In this assignment, I will use WinPlot, a graphing display program.

1. Find an expression for the volume of the cone in terms of r and θ.

The formula for the volume of a cone is: V = 1/3 x height x base area

To find the base area, we must find the radius of the base.

The circumference of the base of the cone = the length of arc ABC
Therefore:
$2\pi \times r_{base} = r\theta$
$r_{base} = r\theta / (2\pi)$.

The area of the base can therefore be calculated:
$A_{base} = \pi \times r_{base}{}^2 = \pi \times (r\theta / (2\pi))^2$

Next, we must find the height of the cone, h.
Notice that for the cone, h r_{base} r is a right angled triangle, with r as the hypotenuse.
Therefore, using the Pythagorean Theorem, we can find h.

$r^2 = h^2 + R^2$
$r^2 = h^2 + (r\theta/2\pi)^2$
$h^2 = r^2 - (r\theta/2\pi)^2$
$h = \sqrt{} [r^2 - (r^2\theta^2/4\pi^2)]$

Therefore, substituting the values for r_{base} and h, we can find the volume of the cone.
V = 1/3 x height x base area
$V = 1/3 \times \sqrt{} [r^2 - (r^2\theta^2/4\pi^2)] \times \pi (r^2\theta^2/4\pi^2)$

2. By using the substitution x = θ/2π , express the volume as a function of x.

$x = \theta/2\pi$
$\theta = x2\pi$

$V = 1/3 \times \sqrt{} [r^2 - (r^2\theta^2/4\pi^2)] \times \pi (r^2\theta^2/4\pi^2)$

$V = 1/3 \times \sqrt{} [r^2 - (r^2(x2\pi)^2/4\pi^2)] \times \pi (r^2(x2\pi)^2/4\pi^2)$
$= 1/3 \times \sqrt{} [r^2 - (r^2 4x^2\pi^2/4\pi^2)] \times \pi (r^2 4x^2\pi^2/4\pi^2)$
$= 1/3 \times \sqrt{} [r^2 - (r^2x^2)] \times \pi (r^2x^2)$

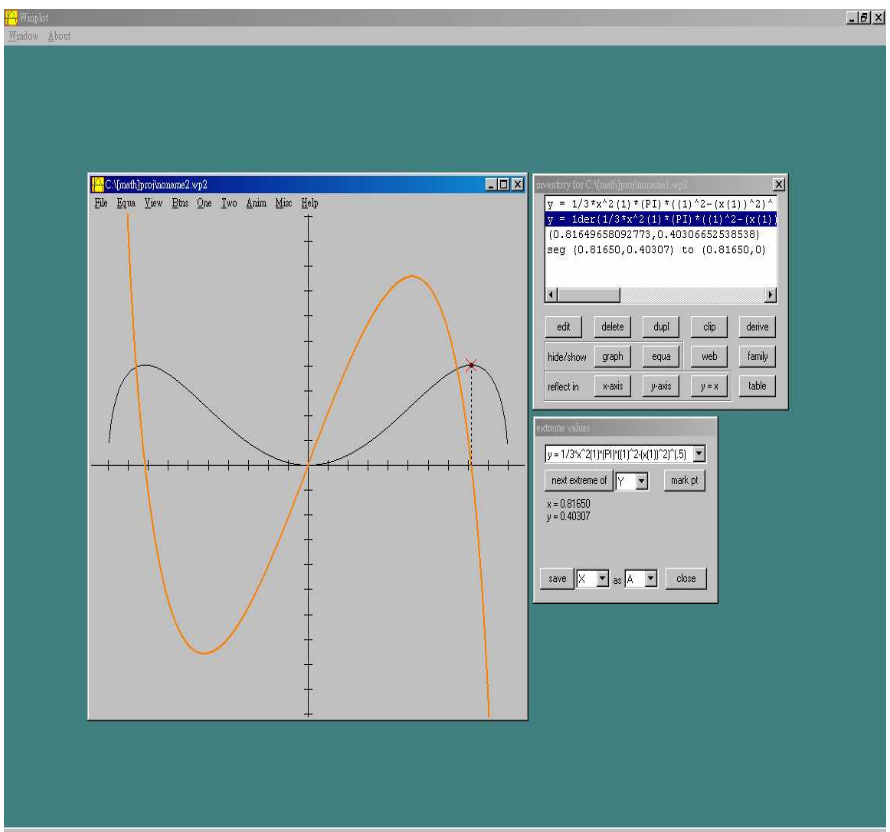
3. Draw the graph of this function using the calculator. Hence find the values of x and θ for which this volume is a maximum. Give your answer for x to four decimal places.

Since $x = \theta / (2\pi)$, the maximum x value will be a constant no matter what the value of r is (x represents the amount per proportion of paper used for creating the cone, which is a constant ratio. The value of r will change the volume of the cone, but x will remain the same). Therefore we can replace the r in the function with any constant number. And in this project, 1 is used for replacing r since it makes the function easier to calculate. Thus:

$V_{cone} = 1/3 \times \sqrt{[r^2 - (r^2x^2)]} \times \pi (r^2x^2)$
$V_{cone} = 1/3 \times \sqrt{[1^2 - (x^2 1^2]} \times \pi x^2 1^2$
$V_{cone} = 1/3 \times \sqrt{(1 - x^2)} \times \pi x^2$

In this function, $1/3\ \pi$ is constant. The shape of the function will be $x^2 \times \sqrt{(1 - x^2)}$

To find the maximum value of the cone, we can either:
Graph $V_{cone} = x^2 \times \sqrt{(1 - x^2)}$ and find the maximum value or
Graph the derivative of V_{cone} and find the zero/root of the function



Graph of V_{cone} (grey) and its derivative (orange)

The two maximum values are at $x = \pm 0.8174$. They both give a maximum cone volume of 0.3871.

