FISHING RODS
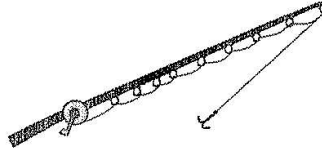
A fishing rod requires guides for the line so that it does not tangle and so that the line casts easily and efficiently. In this task, you will develop a mathematical model for the placement of line guides on a fishing rod.

Leo has a fishing rod with an overall length of 230 cm. The diagram shows a fishing rod with eight guides, plus a guide at the tip of the rod.


The table below shown below gives the distances for each of the line guides from the tip of his fishing rod.

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

The distance of each guide from the tip of the fishing rod is measured in cm. The *x* value, or the independent variable, is the guide number, and the *y* value, or the dependent variable, is the distance from tip. Only positive numbers can fit this scenario, because a fishing rod cannot have negative lengths. We can even further limit the domain, because the fishing rod only has 8 guides, so *x* would have to be greater than or equal to 1 and less than or equal to 8.

The graph of the data would look like this:
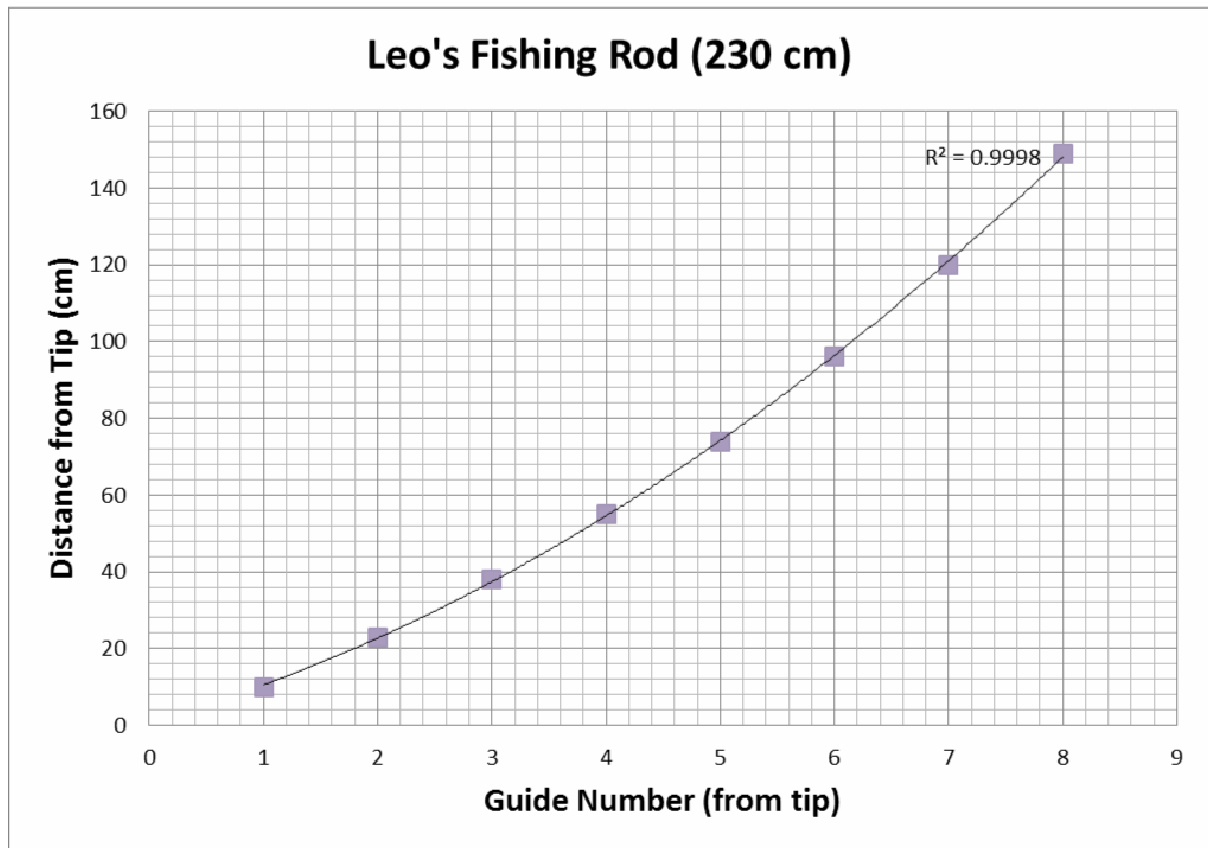
Quadratic Function: (ax^2+bx+c)

