Year 11 IB Maths – Portfolio Type II

Fishing Rods

A fishing rod requires guides for the line so that it does not tangle and so that the line casts easily and efficiently. In this investigation, a mathematical model will be developed using matrix methods, polynomial functions, and technology to calculate functions from the given data points of two fishing rods of lengths 230cm and 300cm. This model will further determine the placement of the line guides on the fishing rod.

In mathematics, a function is a r elation between a given set of elements and another set of elements that associate with each other, algebraically and graphically. In this investigation, an approach using matrices will be attempted to calculate functions for the given data and then be plo tted to verify the results. Furthermore, there will be an effective use of technology, using Graphic calculator and excel, so as to minimize errors and flaws.

The first investigation is of Leo's fishing rod:
Leo has a fishing Rod with overall length 230cm . The table below gives the distance for each of the line guides from the tip of the fishing rod.

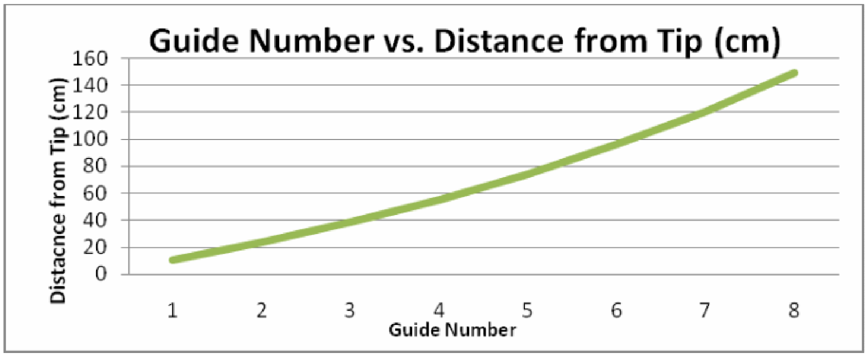
Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

Firstly, before a mathematical model can be formulated, we must o utline and define the variables and constraints associated with the values given above.

Independent variable (x): The guide number from the tip of the fishing rod – let this equal x
Dependent variable (y): The distance from the tip of fishing rod – let this equal y
Parameters/Constraints:

- The distance from tip for each guide number does not follow a particular pattern. Hence it is difficult to achieve a function that satisfies all of the points on Table 1 .
- Model of a real life situation so there must be s pace for a reel and to hold the rod – limits the space the guide's can have between each other
- The overall length of the fishing rod – it cannot be negative or too long as this will reduce efficiency

The data points given above can then be plotted onto a graph using Microsoft excel.



Using Matrix methods we can find a quadratic fu nction to model the situation from the given data points:

A quadratic equation is in the form $y = ax^2 + bx + c$. As there are 3 unknown variables (a, b, c) we can create a 3 x 3 matrix and a 3 x 1 matrix to model the given information.
Using the first 3 data points, 3 equations can be formed:

$10 = a(1)^2 + b(1) + c$
$\therefore 10 = a + b + c$

$23 = a(2)^2 + b(2) + c$
$\therefore 23 = 4a + 2 + c$

$38 = a(3)^2 + b(3) + c$
$\therefore 38 = 9a + 3b + c$

The three equations can then be transformed to form a 3 x 3 and a 3 x 1 matrix with the corresponding coefficients:

We will call this first matrix <u>- Matrix A</u>

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix}$$

We will call the second matrix – <u>Matrix B</u>

$$\begin{pmatrix} 10 \\ 23 \\ 38 \end{pmatrix}$$

We will now take the inverse of Matrix A and multiply it with Matrix B to get Matrix C using the graphics calculator. Matrix C will contain the a, b and c values of the quadratic equation in a 3x1 Matrix.

$$\begin{pmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{pmatrix}^{-1} \times \begin{pmatrix} 10 \\ 23 \\ 38 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 1 \\ 10 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Therefore the respective values of the unknowns a, b and c are a=1, b=10 and c=-1
These values can then be subbed into the original equation of the quadratic $y = ax^2 + bx + c$ to form the quadratic function modelling the situation:

