International Baccalaureate

IB Math Standard Level

Portfolio

Type II

Fishing Rods

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IB Math SL
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Introduction

A fishing rod guides guides of the wire so that it does not change that so that the wire costs ecs by that e?ic ve with.

In this investigation, a mathematical model will be developed that will calculate functions from the given data points of two fishing rods of lengths 230cm and 300cm. This model will further determine the placement of the line guides on the fishing rod.

In mathematics, a function is a relation between a given set of elements and another set of elements that associate with each other, algebraically and graphically. In this investigation, an approach using matrices will be attempted to calculate functions for the given data and then be plotted to verify the results. Furthermore, there will be an effective use of technology so as to minimize errors and flaws.

Variable

Let x= Guide number (from tip)
Let y= Distance from tip (cm)

This investigation takes place under the condition that x > 0
$$y > 0$$
This condition is important because the fishing rod is required to have at least 1 guide, and the distance between two guides has to be a positive integer.
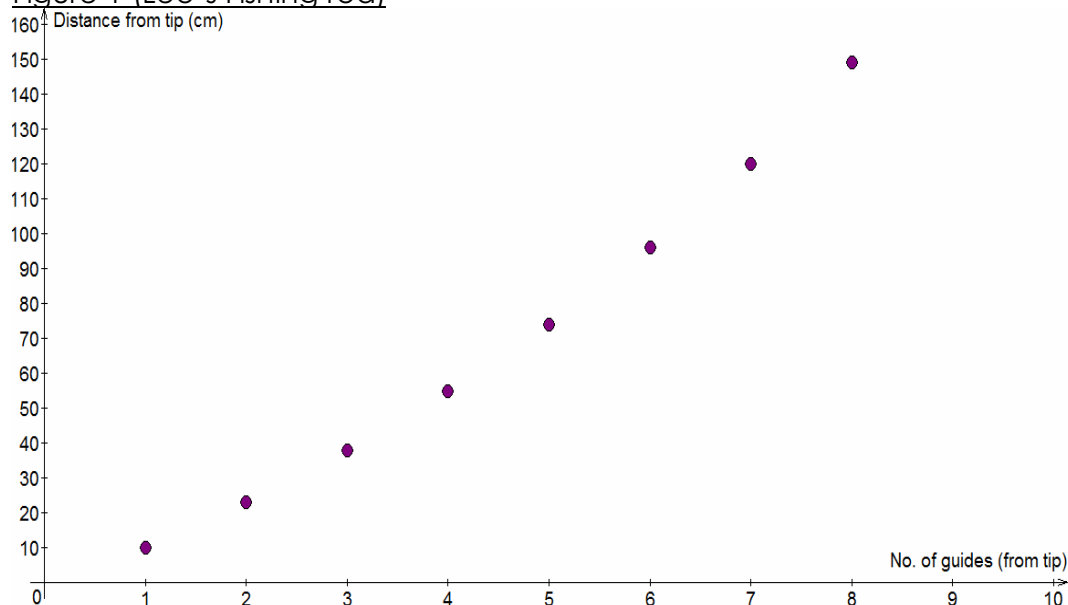
Investigation

The graph below shows Leo's fishing rod with respect to the Distance from tip (cm) to Guide number (from tip). The data points for the graph are plotted from the given data, which can be seen in the table below.

Guide No. (from tip)	Distance from tip (cm)
1	10
2	23
3	38

4	55
5	74
6	96
7	120
8	149

Table 1 (Leo's Fishing Rod)

Figure 1 (Leo's Fishing rod)

The graph in Figure 1 was plotted using an application called ZGrapher with reference to Table 1. As it can be seen in Figure 1, the best fit curve that would go through all the points will be a polynomial function, most likely a quadratic or cubic function.

Quadratic Equation

$$[y = ax^2 + bx + c]$$

To calculate this equation, the first, fourth and eighth data points from Table 1 will be used.

