

Alice Wang  
Fishing Rod Lab

Table 1. Leo's fishing rod

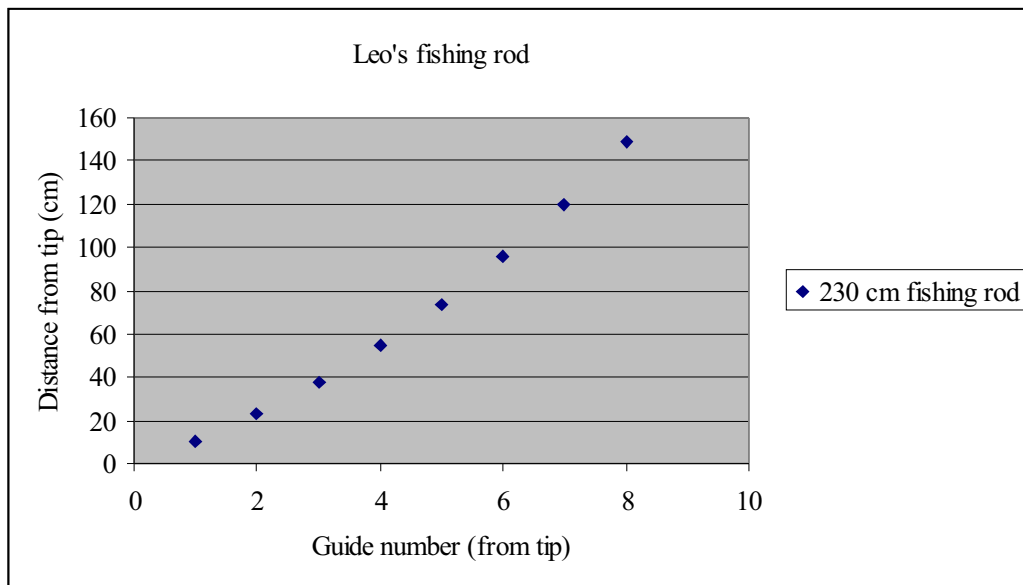
Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

Dependent variable: The distance from the tip of fishing rod

Independent variable: The guide number from the tip of the fishing rod

Parameters/Constraints: The distance from tip for each guide number does not follow a particular pattern. Hence it is difficult to achieve a function that satisfies all of the points on Table 1.

Graph 1. Leo's fishing rod



Consider general form of cubic and quadratic equations (where a,b,c,d are coefficients and where a is never zero):

$$y=ax^2+bx+c \text{ (quadratic)}$$

$$y=ax^3+bx^2+cx+d \text{ (cubic)}$$

$$10 = a + b + c$$

$$23 = 4a + 2b + c$$

$$38 = 9a + 3b + c$$

Put into matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 4 & 2 & 1 & 23 \\ 9 & 3 & 1 & 38 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 4 & 2 & 1 & 23 \\ 9 & 3 & 1 & 38 \end{bmatrix}$$

Original matrix

First, to clear the first 0 of the column, the first row has to be multiplied by 9 and subtracted from the second row.

$$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 0 & 2 & 3 & 17 \\ 9 & 3 & 1 & 38 \end{bmatrix}$$

**EXAMPLE:**  $(9) * (1)(1)(1)(10) =$

$$\begin{array}{r} 9 \ 9 \ 9 \ 90 \\ - \ 4 \ 1 \ 1 \ 23 \\ \hline 0 \ 2 \ 3 \ 17 \end{array}$$

To clear the second 0 of the first column, the second row has to be multiplied by 3 and be subtracted to the third row.

$$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 0 & 2 & 3 & 17 \\ 0 & 6 & 8 & 52 \end{bmatrix}$$

**EXAMPLE:**  $(3) * (0)(2)(3)(17) =$

$$\begin{array}{r} 0 \ 6 \ 9 \ 51 \\ - \ 0 \ 6 \ 8 \ 52 \\ \hline 0 \ 0 \ 1 \ -1 \end{array}$$

To clear the second column's last row to a 0, the second row has to be multiplied by  $\frac{1}{2}$ .