$\boldsymbol{y = x^2 + 10x - 1}$

We will now use similar matrix methods to find a cubic function that models the situation:

A cubic equation is in the form $y = ax^3 + bx^2 + cx + d$. As there are 4 unknown variables (a, b, c, d) we can create a 4 x 4 matrix and a 4 x 1 matrix to model t he given information.
Using 4 different points that represent the spread of the original points, to possibly increase the accuracy of the function, 4 equations can be formed:

$10 = a(1)^3 + b(1)^2 + c(1) + d$
$\therefore 10 = a + b + c + d$

$55 = a(4)^3 + b(4)^2 + c(4) + d$
$\therefore 55 = 64a + 16b + 4c + d$

$74 = a(5)^3 + b(5)^2 + c(5) + d$
$\therefore 74 = 125a + 25b + 5c + d$

$149 = a(8)^3 + b(8)^2 + c(8) + d$
$\therefore 149 = 512a + 64b + 8c + d$

The four equations can then be transformed to form a 4 x 4 and a 4 x 1 matrix with the corresponding coefficients:

We will call this first matrix – Matrix A

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 64 & 16 & 4 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{pmatrix}$$

We will call the second matrix – Matrix B

$$\begin{pmatrix} 10 \\ 55 \\ 74 \\ 149 \end{pmatrix}$$

We will now take the inverse of Matrix A and multiply it with Matrix B to get Matrix C, using the graphics calculat or. Matrix C will contain the a, b, c and d values of the quadratic equation in a 4x1 Matrix.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 64 & 16 & 4 & 1 \\ 125 & 25 & 5 & 1 \\ 512 & 64 & 8 & 1 \end{pmatrix}^{-1} \times \begin{pmatrix} 10 \\ 55 \\ 74 \\ 149 \end{pmatrix} = \begin{pmatrix} 0.071 \\ 0.286 \\ 12.071 \\ -2.429 \end{pmatrix}$$

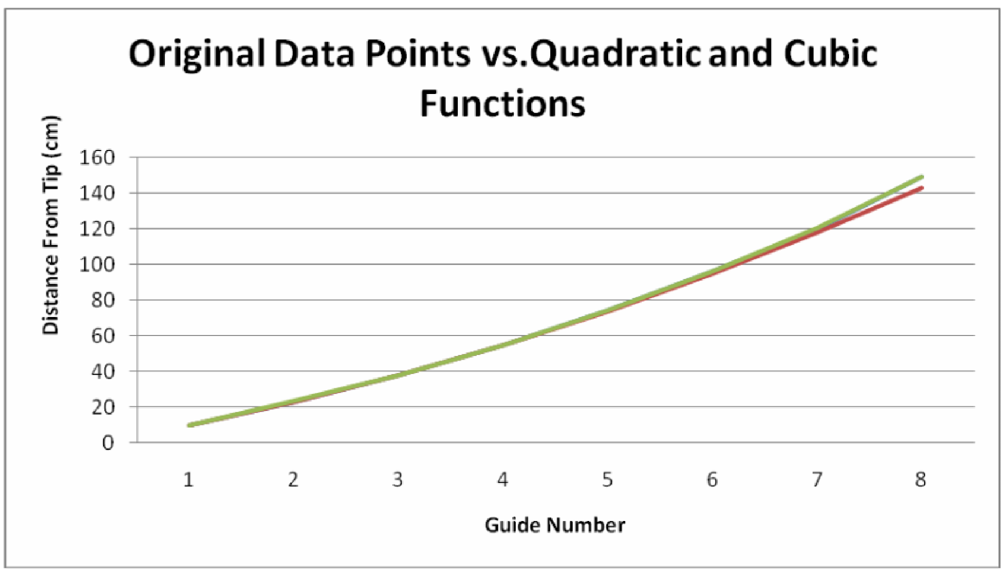
$$\therefore \begin{pmatrix} 0.071 \\ 0.286 \\ 12.071 \\ -2.429 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

Therefore the respective values of the unknowns a,b,c and d are a=0.071, b=0.286, c=12.071 and d=-2.429

These values can then be subbed into the original equation of the cubic $y = ax^3 + bx^2 + cx + d$ to form the quadratic function modelling the situatio n:

$\mathbf{y = 0.071x^3 + 0.286x^2 + 12.071x - 2.429}$

Using Excel, we can now take these two new functions of $y = x^2 + 10x - 1$ and $y = 0.041x^3 + 0.0625x^2 + 10.958x - 1.625$ and plot them on an axes along with the graph of the original data points to assess the accuracy of the functions in modelling the situation.