Table 2

Guide No. (from tip)	Distance from tip (cm)
1	10

4	55
8	149

Substituting the values from Table 2 into a quadratic equation, we get the following equations:

$$10 = a(1)^2 + 1b + c$$
$$10 = a + b + c$$

$$55 = a(4)^2 + 4a + c$$
$$55 = 16a + 4b + c$$

$$149 = a(8)^2 + 8a + c$$
$$149 = 64a + 8b + c$$

Now by transforming the coefficients of the above equations into matrices, we get the matrices as below:

$$\begin{bmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 55 \\ 149 \end{bmatrix}$$

$$\text{Let } \begin{bmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{bmatrix} = [A]$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = [A]^{-1} \begin{bmatrix} 10 \\ 55 \\ 149 \end{bmatrix}$$

$$\text{Let } \begin{bmatrix} 10 \\ 55 \\ 149 \end{bmatrix} \text{ be equal to } [D]$$

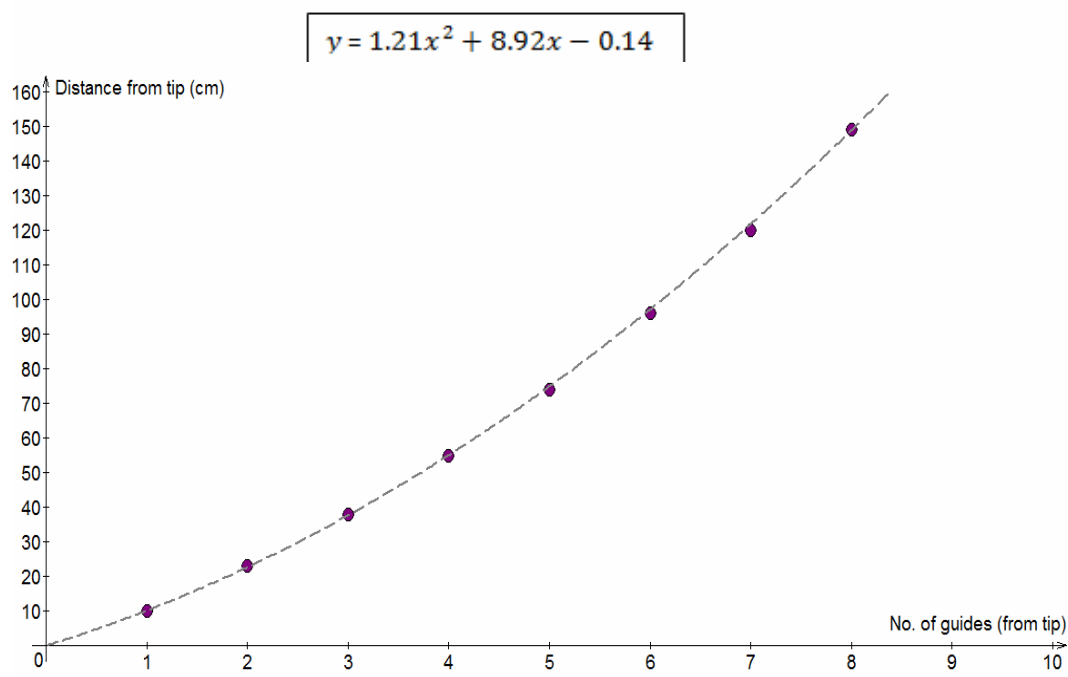$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = [A]^{-1}[D]$$

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[A]⁻¹[D]
[[1.214285714 ]
 [8.928571429 ]
 [-.1428571429]]
■
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$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1.21 \\ 8.92 \\ -0.14 \end{bmatrix}$$

Hence, the equation is $y = 1.21x^2 + 8.92x - 0.14$

The graph below shows the calculated quadratic equation that goes through the data points of Table 2

Figure 3

$y = 1.21x^2 + 8.92x - 0.14$

This model is quite accurate as the curve of the function passes through all the points. Although the curve does not pass through the center of the data points on the graph.

