FISHING RODS **SL TYPE II**

This portfolio deals with Leo's fishing rod which has an overall length of 230 cm together with eight guides that are placed a certain distance from the tip of the fishing rod as shown in Table 1. The task therefore is to develop mathematical models for the placement of the line guides on the fishing rod using quadratic, cubic, polynomial and one other free function. In addition, the quadratic model function that is developed will be further tested by applying it to Mark's fishing rod which has an overall length of 300 cm and eight guides.

Table 1. Number of guides, with respective distances from the tip, on Leo's fishing rod

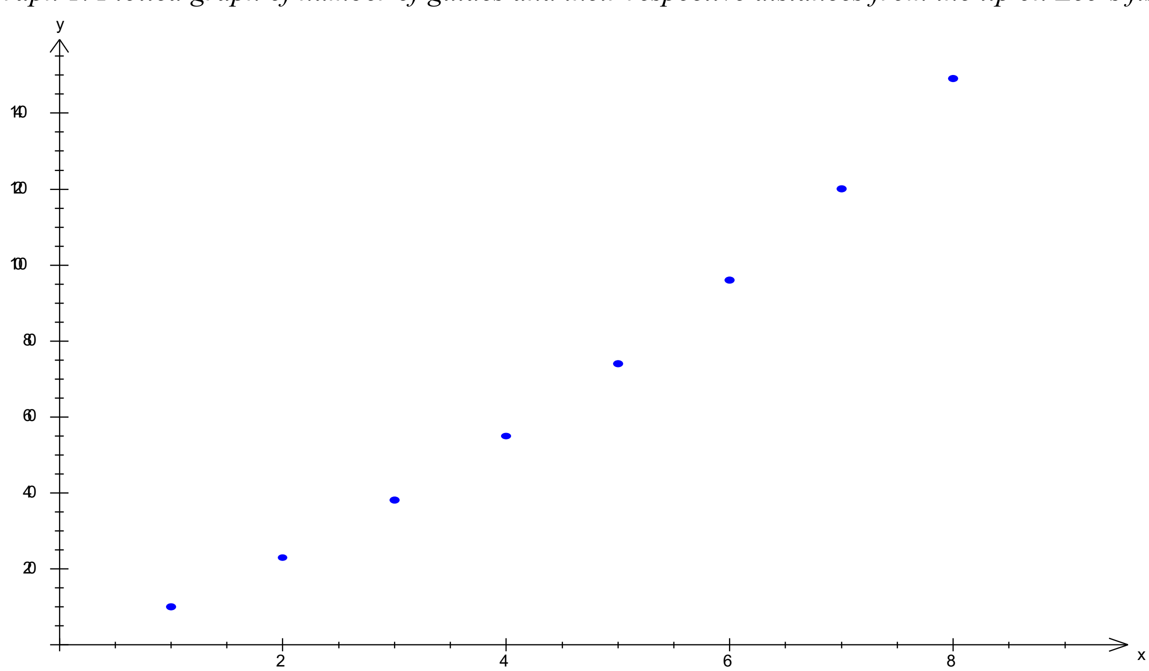
Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

Before beginning the process of formulating different mathematical models, it is possible to mention certain constrains as well as variables. The two variables in this modeling are

g= Guide number (from tip)   and   X= distance from tip (cm)

One constrain of the data presented in *Table 1* is that there cannot be negative number of guides or negative distances from the tip. As a result the plotted graph is limited to the first quadrant as seen below from *Graph 1.* Also the placement of the guides (distance) from the tip does not follow a regular pattern. This might make it difficult to achieve a function that satisfies all the points.

Graph 1. Plotted graph of number of guides and their respective distances from the tip on Leo's fishing rod



it can also be seen from *Graph 1* that the plotted data of Leo's fishing rod begins from (1, 10) and goes up to (8,149) forming an ascending pattern. Thus this pattern indicates that the constant of the highest index on each of the model functions must be positive.