Green Line: Original Data Points
Red Line: $y = x^2 + 10x - 1$
Blue Line: $y = 0.071x^3 + 0.286x^2 + 12.071x - 2.429$

In the graph above, it can clearly be seen that the function $y = x^2 + 10x - 1$ accurately models the original data points given when Leo has a rod of overall length 230cm. However, as the new fu nction approaches the 6th, 7th and 8th guide, its values differ slightly, limiting the overall accuracy of the equation in modelling th e situation. As the blue line representing the function of $y = 0.071x^3 + 0.286x^2 + 12.071x - 2.429$ is not visible behind the graph of the original data points, this demonstrates the near complete accuracy of the function in modelling the situation. Using a graphics calculator, the data points of the two new functions can be found and then compared with the original data points to asses s the differences between the three.

Distance from tip (cm) – Original Points	10	23	38	55	74	96	120	149
Distance from tip (cm) $y = x^2 + 10x - 1$	10	23	38	55	74	95	118	143
Distance from tip (cm) $y = 0.071x^3 + 0.286x^2 + 12.071x - 2.429$	9.999	23.425	38.275	54.975	73.951	95.629	120.440	148.800

As seen in the graph and the table above, the graphs created using the quadratic and cubic fit the situation of Leo's fishing rod, however the cubic function produces a graph and data points much closer to the original data points and therefore is the most accurate representation of the situation.

To further test the modelling of the situation, a polynomial function can be found that passes through every data point. The most accurate form of a polynomial function given the data would be an eighth order function, given there are 8 data points. However this will provide a very long and confusing function that may not be realistic. Therefore, I have chosen to model the situation using a quartic function found by matrix methods. As this is to a higher power than a quadratic or cubic equation, its accurateness and reasonableness will be increased.

Using matrix methods we can find a quartic function to model the situation from the given data points:

A quartic equation is in the form $y = ax^4 + bx^3 + cx^2 + dx + e$. As there are 5 unknown variables (a, b, c,d,e) we can create a 5 x 5 matrix and a 5 x 1 matrix to model the given information.
Using the 10 different data points, that represent the spread of the data, 5 equations can be formed:

$10 = a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e$
$\therefore 10 = a + b + c + d + e$

$38 = a(3)^4 + b(3)^3 + c(3)^2 + d(3) + e$
$\therefore 38 = 81a + 27b + 9c + 3d + e$

$74 = a(5)^4 + b(5)^3 + c(5)^2 + d(5) + e$
$\therefore 74 = 625a + 125b + 25c + 5d + e$

$96 = a(6)^4 + b(6)^3 + c(6)^2 + d(6) + e$
$\therefore 96 = 1296a + 216b + 36c + 6d + e$

$149 = a(8)^4 + b(8)^3 + c(8)^2 + d(8) + e$
$\therefore 149 = 4096a + 512b + 64c + 8d + e$

The three equations can then be transformed to form a 5 x 5 and a 5 x 1 matrix with the corresponding coefficients:

We will call this first matrix <u>- Matrix A</u>

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 81 & 27 & 9 & 3 & 1 \\ 625 & 125 & 25 & 5 & 1 \\ 1296 & 216 & 36 & 36 & 1 \\ 4096 & 512 & 64 & 8 & 1 \end{pmatrix}$$

We will call the second matrix – <u>Matrix B</u>

$$\begin{pmatrix} 10 \\ 38 \\ 74 \\ 96 \\ 149 \end{pmatrix}$$

We will now take the inverse of Matrix A and multiply it with Matrix B to get Matrix C, using the graphics calculator. Matrix C will contain the a,b,c ,d and e values of the quadratic equation in a 5x1 Matrix.