One possible reason for flaw could be that only four data points were used to calculate the function, even though that was the only possibility. If there was a way to use all eight data points to calculate the function, the curve would go through all data points on the graph hence being more accurate.

$$\left[y = ax^3 + bx^2 + cx + d\right]$$

To calculate this equation, the first, third, sixth and eighth data points will be used.

Guide No. (from tip)	Distance from tip (cm)
1	10
3	38
6	96
8	149

Table 3

Now by transforming the coefficients of the above equations into matrices, we get the matrices as below:

$10 = a(1)^3 + b(1)^2 + c(1) + d$
$10 = a + b + c + d$

$38 = a(3)^3 + b(3)^2 + c(3) + d$
$38 = 27a + 9b + 3c + d$

$96 = a(6)^3 + b(6)^2 + c(6) + d$
$96 = 216a + 36b + 6c + d$

$149 = a(8)^3 + b(64)^2 + c(8) + d$
$149 = 512a + 64b + 8c + d$

The coefficients of the above equations can be transformed into matrices for further calculations.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 216 & 36 & 6 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 10 \\ 38 \\ 96 \\ 149 \end{bmatrix}$$

Let $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 216 & 36 & 6 & 1 \\ 512 & 64 & 8 & 1 \end{bmatrix}$ be equal to $[B]$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = [B]^{-1} \begin{bmatrix} 10 \\ 38 \\ 96 \\ 149 \end{bmatrix}$$