The next step involves finding a quadratic function that can be used as a model. Keeping in mind the general expression of a quadratic function, which is in the form of $X = Ag^2 + Bg + C$, it is possible to apply the matrix method to find the unknown variables *A, B, C*. In this case g will represent guide number while X will be equivalent to the distance from the tip.

To find the three unknowns *A, B, C* the matrix method requires three equations to be formed. This can be achieved by choosing three guide lines with their respective distance and forming three equations by substituting them in the place of g and X. So I have chosen to take the first, last guides and their respective distances from the tip since they are the beginning and the ending points of the problem. The fourth guide was also chosen since it is a middle point. The equations are as follows:-

Equation 1: $10 = A(1) + B(1) + C$
Equation 2: $55 = A(16) + B(4) + C$
Equation 3: $149 = A(64) + B(8) + C$

Once the above system of equations are formed it is possible to translate them into matrices as follows

$$\mathbf{A}=\begin{bmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{bmatrix}, \mathbf{Y}=\begin{bmatrix} A \\ B \\ C \end{bmatrix}, \mathbf{B}=\begin{bmatrix} 10 \\ 55 \\ 149 \end{bmatrix}$$

Martix **A** represents the values of the chosen guides in the quadratic equations while matrix **Y** consists of the unknown constants. Matrix **B** is the placement distance for each of the chosen guides. Now it is possible to apply the following matrix formula to find the unknowns,

$A^{-1}AY = A^{-1}B$, where A^{-1} is the inverse of the matrix A

$$\begin{bmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 55 \\ 149 \end{bmatrix}$$

Using technology the answer will be:-

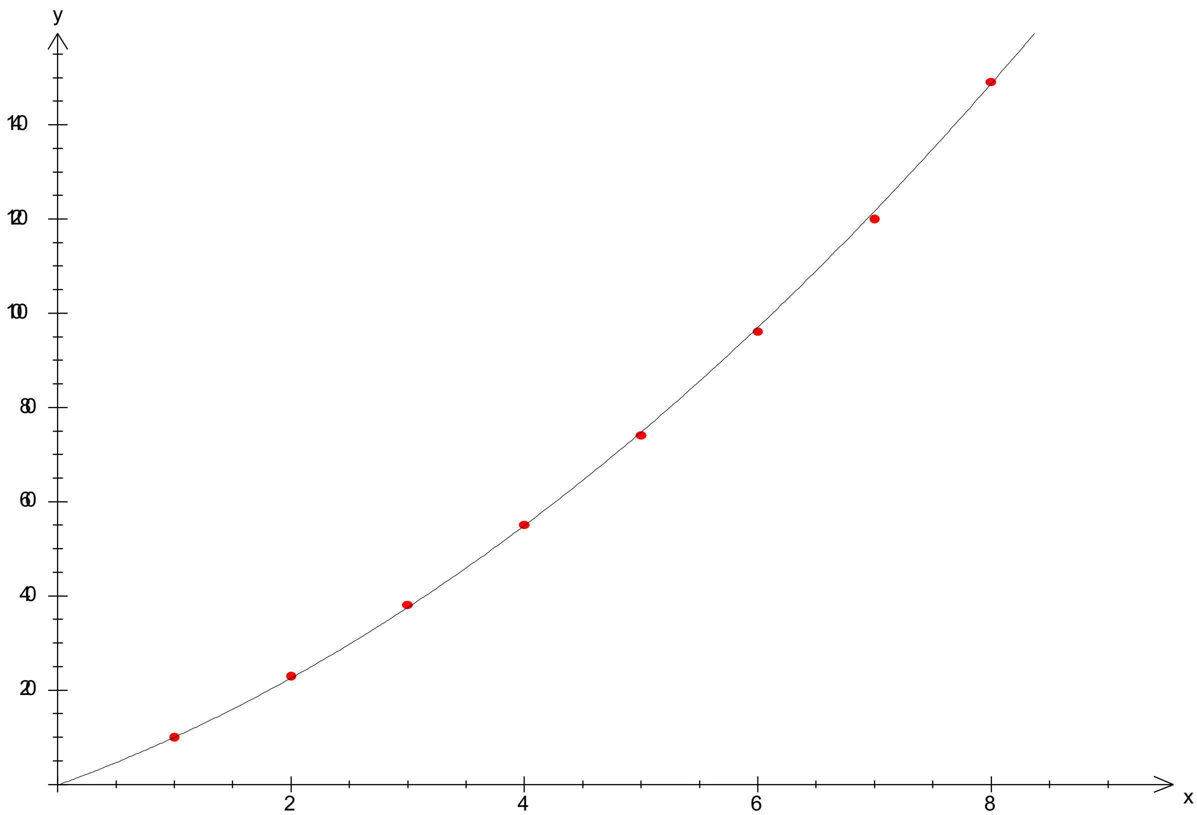
$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1.2142857142857 \\ 8.9285714285714 \\ -0.1428571428571 \end{bmatrix}$$