To find a quadratic function for this data, I used the first three guide numbers and distances from the tip: (1, 10), (2, 23), (3, 38) and used these numbers in the matrix method

$$10 = a(1)^2 + b(1) + c$$
$$23 = a(2)^2 + b(2) + c$$
$$38 = a(3)^2 + b(3) + c$$

$$10 = a + b + c$$
$$23 = 4a + 2b + c$$
$$38 = 9a + 3b + c$$

Hence, a matrix is created:

$$[A] \times [X] = [B]$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 23 \\ 38 \end{bmatrix}$$

To find [X], we could use this formula:

$$[A]^{-1} \times [B] = [X]$$

$$[A]^{-1} = \begin{bmatrix} \dfrac{1}{2} & -1 & \dfrac{1}{2} \\ -\dfrac{5}{2} & 4 & -\dfrac{3}{2} \\ 3 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{1}{2} & -1 & \dfrac{1}{2} \\ -\dfrac{5}{2} & 4 & -\dfrac{3}{2} \\ 3 & -3 & 1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 23 \\ 38 \end{bmatrix} = \begin{bmatrix} 1 \\ 10 \\ -1 \end{bmatrix}$$

$$a = 1, b = 10, c = -1$$

Quadratic function is:
$$y = x^2 + 10x - 1$$

Cubic Function: $ax^3 + bx^2 + cx + d$

The same method was used to find the cubic formula, except that the first *4* guide numbers and distances were used in the matrix method. (1, 10), (2, 23), (3, 38), (4, 55)

$$10 = a(1)^3 + b(1)^2 + c(1) + d$$
$$23 = a(2)^3 + b(2)^2 + c(2) + d$$
$$38 = a(3)^3 + b(3)^2 + c(3) + d$$
$$55 = a(4)^3 + b(4)^2 + c(4) + d$$

$$10 = a + b + c + d$$
$$23 = 8a + 4b + 2c + d$$
$$38 = 27a + 9b + 3c + d$$
$$55 = 64a + 16b + 4c + d$$

Hence, a matrix is created:

$$[A] \times [X] = [B]$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ 64 & 16 & 4 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 10 \\ 23 \\ 38 \\ 55 \end{bmatrix}$$

To find [X], we could use this formula:

$$[A]^{-1} \times [B] = [X]$$

So:

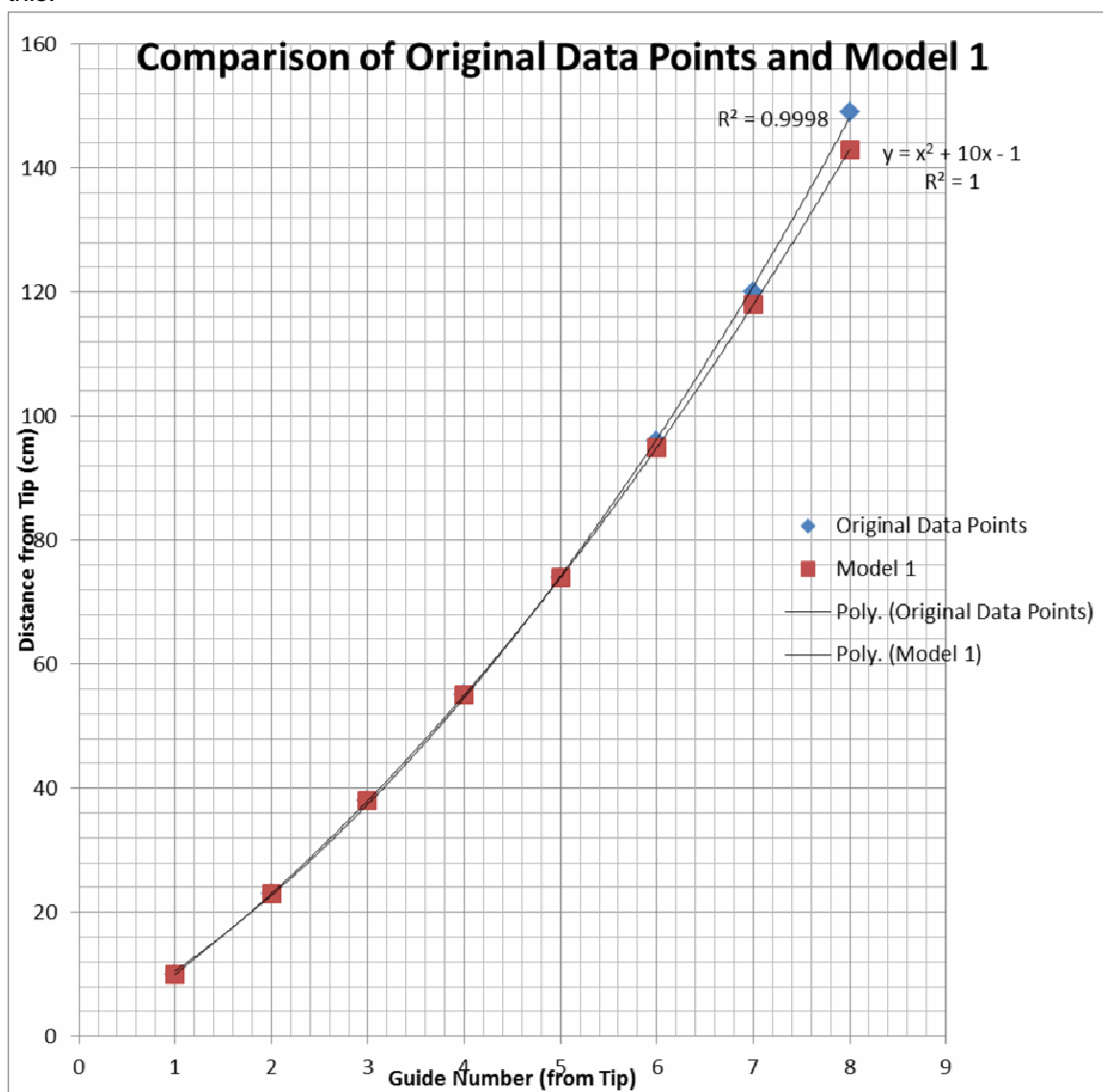$$a = 0, b = 1, c = 10, d = -1$$

So, the Cubic Function is:

$$y = x^2 + 10x - 1$$

In order to graph accurately, I decided to only graph the quadratic and cubic functions only with the domain of the original data points. This domain is: $1 \le x \le 8$. Using this domain, I can determine the distances of the guides from the tip (cm) of the fishing rod.
To find the y values for this domain, on Excel, I created a formula that would allow me to plug in the x values and find the y values. The table loos like this:

Table 2: Distances of guide numbers from tips calculated from Quadratic and Cubic Functions								
Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	95	118	143

The graph illustrating the original data points and the model functions looks like this:

Comparison of Original Data Points and Model 1

$R^2 = 0.9998$

$y = x^2 + 10x - 1$

$R^2 = 1$

- ◆ Original Data Points
- ■ Model 1
- —— Poly. (Original Data Points)
- —— Poly. (Model 1)

Distance from Tip (cm)

Guide Number (from Tip)

The quadratic and cubic functions that I found were the same in the first quadrant. Because the guide number cannot be negative, I chose to only compare these two in the first quadrant. If we looked at the values in the other quadrants, they would be different. Hence, only one graph is used to show both of them. These functions were quite similar to the original and had an r^2 value of 1. This shows that guide number and the distance of the guide number from the tip have a very strong correlation. There were no difference in points in the original and model, until I reached $x=6$ where the distance from the tip (cm) was slightly different (see table 2). The quadratic and cubic functions were both very accurate.