Let $\begin{bmatrix} 10 \\ 38 \\ 96 \\ 149 \end{bmatrix}$ be equal to $[C]$

```
[B]⁻¹*[C]
[[.0523809524 ]
 [.5428571429 ]
 [11.14761905 ]
 [-1.742857143]]
■
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$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = [B]^{-1}[C]$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.54 \\ 11.15 \\ -1.74 \end{bmatrix}$$

Hence the cubic function $y = 0.05x^3 + 0.54x^2 + 11.15x - 1.74$

The graph below shows the calculated quadratic equation that goes through the data points of Table 3.
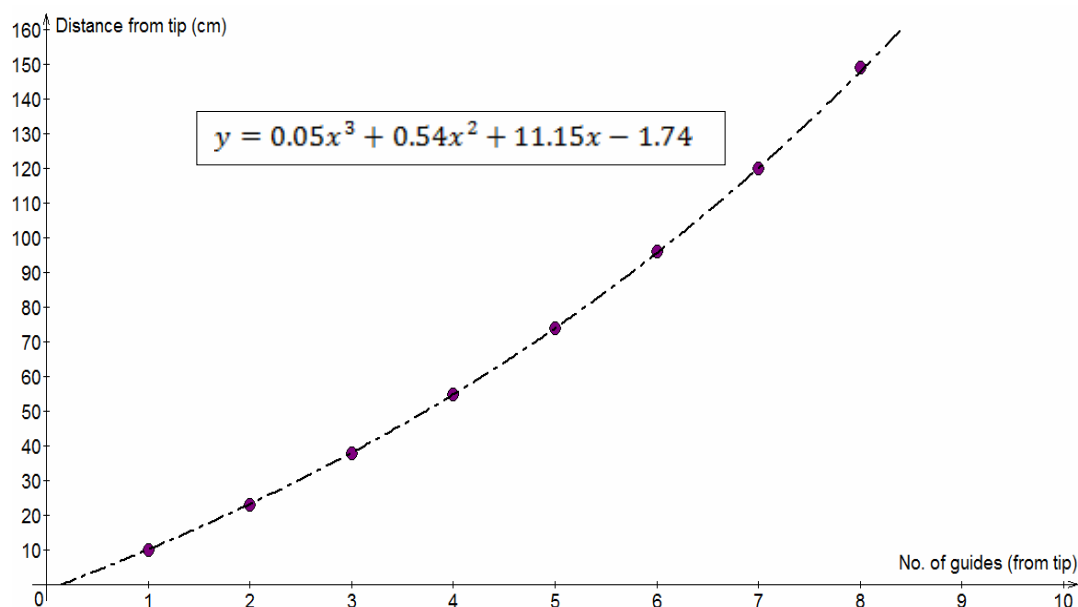
Figure 4

This model is quite accurate as the curve of the function passes through all the points. Although the curve does not pass through the center of the data points on the graph.

One possible reason for flaw could be that only four data points were used to calculate the function, even though that was the only possibility. If there was a way to use all eight data points to calculate the function, the curve would go through all data points on the graph hence being more accurate.

Now, by using the method of trial and error, I tried to determine a function that passes through all data points. Finally I came up with an equation that does so. This is