Then A= 1.21, B= 8.93, C= -0.143 (*three significant figures*). By substituting these values into the general expression of quadratic functions ($X = Ag^2 + Bg + C$), it is possible to formulate a quadratic model for Leo's fishing rod which is :-

$$X = 1.21g^2 + 8.93g - 0.143$$

The pictorial representation of the quadratic model function with the original data points is as follows:-

Graph 2. Plot of the quadratic model function



From the above graph it can be seen that the quadratic function only passes properly through 3 points making it far from being a desired model function.

A cubic model can also be formulated using the same matrix method that was explained above. The only difference is that the general expression for cubic functions is X= $Ag^3 + Bg^2 + Cg + D$ with four unknowns *A, B, C, D* thus requiring that the system of equations consist of four equations instead of three. I have chosen to take guide number one, three, five and eight.

Equation 1: $10 = A\,(1) + B\,(1) + C\,(1) + D$

Equation 2: $38 = A\,(27) + B\,(9) + C\,(3) + D$

Equation 3: $74 = A\,(125) + B\,(25) + C\,(5) + D$

Equation 4: $149 = A\,(512) + B\,(64) + C\,(8) + D$

Then,
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 10 \\ 38 \\ 74 \\ 149 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 38 \\ 74 \\ 149 \end{bmatrix}$$