A polynomial function that passes through every data point would be the quadratic, cubic, or quartic functions. These three polynomial functions have the same distances from the tip when calculated. The table below shows all the values calculated:

	1	2	3	4	5	6	7	8
Original Distances from Tip (cm)	10	23	38	55	74	96	120	149
Quadratic ($y=x^2+10x-1$) Distances from Tip (cm)	10	23	38	55	74	95	118	143
Cubic ($y=x^2+10x-1$) Distances from Tip (cm)	10	23	38	55	74	95	118	143
Quartic ($y=(2.9x10^{-14})x^4+x^2+10x-1$) Distances from Tip (cm)	10	23	38	55	74	95	118	143

These polynomial functions are the again the same when considering the domain mentioned above, therefore any of them could be the most accurate. All the values are very close to the original so these are all reasonable functions to use. The graph would look like this:
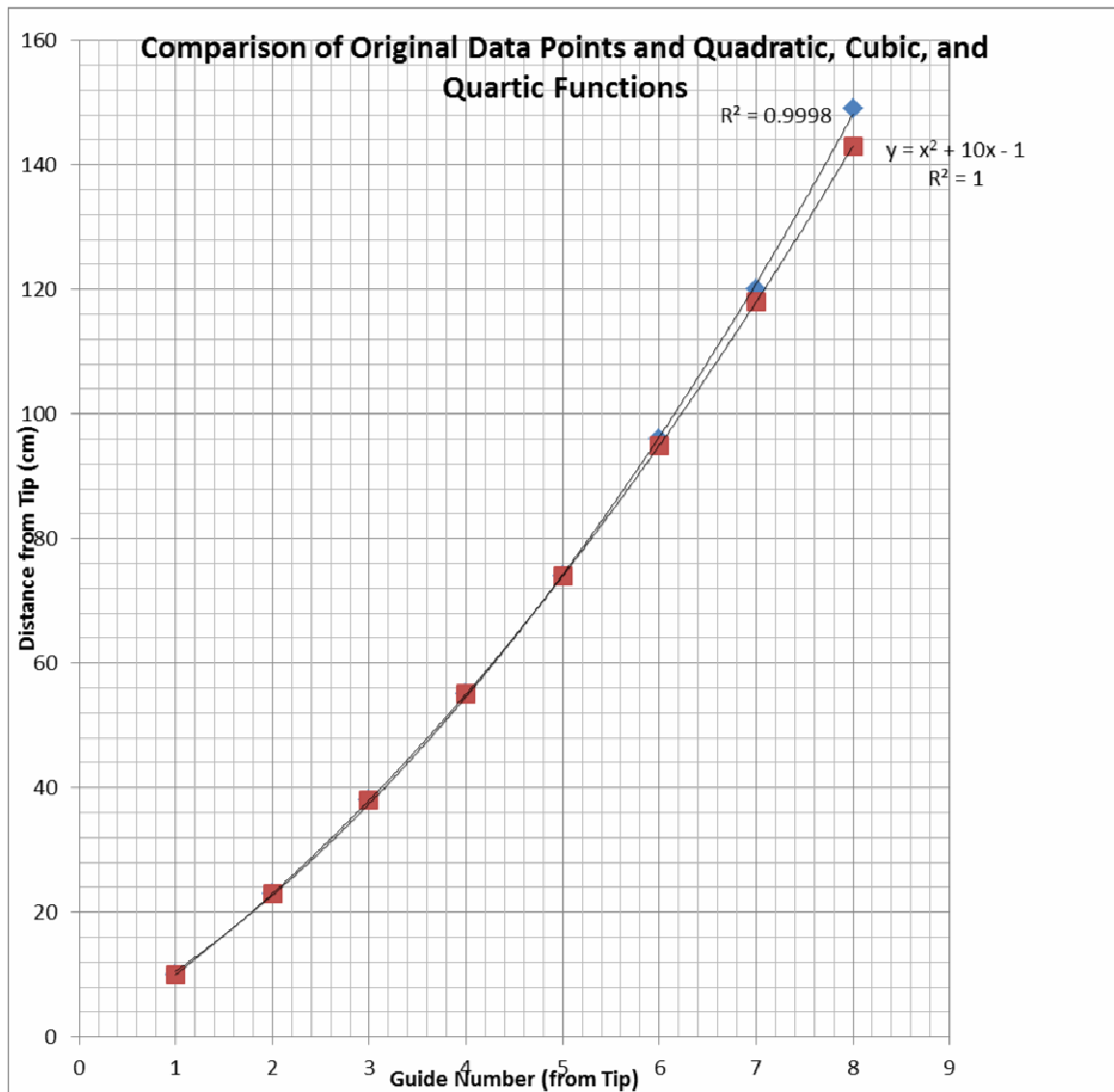
The diamonds represent the original data, while the squares represent the polynomial functions.