$$y = 1.24x^2 + 8.48x - 0.14$$

The graph below shows the equation that was found out using the trial and error method.
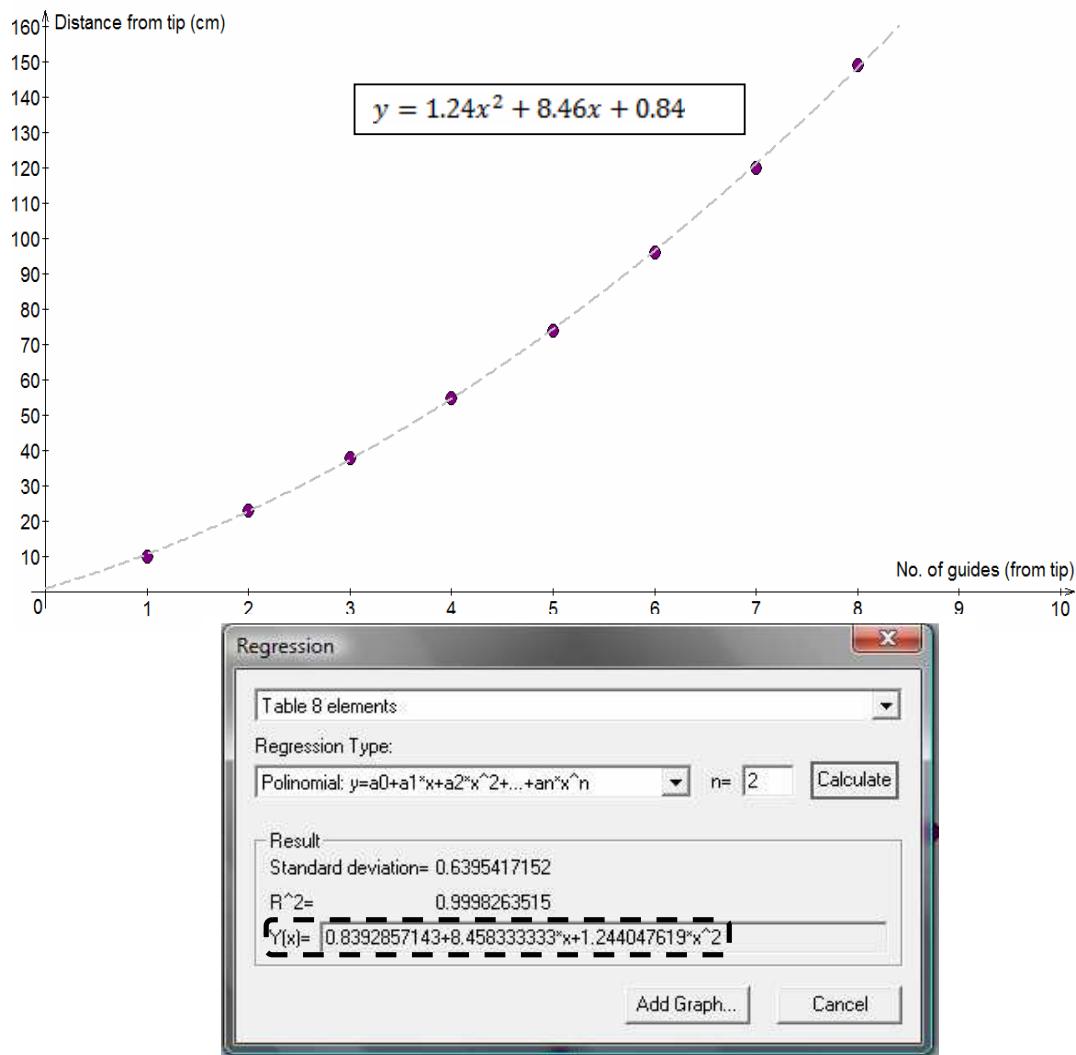
Figure 5

$$y = 1.24x^2 + 8.48x - 0.14$$
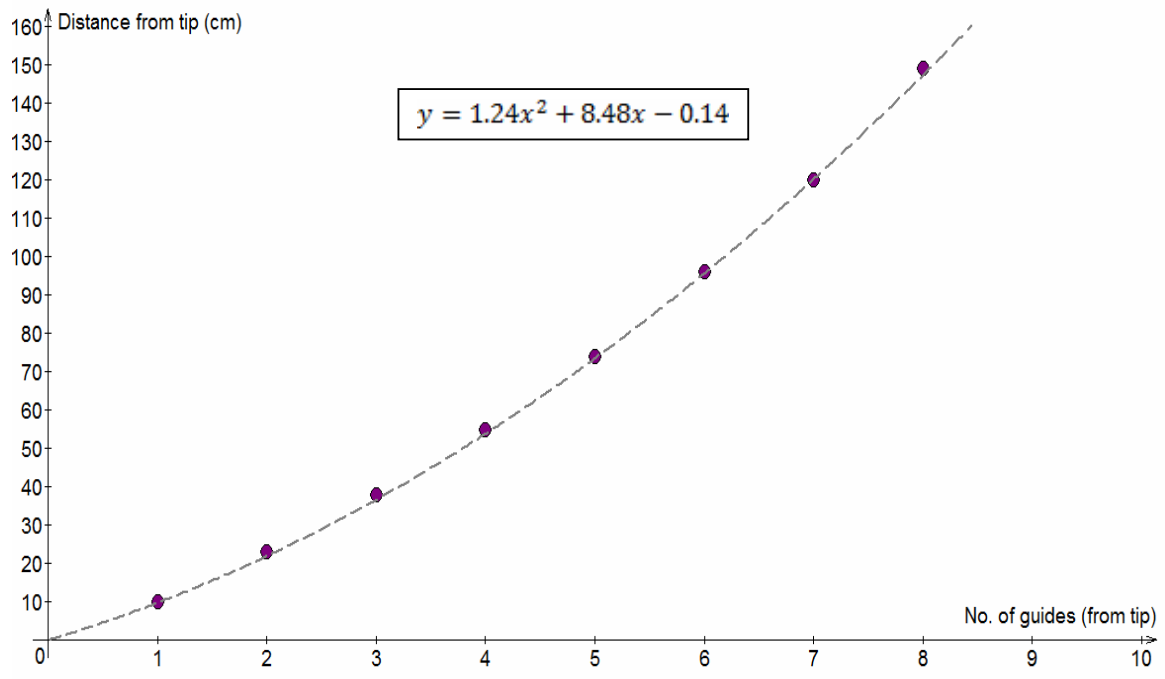
A modified quadratic equation is used in this case as it was needed to further improve the functions calculated earlier. The new function touches all of the data points. Even though the curve starts from 0.14 on the x-axis instead of from the origin of the graph, this is the closest match to accuracy.

Function using Technology

Figure 6

The above trend-line in the graph was generated using ZGrapher's Regression tool. This function is the most accurate of all because it not only passes through all the data points in the graph, it passes through the centers of the data points too.

Hence, Figure 6 shows the function that best suits the data points from Table 1.

Placement of the 9th Guide

If a ninth guide was to be placed, then in the best function, 9 should be substituted in the x-value as x-values are the guide numbers

$$y = 1.24x^2 + 8.46x + 0.84$$