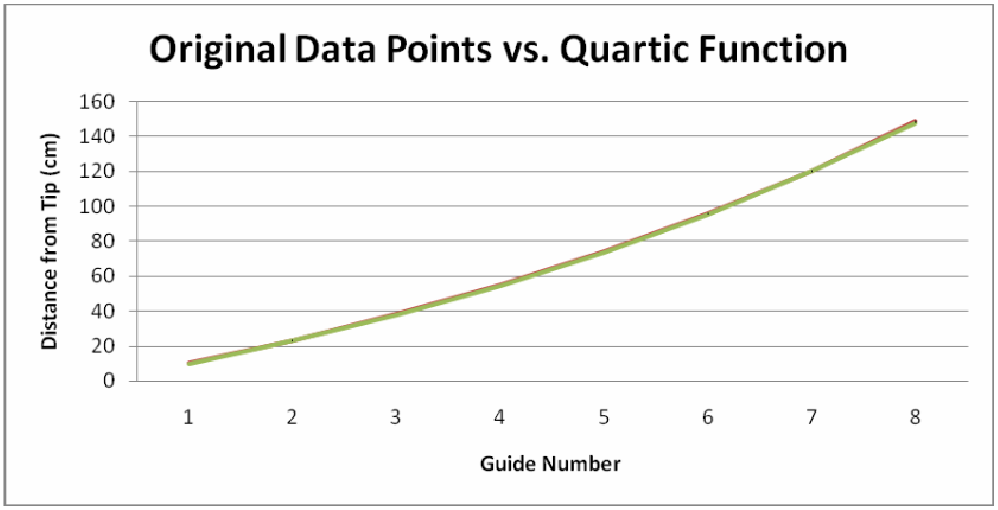
$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 81 & 27 & 9 & 3 & 1 \\ 625 & 125 & 25 & 5 & 1 \\ 1296 & 216 & 36 & 36 & 1 \\ 4096 & 512 & 64 & 8 & 1 \end{pmatrix}^{-1} \times \begin{pmatrix} 10 \\ 38 \\ 74 \\ 96 \\ 149 \end{pmatrix} = \begin{pmatrix} -0.005 \\ 0.138 \\ 0.033 \\ 12.262 \\ -2.429 \end{pmatrix}$$

$$\therefore \begin{pmatrix} -0.005 \\ 0.138 \\ 0.033 \\ 12.262 \\ -2.429 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix}$$

Therefore the respective values of the unknowns a,b,c,d,e are *a= -0.005, b=0.138, c=0.033 d=12.262* and *e=-2.429*. These values can then be subbed into the original equation of the quartic $y = ax^4 + bx^3 + cx^2 + dx + e$ to form the quartic function modelling the situation:

$$\boldsymbol{y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429}$$

Using Excel, we can now take this new function of $y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429$ and plot it on an axes along with the graph of the original data points to evaluate the accuracy of the function in modelling the situation.



Green Line: Original Data Points
Red Line: $y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429$

In the graph above, it can clearly be seen that the function $y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429$ accurately models the original data points given when Leo has a rod of overall len gth 230cm. As the red line representing the function of $y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429$ is not visible behind the graph of the original data points, this demonstrates the near complete accuracy of the function in modelling the situation. Using a graphics calculator, the data points of the function can be found and then compared with the original data points to assess the differences between them.

Distance from tip (cm) –Original Data Points	10	23	38	55	74	
Distance from tip (cm) - $y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429$	9.999	23.251	37.975	54.699	73.831	95.6

As seen in the table above, the values found from the function $y = -0.005x^4 + 0.138x^3 + 0.033x^2 + 12.262x - 2.429$ are almost identical to those of the original data. A small degree of error can be seen as the graph approaches the 7th and 8th guide, however not by a significant amount and so the equation is deemed as extremely accurate.

Using the graphics calculator, we can now find a function that models the situation to a greater degree of accuracy. I have chose n to use the method of Quartic Regression on the calculator, to provide me with a function with a very high degree of accuracy. I have chosen to use Quartic regression as a pose to quadratic or cubic, as when we raise the function to a higher power, the ac curacy is increased. This function can then be compared to the quartic gained through matrix methods above.