Other function that fits the data:

Linear function (*ax+b*):

To test a linear function, I used the matrix method once more, and this time I only used the first two guide numbers and distances: (1, 10), (2,23)

$$10 = a(1) + b$$
$$23 = a(2) + b$$

$$10 = a + b$$

$$23 = 2a + b$$

Hence, this matrix resulted:

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 10 \\ 23 \end{bmatrix}$$

The same method that was used previously was used to calculate *a* and *b:*

$$[A]^{-1} \times [B] = [X]$$

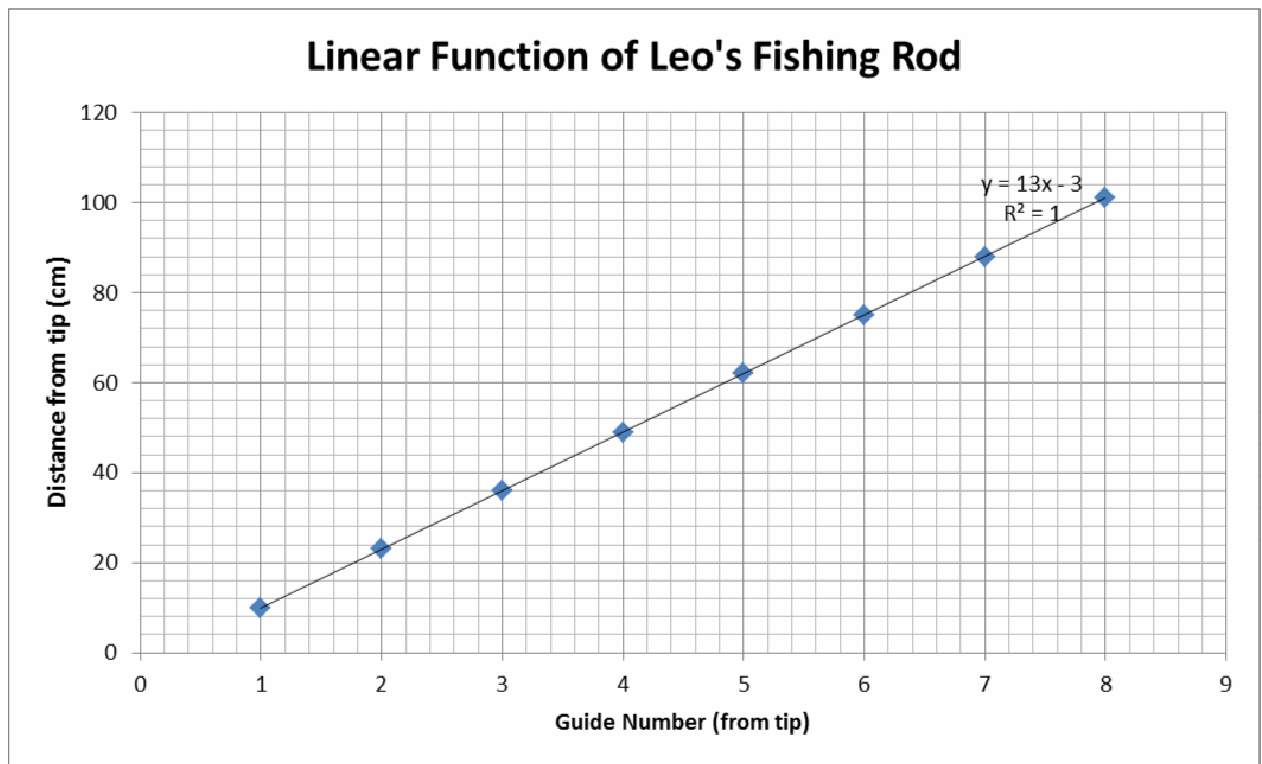$$[A]^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 10 \\ 23 \end{bmatrix} = \begin{bmatrix} 13 \\ -3 \end{bmatrix}$$