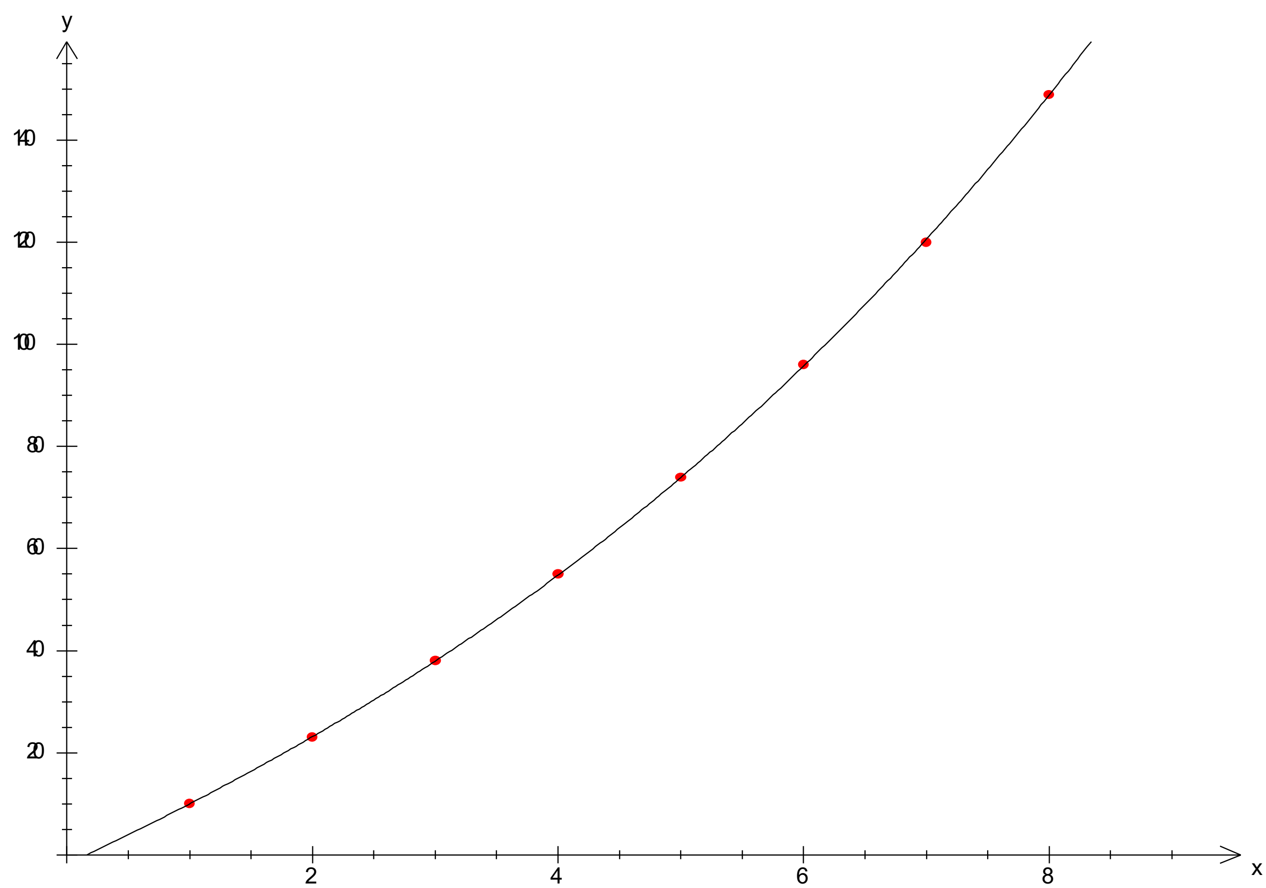
By using technology the answer is:-

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0.0571428571429 \\ 0.4857142857243 \\ 11.3142857142857 \\ -1.8571428571429 \end{bmatrix}$$

By approximation, A= 0.0571, B= 0.486, C= 11.3, D=-1.86, resulting in a cubic model function of

$$X= 0.0571g^3 +0.486g^2 +11.3g -1.86$$

Graph 3. Plot of the cubic model function together with original data points



The cubic model fits better than the quadratic one since it passes through 4 points precisely which is one more than the quadratic model making it more accurate. By analysing the previous two models one can notice that the quadratic model which had three unknowns constants passed through three points properly and the cubic model which had four unknown constants passed through 4 points properly. So if a model function needs to pass through all the points it is necessary to have a function with eight unknowns and that can be solved by the matrix method using 8 set of equations. Hence, a 7^{th} degree polynomial with 8 unknowns enables the formation of 8 equations that can be solved by the matrix method to result in a model that passes through all data points. The general expression for a 7^{th} degree polynomial is

$$X = Ag^7 + Bg^6 + Cg^5 + Dg^4 + Eg^3 + Fg^2 + Gg + H$$

Equation 1: $10 = A(1)^7 + B(1)^6 + C(1)^5 + D(1)^4 + E(1)^3 + F(1)^2 + G(1) + (1)$
Equation 2: $23 = A(2)^7 + B(2)^6 + C(2)^5 + D(2)^4 + E(2)^3 + F(2)^2 + G(2) + (2)$
Equation 3: $38 = A(3)^7 + B(3)^6 + C(3)^5 + D(3)^4 + E(3)^3 + F(3)^2 + G(3) + (3)$
Equation 4: $55 = A(4)^7 + B(4)^6 + C(4)^5 + D(4)^4 + E(4)^3 + F(4)^2 + G(4) + (4)$
Equation 5: $74 = A(5)^7 + B(5)^6 + C(5)^5 + D(5)^4 + E(5)^3 + F(5)^2 + G(5) + (5)$
Equation 6: $96 = A(6)^7 + B(6)^6 + C(6)^5 + D(6)^4 + E(6)^3 + F(6)^2 + G(6) + (6)$
Equation 7: $120 = A(7)^7 + B(7)^6 + C(7)^5 + D(7)^4 + E(7)^3 + F(7)^2 + G(7) + (7)$
Equation 8: $149 = A(8)^7 + B(8)^6 + C(8)^5 + D(8)^4 + E(8)^3 + F(8)^2 + G(8) + (8)$