**EXAMPLE:**  $(\frac{1}{2}) * (0)(2)(3)(17) =$

$$\begin{array}{r} 1 \ 1 \ 1 \ 10 \\ - \ 0 \ 1 \ 3/2 \ 11/2 \\ \hline 1 \ 0 \ -1/2 \ 3/2 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 10 \\ 0 & 2 & 3 & 17 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

To clear the second column's first row to a 0, the last row has to be multiplied by  $-\frac{1}{2}$ .

$$\begin{bmatrix} 1 & 0 & -1/2 & 3/2 \\ 0 & 2 & 3 & 17 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

**EXAMPLE:**  $(1/2) * (0)(2)(3)(17) =$

$$\begin{array}{r} 1 \ 1 \ 1 \ 10 \\ - \ 0 \ 1 \ 3/2 \ 11/2 \\ \hline 1 \ 0 \ -1/2 \ 3/2 \end{array}$$

To clear the third column's third row to a 0, the last column should be multiplied by 3

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 3 & 17 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

**EXAMPLE:**  $(3) * (0)(0)(1)(-1) =$

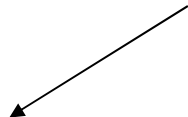
$$\begin{array}{r} 0 \ 2 \ 3 \ 17 \\ - \ 0 \ 0 \ 3 \ -3 \\ \hline 1 \ 0 \ 0 \ 20 \end{array}$$

Finally to make the second column contain a 1, the second row is divided by 2.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 20 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Ex:  $(1/2) * (0)(2)(0)(20) =$

$$(0)(1)(0)(10)$$



$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

All that is left is the coefficients of the equation:

$$a=1, b=10, c=-1$$

therefore the quadratic equation:  $y=x^2+10x-1$

Table 3. Data points of Quadratic Function

1	10
2	23
3	38
4	55
5	74
6	95
7	118
8	143

The process that I used for the cubic function was by calculator.

$$y= ax^3 +bx^2 +cx+ d \text{ (cubic)}$$

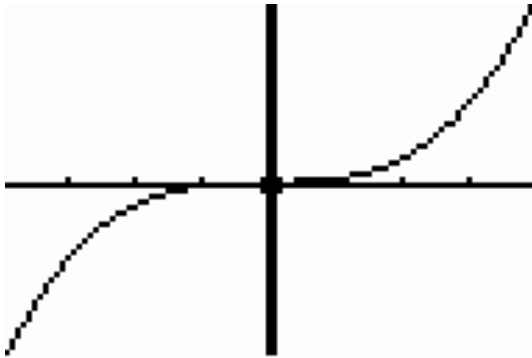
```
CubicReg
y=ax3+bx2+cx+d
a=.0631313131
b=.3917748918
c=11.70959596
d=-2.285714286
```

Table 2. Data points of Cubic Function

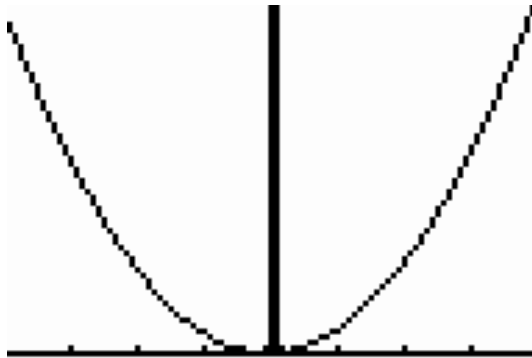
1	9.869
2	23.186
3	38.043
4	54.818
5	73.889
6	95.634

7	120.43
8	180.69

Thus, the equation of the cubic equation is  $y = .063x^3 + .392x^2 + 11.7x - 2.286$



Cubic Function Graph (above)



Quadratic Function (above)

The main difference between quadratic and cubic functions is that a quadratic equation is a polynomial fit using three points while a cubic function uses four or more points and is a polynomial regression. With a higher degree for a polynomial function, there is a less degree of accuracy, especially when the guide number increases. The quadratic model is more precise than the quadratic function when the guide number reached 7 and 8.

I used the function  $p(x) = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$  to find a function which passes through every data point in Table 1. With the 7<sup>th</sup> degree in a polynomial, it will pass through every data point of Leo's fishing rod.