$$y = 1.24(9)^2 + 8.46(9) + 0.84$$

$$y = 100.44 + 76.14 + 0.84$$

$$y = 177.42$$

$$\boxed{y = 177\ cm}$$

Adding a ninth guide to the rod would mean that the rod would be longer. The positive effect of adding a ninth guide to a longer rod would be that it would ensure that the line does not become tangled.

The table below shows the given set of data, which is Mark's fishing rod that h as a 300cm line.

Table

Guide No. (from tip)	Distance from tip (cm)
1	10
2	22
3	34
4	48
5	64
6	81
7	102
8	124

The graph below shows the calculated quadratic function passing through the second set of data points (Mark's fishing rod) from Table
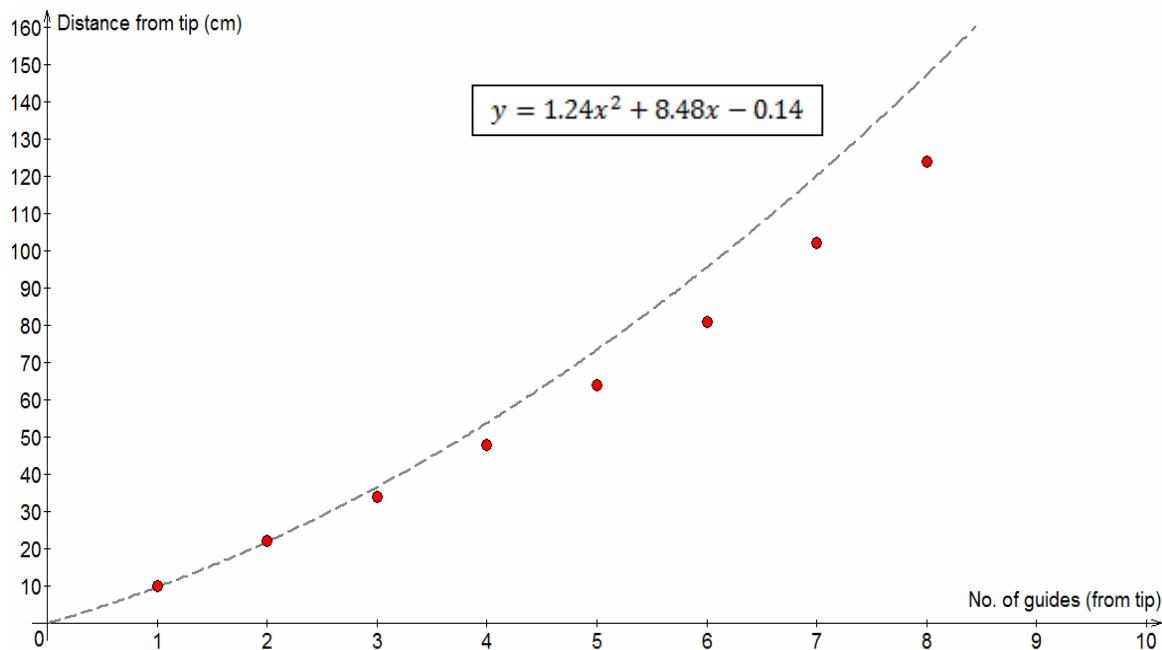
It can be clearly seen that the quadratic function does not fit this new set of data. In order for that model to fit this new data, an enhanced quadratic would need to be recalculated using these new data points i.e. the same approach of matrices will be required. One limitation with this model is that it disregards the length of the fishing rod. A fishing rod with a different length would result in different data points. Different data points, would result in a new curve, which the original model does not fit.