Fishing Rods

SL Type II

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An efficient fishing rod requires numerous guides for the line or else it will get tangled. Companies must find a strategic position for them or else the rod will become obsolete. As a result, a mathematical model can help companies predict where to place these line guides in order for a successful launch. Imagine customers launching a fishing rod that gets caught up before landing in the water! Not a very pleasant experience, is it? In this portfolio, I will be finding an appropriate model that can mathematically show the placement of line guides on a fishing rod. To do so, I will be using a given diagram, whereas: a fishing rod has an overall length of 230cm. The distances for each of the guides from the tip of the fishing rod is shown in a table on the back, but I will be copying it out, so one can refer to it while reading.

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

Ninth guide on the tip, (0,0)

Before beginning, I will be identifying any parameters or constraints in relation to the fishing rod. First of all, it is common sense that a negative number of guides cannot exist in reality, just like there cannot be negative numbers of apples, and as a result, negative distances are neglected. As a result, the graph must be limited to the first quadrant because the second, third, and fourth ones contain negative numbers which don't exist in a fishing rod. Similarly, we are limited in our calculations to the fact that all fishing rods have a certain length; thus, the addition of infinite guides is not possible. The ninth guide on the tip will be included in the calculations because it is technically part of the rod. If we want to model an accurate situation, it is evident that the fishing line must pass by the ninth guide one way or another in order for the mechanism to work. In this portfolio, the variables I will use are:

g = Guide number (from tip)

X = Distance from tip (cm)

Now, that I have listed some possible restraints and variables of the model, I will be plotting the data points on graph to provide a pictoral representation of the general shape of the fishing rod.

Although we know the general shape of the fishing rod and the location of the guides, companies need a mathematical model that can help predict the most efficient placement of other guides. By using matrix methods, I will describe a way to model this situation using a quadratic function and a cubic function. To those who are not familiar with matrices, they are a rectangular table of numbers, or more generally, any abstract quantity that can be added or multiplied. Matrices can be used to depict linear equations, trace various transformations and to record data that depend on multiple parameters. They can also be used to solve a systems of equation by using this general formula:

$A^{-1}AX = A^{-1}B$, whereas A^{-1} is the inverse of the matrix A.

To begin the model, it is crucial to realize the general equation of a quadratic function which is:

$0 = Ax^2 + Bx + C$, which will be referred as $X = Ag^2 + Bg + C$ in the portfolio whereas g is equal to the guide number (from tip) and X is equivalent for the distance from tip (cm).

Now, that I have established the premises of what is necessary in order to create a mathematical model that represents the placement of guides on a fishing rod, I will begin building it step by step.

Since there are three variables (A,B,C) that must be present in a quadratic function, I will use three equations (requires three points) represented in matrix form and solve it using the general equation as mentioned before.

First of all, the ideal mathematical model would show a line passing through the first guide and the last guide. This is important because the first guide acts as the starting guide and the last guide acts as the ending guide; therefore, it is crucial that these points are utilized in order to create a realistic model. Since two points are already set, the third one can be randomly selected in between the start and end guides, but for the purposes of stability, I decided to use the median. To find this median, I went three up from the start guide and landed on guide # 4. I tried going up by a sequence of four, which would have result in guide # 5 as the median, but after I went up 4 more, it landed on guide number #9, which does not exist. Why the median? Well, let's say that a point was chosen right next to the starting guide; as a result, the model may falter in validity because there is a huge distance between the second guide and the last guide.

With that in mind, let's begin.

Guides used for Quadratic function:
Number 1, Number 4, and Number 8
Distance from tip:
10cm, 55cm, 149cm

The points are: (1,10), (4,55), (8,149) in the form of **(g,X)**

Now, I will substitute these numbers in three quadratic equations:

$$X = Ag^2 + Bg + C$$

Equation 1: $10 = A(1) + B(1) + C$

Equation 2: $55 = A(16) + B(4) + C$

Equation 3: $149 = A(64) + B(8) + C$

Now, to represent these equations in matrix form:

$$\left(\begin{pmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{pmatrix}\right)\left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right) = \left(\begin{pmatrix} 10 \\ 55 \\ 149 \end{pmatrix}\right)$$

Next, isolate the variables by multiplying $\begin{pmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{pmatrix}$ with its inverse which will result in the identity matrix. What is done on one side will result in the other side, so the final equation will look like this:

$$\left(\begin{pmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{pmatrix}\right)^{-1} \left(\begin{pmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{pmatrix}\right)\left(\begin{pmatrix} A \\ B \\ C \end{pmatrix}\right) = \left(\begin{pmatrix} 1 & 1 & 1 \\ 16 & 4 & 1 \\ 64 & 8 & 1 \end{pmatrix}\right)^{-1} \left(\begin{pmatrix} 10 \\ 55 \\ 149 \end{pmatrix}\right)$$

After that, matrix "A" turns into the identity matrix and the inverse of matrix "A" is used to multiply with matrix "B". Using technology, the answer is:

$$\left(\begin{pmatrix} A \\ B \\ C \end{pmatrix}\right) = \left(\begin{pmatrix} 1.2142857142857 \\ 8.9285714285714 \\ -0.1428571428571 \end{pmatrix}\right)$$

 A = 1.21, B = 8.93, C = -0.143, rounded to three significant figures

Let's revisit the general quadratic equation and substitute these three values in.