The resulting matrices are:-

$$\mathbf{A}=\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2 & 1 \\ 3^7 & 3^6 & 3^5 & 3^4 & 3^3 & 3^2 & 3 & 1 \\ 4^7 & 4^6 & 4^5 & 4^4 & 4^3 & 4^2 & 4 & 1 \\ 5^7 & 5^6 & 5^5 & 5^4 & 5^3 & 5^2 & 5 & 1 \\ 6^7 & 6^6 & 6^5 & 6^4 & 6^3 & 6^2 & 6 & 1 \\ 7^7 & 7^6 & 7^5 & 7^4 & 7^3 & 7^2 & 7 & 1 \\ 8^7 & 8^6 & 8^5 & 8^4 & 8^3 & 8^2 & 8 & 1 \end{bmatrix}, \mathbf{Y}=\begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{bmatrix}, \mathbf{B}=\begin{bmatrix} 10 \\ 23 \\ 38 \\ 55 \\ 74 \\ 96 \\ 120 \\ 149 \end{bmatrix}$$

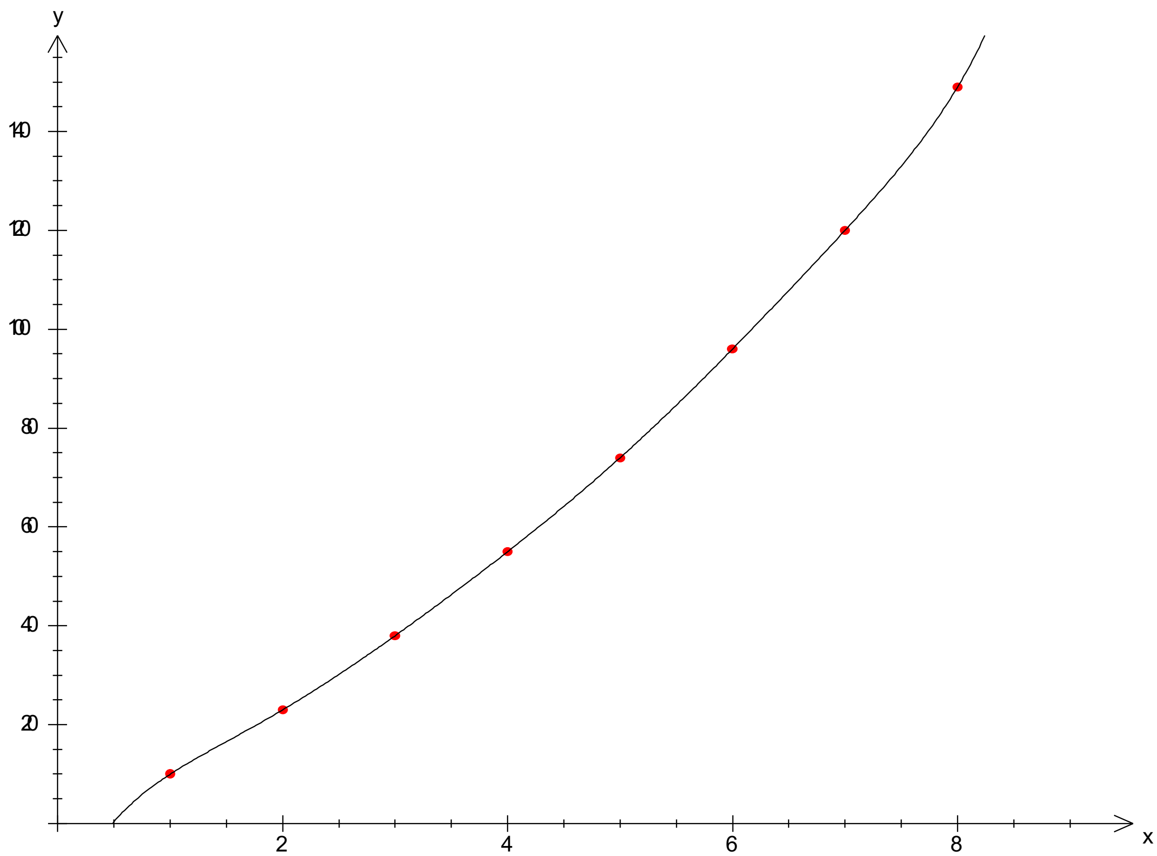
By multiplying **A** and **B** with the $\mathbf{A}^{-1}$ it is possible to isolate **Y** and then find the unknowns. Using technology the result is