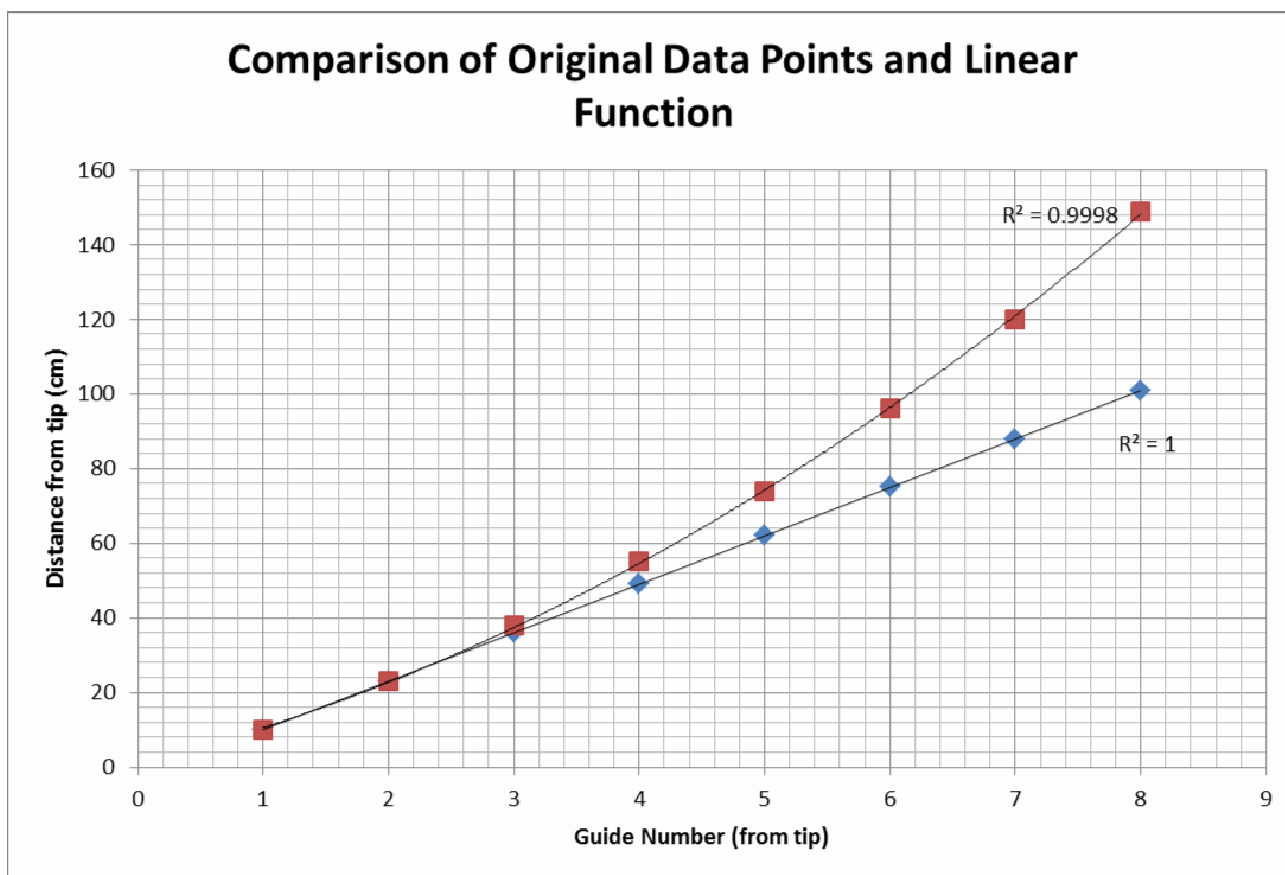
$$a = 13, \quad b = -3 \quad so \quad y = 13x - 3$$

Again I plugged in the *x* values that fit my domain, and graphed those values:

Table 3: Distances of guide numbers from tips calculated from Linear Functions								
Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	36	49	62	75	88	101

Linear Function of Leo's Fishing Rod

$y = 13x - 3$
$R^2 = 1$

(Chart axes: Distance from tip (cm) vs Guide Number (from tip))

Comparison of Original Data Points and Linear Function

In this graph, the square points represent the original data, while the diamonds represent the linear function.