The method using the graphics calculator is as follows:

Quartic function: $\boldsymbol{y = ax^4 + bx^3 + cx^2 + dx + e}$

1. Press STAT button
2. Press EDIT
3. In L1 column – enter x values {1,2,3,4,5,6,7,8}
4. In L2 column – enter y values {10,23,38,55,74,96,120,149}
5. Press STAT
6. Press → arrow and select CALC
7. Press 7 – QuartReg
8. Press Enter

<u>GRAPHICS CALULATOR SCREEN</u>

$y = ax^4 + bx^3 + cx^2 + dx + e$

a = .0123106061
b = -.158459596
c = 1.730113636
d = 8.638979077
e = -.1964285714
R^2 = .9999865758

Therefore, by rounding the values of a,b,c,d and e given above to 4sf we can find the values of the unknowns:

$a = .012$

$b = -.158$

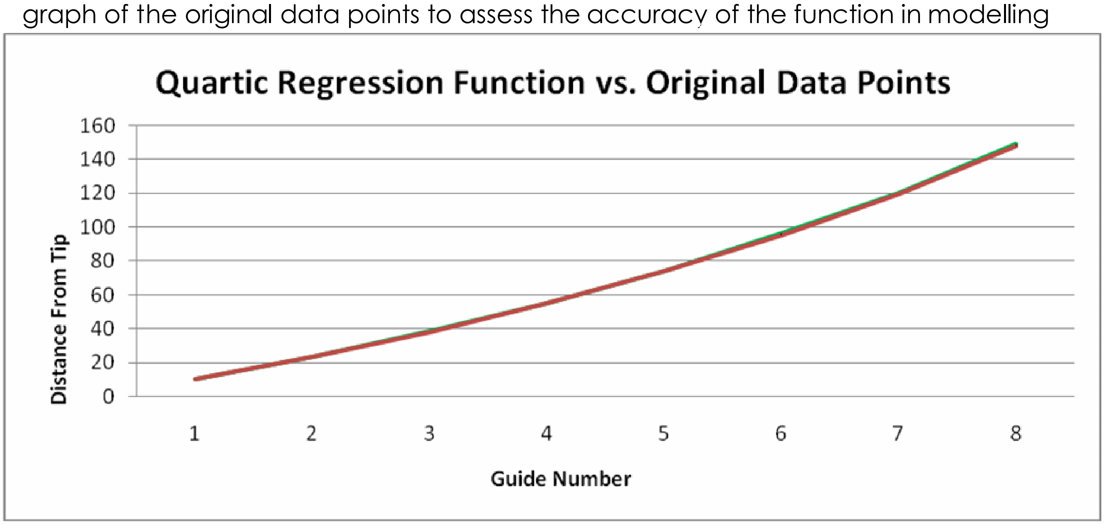
$c = 1.730$

$d = 8.639$

$e = -.196$

$\therefore \boldsymbol{y = 0.012x^4 - 0.158x^3 + 1.730x^2 + 8.639x - 0.196}$

Using Excel, we can now take this new function of $y = 0.012x^4 - 0.158x^3 + 1.730x^2 + 8.639x - 0.196$ and plot it on an axes along with the graph of the original data points to assess the accuracy of the function in modelling



the situation:

Green Line = Original Data Points
Red Line = $y = 0.012x^4 - 0.158x^3 + 1.730x^2 + 8.639x - 0.196$

In the graph above, it can clearly be seen that the function $y = 0.012x^4 - 0.158x^3 + 1.730x^2 + 8.639x - 0.196$ accurately models the original data points given when Leo has a rod of overall length 230cm. As the green line is barely visible, the correlation between the two is almost identical, although varying slightly at the 7th and 8th guide and this can be seen by obtaining the data points of the function using a graphics calculator and comparing them with the original data points.