$$\mathbf{Y}=\begin{bmatrix} 0.0025793651 \\ -0.0777777777 \\ 0.95555552 \\ -6.152777775 \\ 22.25138888 \\ -43.76944443 \\ 55.79047617 \\ -18.999999999 \end{bmatrix}$$

By substituting the values of **Y** in the general expression, the polynomial model function will be

$X=0.0025793651g^7 - 0.0777777777g^6 +0.95555552g^5 – 6.152777775g^4 +22.25138888g^3 -43.76944443g^2 +55.79047617g – 18.999999999$

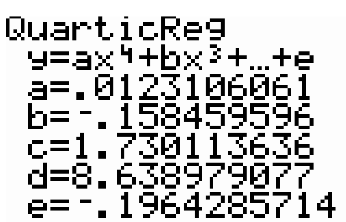
Graph 4. Polynomial model plot together with original data points



As seen from *Graph 4* the polynomial model fits all the data points making it a desired model function. Using the GDC it is possible to form a quartic function which can model Leo's fishing rod. The general expression for a quartic function is

$$X=Ag^4 + Bg^3+ Cg^2+ Dg +E$$

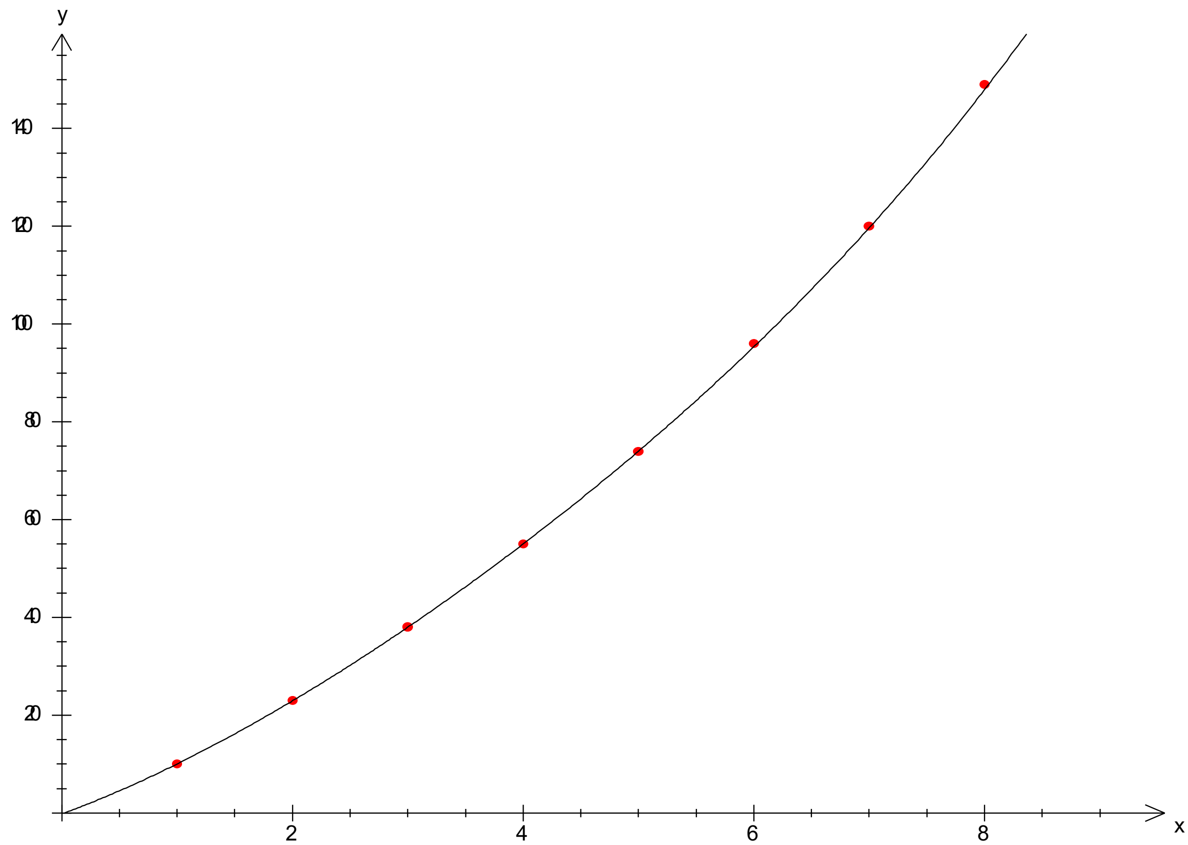
Using the GDC the unknown constants are found as seen below:-