The quadratic model of the guide placements of a fishing rod can be represented as:

$$X = 1.21\, g^2 + 8.93\, g - 0.143$$

On the next page, there will be a comparison of the quadratic model with the data points.

For a cubic model, I will use the same methods except, use the general equation of a cubic equation:

$$X = A\, g^3 + B\, g^2 + C\, g + D$$

This time, 4 ordered pairs will be required and by using the same method as the previous model, I chose them in an interval of two (intervals of three does not work). The start guide is the first ordered pair, and then two up; the third guide, two up again; the fifth guide, and lastly, the end guide.

Substitution:

Equation 1: 10 = A (1) + B (1) + C (1) +D

Equation 2 : 38 = A (27) + B (9) + C (3) + D

Equation 3: 74 = A (125) + B (25) + C (5) + D

Equation 4: 149 = A (512) + B (64) + C (8) + D

Now, to represent these equations in matrix form:

$$\left(\begin{pmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{pmatrix}\right)\left(\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}\right) = \left(\begin{pmatrix} 10 \\ 38 \\ 74 \\ 149 \end{pmatrix}\right)$$

Using the same methods to isolate the unknown variables (A,B,C,D), I will multiply each side with the inverse of matrix "A". Using technology, the answer is:

$$\left(\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix}\right) = \left(\begin{pmatrix} 0.0571428571429 \\ 0.4857142857143 \\ 11.3142857142857 \\ -1.8571428571429 \end{pmatrix}\right)$$

 A = 0.0571, B = 0.486, C = 11.3, D = -1.86, rounded to three significant figures

By revisiting the general cubic equation, we can substitute these values in and get a mathematical model:

$$X = 0.0571\, g^3 + 0.486\, g^2 + 11.3\, g - 1.86$$

On the next page, there will be a comparison graph of this model with the original data points.

Next, I will be exploring a model that can pass through every data point correctly without any differences. In order to this, I decided that a function to the power of seven would correctly demonstrate this. Solving four equations allows the model to pass through 4 points precisely, as proven from the previous diagram. By analysing the previous models, I came to a conclusion that by solving eight equations, the line would pass through all 8 points correctly. Therefore, a function raised to the seventh power must be utilized as it requires eight unkowns; thus, eight equations.

General Equation: $Ag^7 + Bg^6 + Cg^5 + Dg^4 + Eg^3 + Fg^2 + G(g) + H$

This time, nine ordered pairs are required (nine unknown variables); therefore, all data points are utilized instead of just a select few.

Substitution:

Equation 1: $10 = A(1^7) + B(1^6) + C(1^5) + D(1^4) + E(1^3) + F(1^2) + G(1) + H$

Equation 2: $23 = A(2^7) + B(2^6) + C(2^5) + D(2^4) + E(2^3) + F(2^2) + G(2) + H$

Equation 3: $38 = A(3^7) + B(3^6) + C(3^5) + D(3^4) + E(3^3) + F(3^2) + G(3^1) + H$

Equation 4: $55 = A(4^7) + B(4^6) + C(4^5) + D(4^4) + E(4^3) + F(4^2) + G(4) + H$

Equation 5: $74 = A(5^7) + B(5^6) + C(5^5) + D(5^4) + E(5^3) + F(5^2) + G(5) + H$

Equation 6: $96 = A(6^7) + B(6^6) + C(6^5) + D(6^4) + E(6^3) + F(6^2) + G(6) + H$

Equation 7: $120 = A(7^7) + B(7^6) + C(7^5) + D(7^4) + E(7^3) + F(7^2) + G(7) + H$

Equation 8: $149 = A(8^7) + B(8^6) + C(8^5) + D(8^4) + E(8^3) + F(8^2) + G(8) + H$

In matrix form, these nine equations can be represented as:

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 1 \\ 3^7 & 3^6 & 3^5 & 3^4 & 3^3 & 3^2 & 3 & 1 \\ 4^7 & 4^6 & 4^5 & 4^4 & 4^3 & 4^3 & 4 & 1 \\ 5^7 & 5^6 & 5^5 & 5^4 & 5^3 & 5^2 & 5 & 1 \\ 6^7 & 6^6 & 6^5 & 6^4 & 6^3 & 6^2 & 6 & 1 \\ 7^7 & 7^6 & 7^5 & 7^4 & 7^3 & 7^2 & 7 & 1 \\ 8^7 & 8^6 & 8^5 & 8^4 & 8^3 & 8^2 & 8 & 1 \end{pmatrix}$$

Next, isolate the variables by multiplying the inverse of matrix “A” on both sides. Using technology, the answer is:

$$\begin{pmatrix} 0.0007015734776 \\ -0.0176884465228 \\ 0.1706386660374 \\ -0.7747826304575 \\ 1.6088258112031 \\ 0.0056333748052 \\ 9.0785323061397 \\ -0.0718606546827 \end{pmatrix}$$

 A = 0.000702, B = -0.0177, C = 0.171, D = -0.775, E = 1.61, F = 0.00563, G = 9.08, H = -0.0719, rounded to three significant figures

After substituing all these values, the final equation came out to be:

$$X = 0.000702\, g^7 - 0.0177\, g^6 + 0.171\, g^5 - 0.775\, g^4 + 1.61\, g^3 + 0.00563\, g^2 + 9.08\, g - 0.0719$$

The comparison graph between this model with the data points is on the next page.

After making various models without using technology, I will now explore regressions (or best-fit lines) and use their graphs for comparison. The three types of regressions that will be used are: quadratic, cubic, and quartic. Two of them, the quadratic and cubic, will be compared to the models that have already been made up and the purpose of this is to identify any differences or similarities. Finally, all the regression graphs will be placed on the same page and by analysing how the line passes through the data points, the best one can be figured out. With technology, I will be able to find three equations that describe a quadratic, cubic, and quartic function which passes through as many data points as possible.

Quadratic Regression Equation:

$$X = 1.24g^2 \square\ 8.46g \square\ 0.839$$

Cubic Regression Equation:

$$X = 0.0631g^3 \square\ 0.392g^2 \square\ 11.7g - 2.29$$

Quartic Regression Equation:

$$X = 0.0123g^4 - 0.158g^3 \square\ 1.73g^2 \square\ 8.64g - 0.200$$

Note: These values were calculated using a graphics calculator. The graphs are on the next page.

By analysing all these functions and commenting on their differences, it is evident that the one who models the placements of a guide on a fishing rod the best is the 7th powered function. None of the points are off by a significant amount, unlike the other models; however, in real life, this function may not be desirable because of its length. The quartic regression (model) requires less variables to solve and it almost matches the 7th powered function, so without a doubt, it would be more desirable in real life especially in a fast-paced environment. It would be very troublesome solving a system of seven equations while manufacturing thousands and thousands of fishing rods, but when it comes to perfection, the 7th powered model is definitely the most accruate.

As stated above, both a quadratic and cubic model was solved by using matrix methods. Here's a comparison of the quadratic and cubic regression equations in relation to the model functions.

$\boldsymbol{X = 1.21\, g^2 + 8.93\, g - 0.143}$ Model Function – Quadratic

$X = 1.24g^2 + 8.46g + 0.839$ Regression – Quadratic

$\boldsymbol{X = 0.0571\, g^3 + 0.486\, g^2 + 11.3\, g - 1.86}$ Model Function – Cubic

$X = 0.0631g^3 + 0.392g^2 + 11.7g - 2.29$ Regression – Cubic

As one can see, these two equations do not differ greatly as expected because essentially, a regression is a best fit line which should be equivalent to a model that passes through most of the points. Of course, using technology would result in a more accurate answer, but the use of matrix methods is crucial if one does not have knowledge about regressions.

After exploring a fishing rod with eight guides, it would be interesting to guess where would the placement of a ninth guide be (excluding the tip) and measure the effects of this phenomenon. By using my quadratic model, it's quite simple to find out where a ninth guide could possibly be. Since it's just a general idea, the use of complicated functions are not needed; thus, a quadratic model should be sufficient.

Let's revisit the quadratic model:

$$\boldsymbol{X = 1.21\, g^2 + 8.93\, g - 0.143}$$

As mentioned before, the variables: X is the distance from tip (cm) and g is the guide number (from tip). So, to find the ninth guide, all we need to do is substitute g with the number nine.

$X = 1.21(9)^2 - 8.93(9) + 0.143$, which ends up to around 178cm.

Therefore, if a ninth guide was to be added to the fishing rod, it would be located approximately 178cm from the tip (0,0). The effects of having this ninth guide would shorten the line and therefore, it may cause difficulty fishing as it can't reach the water.