Distance from tip (cm) – Original Data Points	10	23	38	55	74	96	
Distance from tip (cm) - $y = 0.012x^4 - 0.158x^3 + 1.730x^2 + 8.639x - 0.196$	10.027	22.93	37.997	55	73.999	95.342	

Now that we have found four functions to model the situation, it is relevant to assess which function most accurately models the situation. Using the four functions we can calculate the percentage error per equatio n to show the degree of accuracy in which the data points found using the function, correlate with the original data points. The lower the percentage error, the closer the function's points are to the original, and similarly with a higher percentage error the accuracy declines.

Using an excel spreadsheet, the average percentage error can be found:

The spread sheet was set up as below

X	Y(O)	Y1	% error	Y2	% error	Y3	% error	Y4	% error
1.00	10.00	**10.00**	0.00	**9.99**	0.01	**9.99**	0.01	**10.03**	0.27
2.00	23.00	**23.00**	0.00	**23.43**	1.85	**23.25**	1.09	**22.93**	0.30
3.00	38.00	**38.00**	0.00	**38.28**	0.72	**37.28**	1.91	**38.00**	0.01
4.00	55.00	**55.00**	0.00	**54.98**	0.05	**54.98**	0.05	**55.00**	0.00
5.00	74.00	**74.00**	0.00	**73.95**	0.07	**73.95**	0.07	**74.00**	0.00
6.00	96.00	**95.00**	1.04	**95.63**	0.39	**95.63**	0.39	**95.34**	0.69
7.00	120.00	**118.00**	1.67	**120.44**	0.37	**120.35**	0.29	**119.67**	0.27
8.00	149.00	**143.00**	4.03	**148.80**	0.13	**147.96**	0.70	**147.89**	0.74
Average % error			**0.84**		**0.45**		**0.56**		**0.29**

For each Y column (Y1,Y2,Y3,Y4) th e values were found by entering the function into a Graphics Calculator and finding the corresponding points using the 2 ND – TABLE function.

To calculate the percentage error using excel:

= ABS((Y (1,2,3,4) value – Y(O) value)/Y(O) value) x 100

Example multiplication: % error for Y3 when X = 2

$= ABS((23.25 - 23) \div 23) \times 100$
$= 1.09$

To calculate the average percentage error using excel:

Highlight values in % error column
Select Formulas
Select AutoSum – Average

Once the average percentage errors are calculated, by comparing the respective values it is clear which function most accurately represents the situation. As the smaller the % error, the more accurate the function, for this reason it is clear that the quartic regression function of $y = 0.012x^4 - 0.158x^3 + 1.730x^2 + 8.639x - 0.196$ best models the situation, with a percentage error of 0.29%. The quadratic function $y = x^2 + 10x - 1$ least accurately models the situation, with a percentage error of 0.84%.

To further test the applicability of the functions to the real life situation of Leo's fishing Rod, we can use the quadratic model of $y = x^2 + 10x - 1$ to find a ninth guide. By finding the placement of the 9th guide (in cm) from the tip, the fishing rod may be more effective in ensuring that the line does not tangle and that it casts easily and efficiently.