Then by substituting the constants into the general expression, the model function will be

$$X= 0.012g^4-0.158g^3 +1.73g^2 + 8.64g-0.196$$

Graph 5. Plot of the quartic model function



The quartic model function passes through 5 points effectively. So it is more accurate than the quadratic or cubic model functions. By analyzing the four different model functions, it can be concluded that the higher the function is powered, the more it fits to the data of Leo's fishing rod. As seen from the graphs of all the model function formed using quadratic, cubic, quartic and polynomial functions , the 7^{th} degree polynomial is the best fit model since it passes through all the data points without being off by a significant making it the desired model.

The application used by the GDC to find the quartic function is called regression which finds the best fit line of a certain data in a graph. Thus it is possible to compare the quadratic and cubic model functions found using the matrix method to their respective regression found using the GDC.

Table 2. comparison of matrix method and best fit line (regression)

Function type	Matrix method	Regression (GDC)
Quadratic	$X = 1.21g^2 + 8.93g - 0.143$	$X = 1.24g^2 + 8.46g - 0.839$
Cubic	$X= 0.0571g^3 + 0.486g^2 + 11.3g - 1.86$	$X= 0.631g^3 + 0.392g^2 + 11.7g - 2.29$

As seen from Table 2 there is a slight difference between the functions formed using the matrix method and using the calculator. Even though the GDC is more precise, thus making it a reliable device, the matrix method is always handy in situations where technology is not available.

It is also possible to determine where a ninth guide can be placed on Leo's fishing rod using the quadratic function.

$$X = 1.21(9)^2 + 8.93(9) - 0.143$$
$$X = 178.2 \text{ cm}$$

One of the implications of adding a ninth guide is that it can shorten the line causing difficulty in fishing.

