the sine/cosine graph and maybe a bit of Calculus.



Considering the shape of the curve, if we draw gradients it will be possible to determine the state of the rate of change, which is in fact the derivative of the graph.

The purple gradient indicates the rate of change somewhere in early June, let's say June 5th. According to my sources, the rate of change of the sunlight will be + 54s.

Now, the dark red gradient indicates a the rate of change somewhere near March, the Spring Equinox, where interestingly the median sunlight is reached. Let's say March 16th, which is 2m 33s.

Now looking at the shape of the gradients, this suddenly becomes amazingly logical. The purple gradient has a smaller slope as it leans horizontally, so naturally, its rate of change is going to be less than the red slope, which is leaning more vertically and has a larger rate of change. And so near the maximum and the minimum of the sine curve, the days will have less change of sunlight than in the middle seasons of Spring and Autumn near March and September, when the change of sunlight will be greater. In terms of Calculus, the fact is that the gradient is the derivative of the curve at a certain point, and will reflect the change in that certain period.

Now nearing the end, it is possible to determine how the Equinoxes and the Solstices fit into the entire system. The most amount of sunlight in the year will correspond to the time when the sun is directly facing a part of the earth, which is the Summer Solstice. The Winter Solstice is the exact opposite, when the earth is most inclined away from the sun. The Spring and Autumn equinoxes are when the earth is not inclined toward or away from the sun, and is the exact median of the amount of sunlight a certain region will get. Thus the Summer (06/21) and Winter Solstices (12/21) will theoretically occur in the days when the most sunlight is received and the least sunlight is received respectively, and the Equinoxes will occur when the rate of change in the amount of sunlight received peaks in Spring (03/20) and Autumn (9/23).

PART IV. The Sine Regression

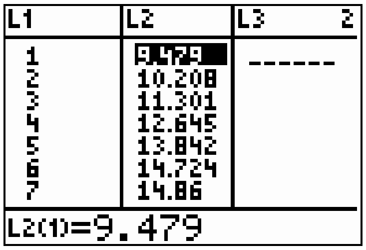
Finally, using the website www.timeanddate.com, I was able to find the hours of sunlight for each day of the month. Putting it simply in a graph it was:

Date	Time (in H:M:S)	Time (Hours)
1-Jan	9:28:45	9.479
1-Feb	10:12:29	10.208
1-Mar	11:19:51	11.301
1-Apr	12:38:44	12.645
1-May	13:50:30	13.842
1-Jun	14:43:27	14.724
1-Jul	14:51:36	14.86
1-Aug	14:10:51	14.181
1-Sep	13:01:55	13.032
1-Oct	11:46:59	11.783
1-Nov	10:32:08	10.536

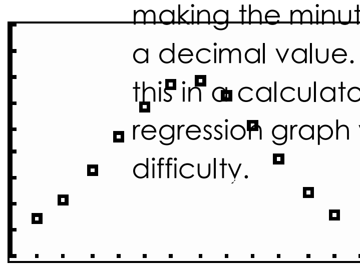
1-Dec	9:38:16	9.638

The time in the original hour/minute/second format was given by the website, I then converted this time into only hours, making the minutes and seconds into a decimal value. So now I could put this in a calculator to get a sine regression graph with no real difficulty.

(1) The next step was to input these values into a list in my TI-84 Plus Calculator.

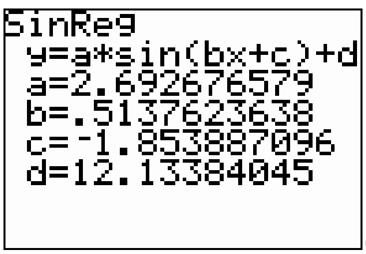


(1)

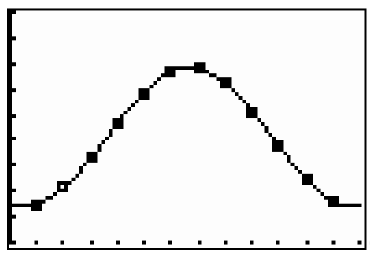

(2)

(2) Then I plotted the values onto the graph, getting a series of points indicating the amount of sunlight.

(3) Now it was time for the sine regression graph. Using the STAT- CALC function, I found the sine regression.



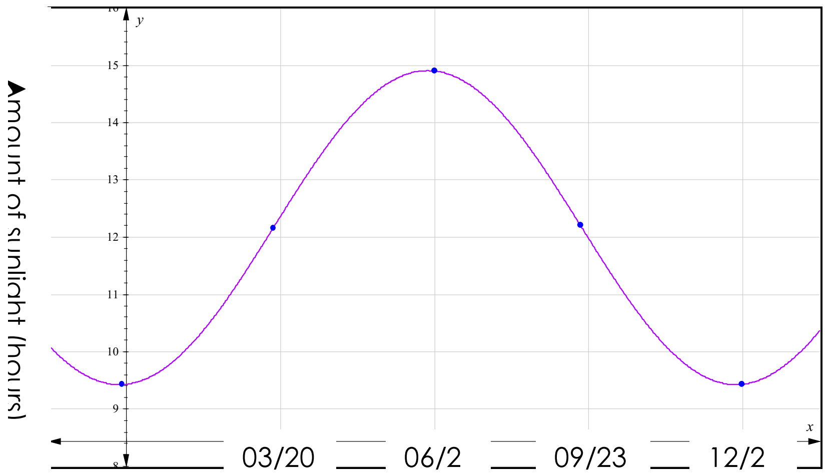
(3)


(4)

(4) Finally I let the calculator graph this equation, rendering a nice sine regression function concerning all the monthly values of sunlight amount.

Comparison:

This last graph I got from the calculator almost amazingly similar to the "theoretical" sine graph I had made using only the shortest and longest days of the year. However, for the one I made, for the sake of facility I used 4 as the input number. Thus the x was actually seasonal rather than monthly. However, that didn't affect the results all that much because to calculate the values I divided all my results by 4 to make it compatible with my graph.



For convenience here is the "theoretical graph that I made with the most and least amounts of sunlight.

What is interesting with the calculator's graph is the median amount of sunlight, which is given by the "d" or the vertical shift value in Figure (3). The median sunlight I calculated with my own graph was very close, which was 12.169, compared to the calculators 12.134. That, I feel, gives credibility to my own results. Since the vertical shift was similar between the two graphs, no doubt the amplitude would be similar, and it was. The calculator gave it at 2.693, while mine was 2.741.

Ultimately, other than the input increment, which mine was at 4 and the graph's was at 12, there was no great difference, making this an overall worthwhile effort.

Bibliography:

The amazingly vital data: Time and Date.com

http://www.timeanddate.com/worldclock/astronomy.html?n=205&month=3&year=2008&obj=sun&afl=-11&day=1

Information on Solstices and Equinoxes:

http://en.wikipedia.org/wiki/Solstice

http://en.wikipedia.org/wiki/Equinox

TI-84 Plus Calculator for calculations

GraphingPackage program for graphs

Microsoft Excel for Charts