To find the distance of the ninth guide from the tip

$y = (9)^2 - 10(9) - 1$

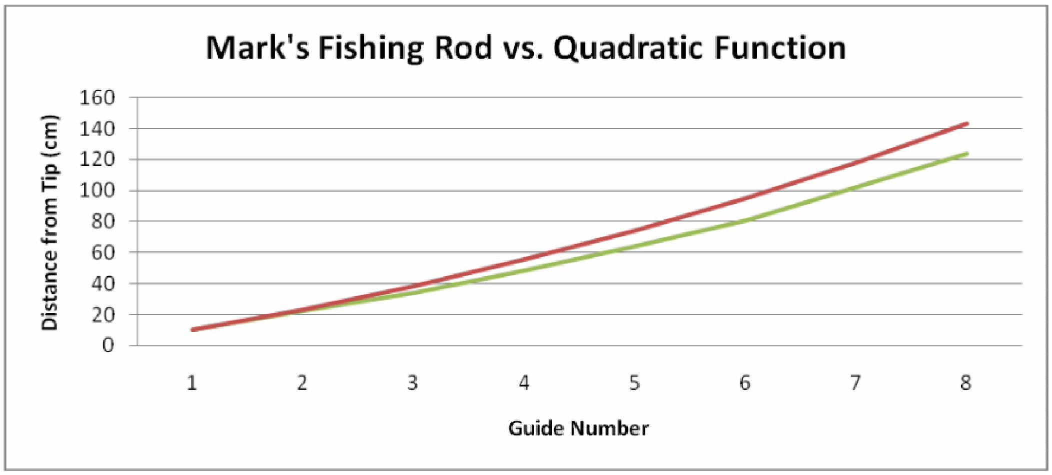
$\therefore \boldsymbol{y = 170cm}$

It can therefore be seen that the distance of the ninth guide from the tip of the fishing rod is 170cm. However, before a ninth guide can be added we must assess the implications upon the rod and ensure that its addition does not limit the functionality of the rod. As the overall rod is of length 230cm, by adding a ninth guide at distance 170cm, this will allow only 60cm for the reel and placement of hands for fishing. This may limit the ability to hold the rod and cast the reel easily and limit the fisherman's ability. Similarly if a ninth guide where added, instead of increasing the effectiveness of the line cast, it may have an opposite effect and make it more difficult to cats the line. Therefore before adding a ninth guide testing would need to be carried out on a realistic model to assess if the implications where negative or positive in their nature.

As this is a model of a real life situation, to test the applicability of the functions, we must apply it to a new set of data and assess if the function is applicable to further real life situations. Mark has a fishing rod with overall length 300cm. The table shown below gives the distances for each of the line guides from the tip of the fishing rod.

Distance from tip (cm)	10	22	34	48	64	81	102	124

Using excel we can generate a graph of the data points of Marks's fishing rod and those found using the quadratic function of $y = x^2 + 10x - 1$ to assess the degree to which the function models the new data.



Green Line: Mark's Fishing Lin Data Points
Red Line: Quadratic Function $y = x^2 + 10x - 1$

As seen above, the quadratic model fits the data for the first two guide points, however beyond this point, the graphs separate significant ly and the function does not fit the new data accurately. This can also be seen in the table below, which shows the data points of mark's fishing rod and the data points found using the quadratic equation:

Distance from tip (cm) Mark's Fishing Rod	10	22	34	48	64	81	102	124
Distance from tip (cm) $y = x^2 + 10x - 1$	10	23	38	55	74	95	118	143

As see on the previous page, as the guide numbers increase, the data points of the quadratic become more inaccurate and by guide 8, differ by around 20cm. So it can be seen that the limitations of our quadratic model is that it is limited by the size of the fishing rod, as well as being only applicable to the original set of data provided by Leo's fishing rod. As the data points of mark's fishing rod are significantly different from those of Leo's, the model found of Leo's fishing rod will be limited in its applicability to the distance of the guide's of Mark's fishing rod. The quadratic function was found using Leo's data points and to model these points. As the Y values of Mark's points vary greatly from these, it would be near impossible for the quadratic model found for Leo's fishing rod to match that of Mark's. Therefore it would be necessary to significant changes to the quadratic function to fit the new data, as when the size of the fishing rod is altered, the function is no longer applicable.

In order to obtain a function that models this new data accurately, b y using the QuadReg function on the graphics calculator, we can find a quadratic equation that is altered to fit the model of Mark's fishing rod.

Using the QuadReg function and entering in the respective Y values into the L1 column we find the function:

A Quadratic function: $\boldsymbol{y = ax^2 + bx + c}$

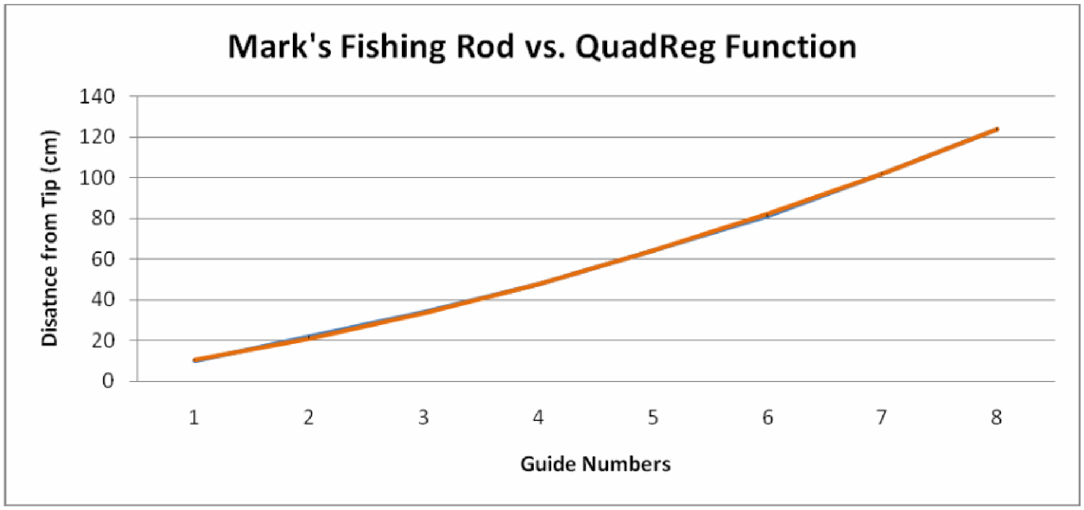
1. Press STAT button
2. Press EDIT
3. In L1 column – enter x values {1,2,3,4,5,6,7,8}
4. In L2 column – enter y values {10,22,34,48,64,81,102,124}
5. Press STAT
6. Press → arrow and select CALC
7. Press 5 – QuadReg
8. Press Enter

Graphics Calculator screen:

$y = ax^2 + bx + c$
a = 0.9345238095
b = 7.720238095
c = 2.053571429

$\therefore \boldsymbol{y = 0.935x^2 + 7.720x + 2.054}$

We can then compare this to the original quadratic function of $y = x^2 + 10x - 1$ and it is evident that significant chang es must be made to the values of a, b and c to model this new data. By adjusting the quadratic value of a=1 slightly to around 0.935 the model will be improved. With the values of b and c however, significant changes would be required, lowering the b value of the quadratic (10) to 7.72 and changing the quadratic c value of -1 to +2.054. These changes would formulate a function which would better fit the data points and more accurately model the data. This can be seen in the graph generated by excel modelling the data points of mark's Fishing rod and the new equation of $y = 0.935x^2 + 7.720x + 2.054$.



Orange Line: Marks Fishing Rod
Purple Line: $y = 0.935x^2 + 7.720x + 2.054$

The accuracy of the data can also be seen in the table of the data below:

Distance from top (cm)	10.709	21.234	33.629	47.894	64.029	82.034	101.909	123.654

As seen above, as the purple line of the new function is not visible behind the graph of mark's fishing rod, the newly adapted function accurately models the situation of Mark's fishing rod. By finding this new function, it is clear that the quadratic function found for Leo's fishing rod is not applicable to that of Mark's and the two situations cannot be compared due to the differing size of the fishing rods.

Throughout this investigation, we have assessed various models of quadratic, cubic and quartic function, found through both matrix methods and technology to model the situation's of both Leo and Marks fishing Rods. From the different values we have found for each equation, it is possible to determine that the most accurate graph that could be found is that of an eighth order function given we were given 8 data points. However, as this was not possible with a graphics calculator , by determining the values of quadratic, cubic and quartic functions, we were able to assess that the function found using technology, and with the highest power, the quartic regression equation, was the most accurate.

As the investigation requires an application to a real life situation, our meth od carried out allows for various conclusions to be drawn as the placement of guides on a fishing rod. For Leo's rod of 230cm, our quadratic functions found modelled the situation, but with a degree of error that meant it could not be deemed totally accurate in relation to the data points given. As our next functions, the cubic and quartic modelled the situation with a lesser degree or error we can conclude that as the power of the polynomial increases, the accuracy of the function increases . Secondly, it can also be seen that by adding a ninth guide, the implications could be positive in nature, easier cast and less tangle, or negative, decreased ability to cast and increased difficulty for the fisherman in the use of the rod. Our final conclusion is drawn from our inability to apply the quadratic function of Leo's rod to fit the data of Mark's. This shows that when the length of the fishing rod is changed, so too must the equation be changed.