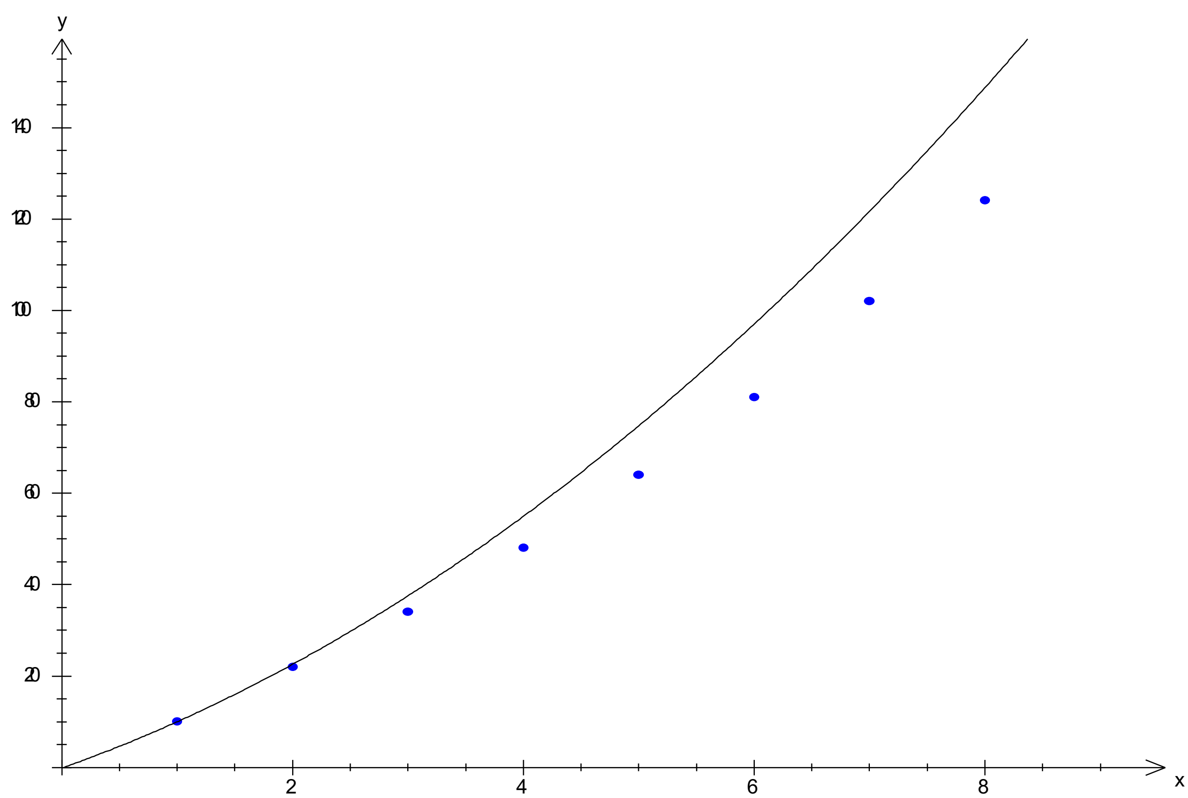
To test the quadratic model function further it can be applied to Marks's fishing rods which is presented in the table below

Table 3. Number of guides, with respective distances from the tip, on Mark's fishing rod

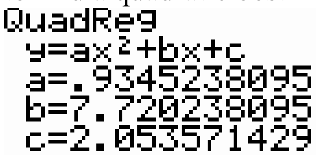
Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	22	34	48	64	81	102	124

To see how well the quadratic model fits the new data it can be plotted as follows:-

Graph 6. Quadratic model, $X = 1.21g^2 + 8.93g - 0.143$, versus marks's fishing rod



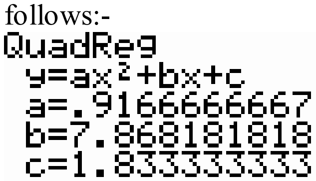
The quadratic model does not fit very well to the new data. It only passes through the first 2 points and missing the rest. In order to see the changes that are needed to improve the model the GDC can be used to find a quadratic best fit line for Mark's fishing rod. The result is


QuadReg
y=ax²+bx+c
a=.9345238095
b=7.720238095
c=2.053571429

The improved quadratic function is

$$X = 0.93g^2 + 7.72g + 2.05$$

One of the limitations of the model function is that it has to be modified every time an addition guide is added to the fishing rod to make it accurate. According to the improved quadratic function, $X = 0.93g^2 + 7.72g + 2.05$, if a ninth and tenth guides are to be added to Mark's fishing rod, they would be placed at a distance of 147 cm and 172 cm from the tip respectively. Then if the GDC is asked to give the best fit line, it would not give the one that is seen above. Instead it will modify it and comes with a new set of values as follows:-


QuadReg
y=ax²+bx+c
a=.9166666667
b=7.868181818
c=1.833333333

The new modified function for Mark's fishing rod with ten guides is:-

$$X = 0.91g^2 + 7.87g + 1.83$$