The linear function is the function that least fits the data points. It has the most difference between original points and the function points.

The function that best models this situation is the quadratic function. It has a very high positive correlation, and passes through every point. There is hardly any difference in the original distances from the tip and the distances I found from the quadratic function when I calculated them (see table 2 above).

Using quadratic model to decide where to place the 9th guide:

Quadratic Function: $y = x^2 + 10x - 1$

x is the number of guides so $x = 9$

$$y = (9)^2 + 10(9) - 1$$

$$y = 81 + 90 - 1$$

$$y = 170 \, cm$$

The 9th guide should be placed at 170 cm. Adding a 9th guide would shorten the handle of the fishing rod, because it can only be 230 cm. If a 9th guide was added, then a total of 10 guides would be on the rod including the guide at the tip of the rod.

Mark's fishing rod:

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	22	34	48	64	81	102	124

My quadratic model does not quite fit this data set because there are some differences in the *y* value. Looking at the graph of the data, it seems that it fits the model but when looking at the equation from the graph, we can see that it is slightly different:

$$y = 0.9345x^2 + 7.7202x + 2.0536$$
$$R^2 = 0.9998$$
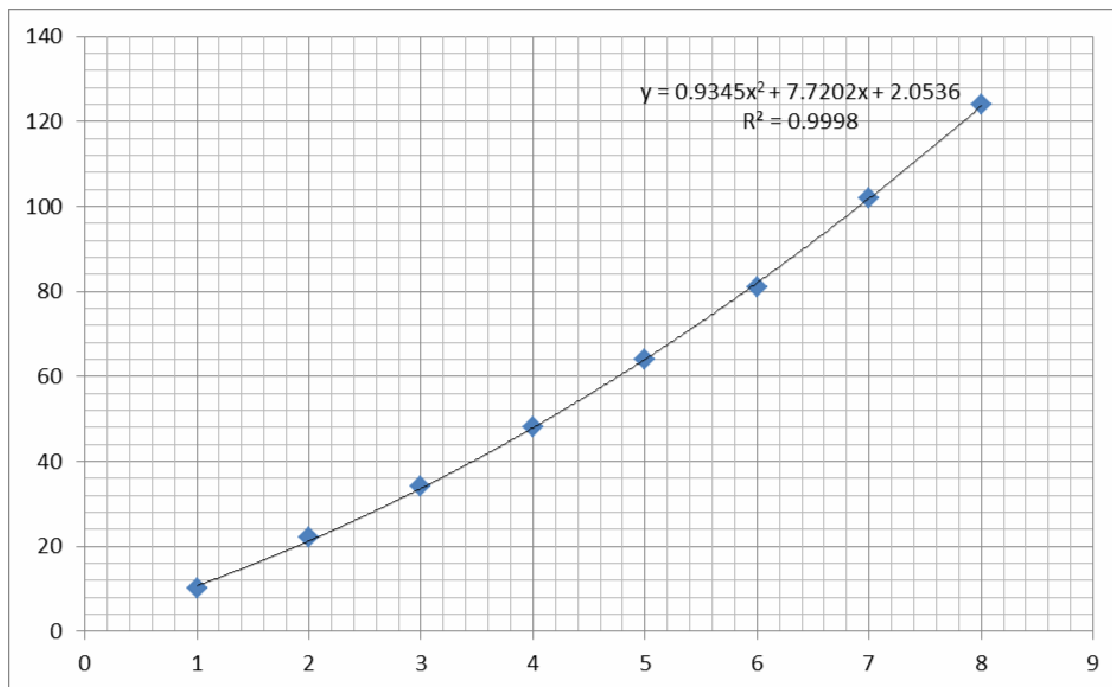
The r^2 value for this graph is 0.998, while for my quadratic model it is 1. I would need to change the equation that I obtained. To do this I could use the matrix method, but instead use these three data points: (1, 10), (2, 22), (3, 34) Using these points in the matrix method, I could calculate the new quadratic model, and then compare it to the original data points. My equation would also change once I do this. The new values for a, b, c, would be 0, 12, -2 respectively. So the new quadratic model that would fit this data is:

$$y = 12x - 2$$