Substitution of Table 1's x and y values

to  $p(x) = ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h$  is as followed:

$$10 = a + b + c + d + e + f + g + h$$

$$23 = 128a + 64b + 32c + 16d + 8e + 4f + 2g + h$$

$$38 = 2187a + 729b + 243c + 81d + 27e + 9f + 3g + h$$

$$55 = 1681a + 4096b + 1024c + 256d + 64e + 16f + 4g + h$$

$$74 = 8125a + 15625b + 3125c + 625d + 125e + 25f + 5g + h$$

$$96 = 27936a + 46656b + 7776c + 1296d + 216e + 36f + 6g + h$$

$$120 = 82353a + 117649b + 16807c + 2401d + 343e + 49f + 7g + h$$

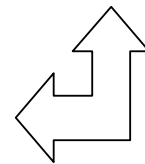
$$149 = 207152a + 32144b + 3738c + 496d + 512e + 64f + 8g + h$$

Using the coefficients of each equation, I plugged it in the calculator by a 8\*9 matrix. Then using the reduced row echelon, I found the coefficient of the polynomical function which passes through every data point.

Table 3. Reduced Row Echelon of Polynomial Function

1	0	0	0	0	0	0	0	.00258
0	1	0	0	0	0	0	0	-.0778
0	0	1	0	0	0	0	0	.9555
0	0	0	1	0	0	0	0	-6.153
0	0	0	0	1	0	0	0	22.351
0	0	0	0	0	1	0	0	-43.769
0	0	0	0	0	0	1	0	55.790
0	0	0	0	0	0	0	1	-19

Values of last line is used to solve polynomial function.



$$y = .02x^7 - .077x^6 + .95x^5 - 6.15x^4 + 22.35x^3 - 43.79x^2 + 55.79x - 19$$

The polynomial function above should be the most accurate out of all the equations because it is based on all of the guide numbers of the fishing rods. With a higher degree of power for a polynomial, there should be a more accurate and fitting equation.

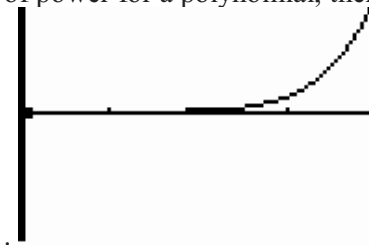


Table 4. Polynomial function table of values.

1	10.099
2	23.752
3	39.839
4	54.448
5	51.475
6	-12.98
7	-241.3
8	-830.2

I found the table of values for the polynomial function did not provide accurate results. When the guide number reached 5, the distance from tip dramatically decreased.

Using technology, I found the quadratic regression as another function that fit the data.

```
QuadReg
y=ax2+bx+c
a=1.244047619
b=8.458333333
c=.8392857143
```

Thus the equation figures is  $1.24x^2 + 8.458x + .839$



The data points of the quadratic regression formula is extremely accurate. The distance from tip points (y values) are only off by a tenth or so compared to the original data points.

Ironically, I found the quadratic model the most accurate out of the three functions. The data points on the table were closely associated to the original data points than the cubic or high degree of polynomial function. When both the degree of a function and the guide number gets higher, the accuracy and precision decreases dramatically.

Based on the quadratic model of  $y=x^2+10x-1$ , adding a ninth guide from the the tip of the fishing rod could be accomplished easily. Using the table of values of the graph, it was found that the distance from the tip would be 170 centimeters, which is 60 centimeters from the end of the fishing rod (shown below).

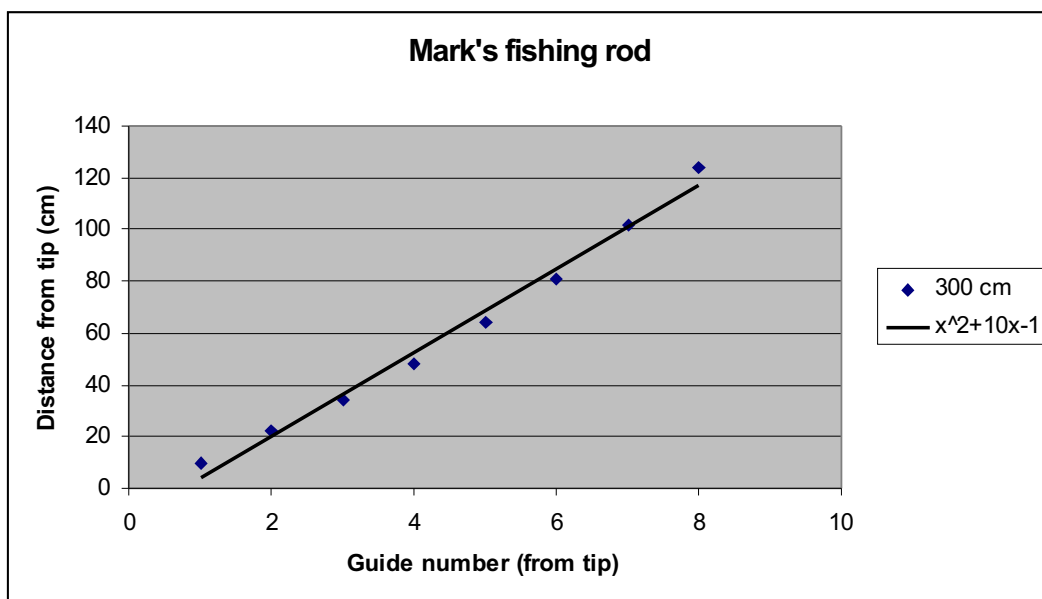
X	Y <sub>1</sub>	
5	74	
6	95	
7	118	
8	143	
9	170	
10	199	
11	230	