To calculate the value of θ, for which the cone volume is a maximum:

$x = \theta / (2\pi) = \pm 0.8174$
Thus, $\theta = 2\pi x = 2\pi \times \pm 0.8165 = \pm 5.136$ rad

When the circle is cut there are two sectors so it is possible to make two cones.

4. Find the value or values of x for which the sum of the volumes of the two cones is a maximum.

To find the sum of the volumes of the two cones, we must find the volume of each individual cone. We will call them Cone A and Cone B.

From Question 1, we know the volume of Cone A:
$V_{cone\ (A)} = 1/3 \times \sqrt{}(r^2 - (x^2r^2) \times \pi\ x^2r^2$

The angle of Cone B = $2\pi - \theta$
$x_B = \theta / 2\pi$
$= 2\pi - \theta / 2\pi$
$= 1 - \theta / 2\pi$
$= 1 - x$

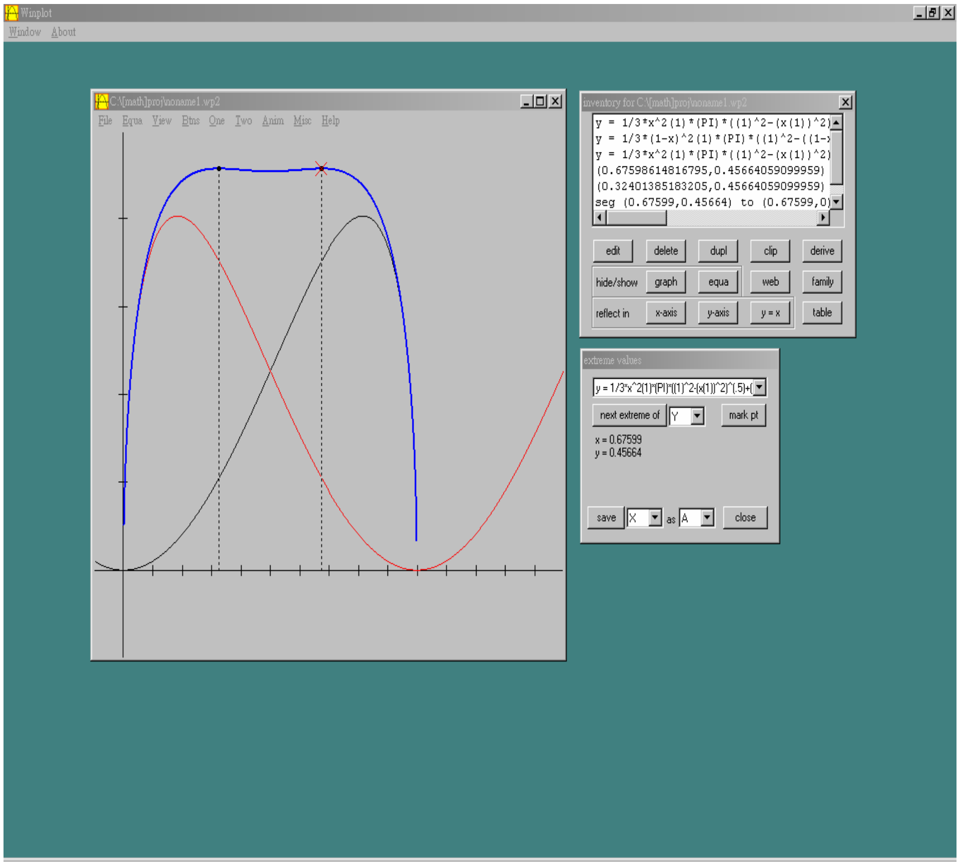
The volume of Cone B can be expressed as:
$V_{cone\ (B)} = 1/3 \times \sqrt{}[r^2 - (1-x)^2\ r^2] \times \pi\ (1-x)^2\ r^2$

Finding the sum of the two cones:

$V = V_{cone\ (A)} + V_{cone\ (B)}$
$= 1/3 \times \sqrt{}(r^2 - x^2r^2) \times \pi\ x^2r^2 + 1/3 \times \sqrt{}[r^2 - (1-x)^2\ r^2] \times \pi\ (1-x)^2\ r^2$

Like Question 3, we can replace the constant r with 1 for easier calculation.
We get: $V = 1/3\ \sqrt{}(1 - x^2) \times \pi\ x^2 + 1/3 \times \sqrt{}[1 - (1-x)^2\ ] \times \pi\ (1-x)^2$

Graphing this equation, we can find the maximum sum of the volumes of the two cones.



Graph of $V_{cone\ (A)}$ (grey), $V_{cone\ (B)}$ (red), and V_{sum} (blue).

The two maximum cone volumes we get are both 0.4566. The x values that give this maximum cone volume are 0.6760 and 0.3240.

Notice that these two x values are just complements of each other (they add up to 1). This means that x = 0.3240 and x – 1 = 0.6760. The maximum volume = 0.6760.

Similarly, this answer can also been found by finding the derivative of V_{sum} and equating it to zero. The zero/root of the function will be the maximum value.