X=9

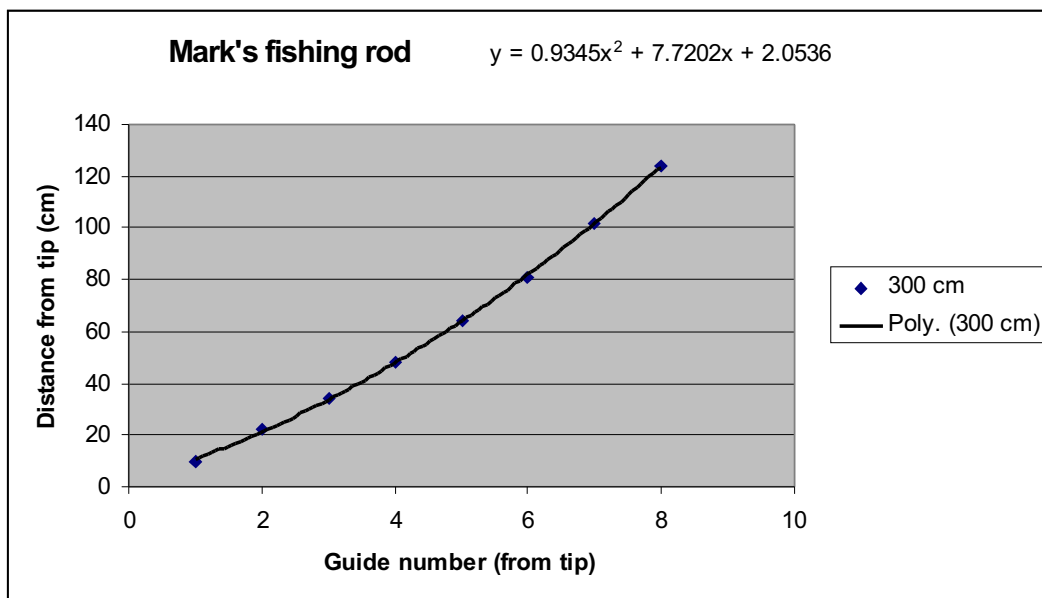
Table 5. Mark's fishing rod

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	22	34	48	64	81	102	124





The quadratic model of  $y = x^2 + 10x - 1$  fits the new data adequately. Although the fit of the quadratic function is not perfect, it closely represents the data provided.



Using Excel, the line of best fit was considered  $y = 0.9345x^2 + 7.7202x + 2.0536$  as a polynomial function. The quadratic model of  $y = x^2 + 10x - 1$  and the line of best fit are extremely similar (demonstrated on the graph below). The narrower function is the line of best fit and the wider function is the quadratic model. My quadratic model could be more vertically compressed by multiplying the value of  $a$  (from  $y = a^2 + bx + c$ ) by a positive integer. The limitations of the quadratic model is that it is only suited for fishing rods that are similar to Leo's. If the distance from tip of each guide number of a fishing rod is substantially different that Leo's fishing rod, the quadratic model of  $y = x^2 + 10x - 1$  will not work. In this case, Mark's distance from tip of each guide number was similar to Leo's. Thus, the quadratic model worked for Mark's fishing rod.

