Min Hua Ma

5-1-09

IB SL MATH

Internal Assessment

Type II

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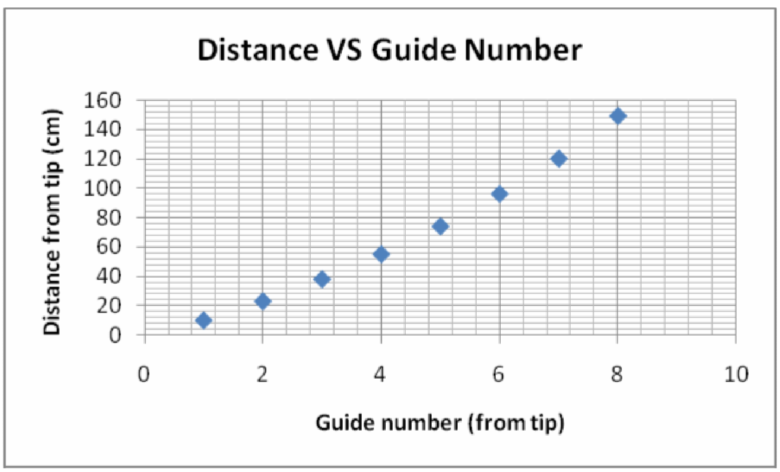
This assignment is an investigation to find different methods that model a given set of data. By using matrix methods, polynomial functions, and technology to find different equations, we can discover which equation best models the data. Leo has a fishing rod that has a length of 230cm, and the given data about his rod is:

Guide #	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

To begin the investigation, we began by plotting the given points in a scatter plot:

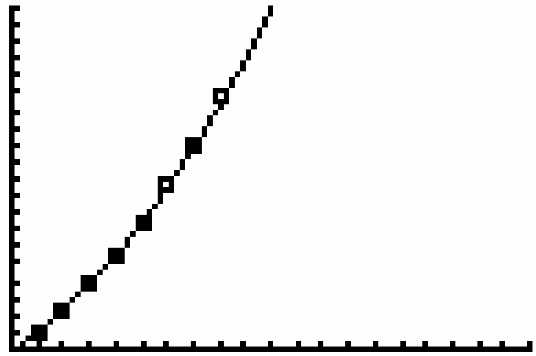


The first method is using matrices to find a quadratic function. A quadratic equation is: $ax^2+bx+c=y$, since there is only 3 variables (a, b, c) we can only create a 3 by 3 matrix and a 3 by 1 matrix. So I choose the first six data and separated them into two sets of three to make these equations to model the data.

$$\begin{bmatrix}1 & 1 & 1\\4 & 2 & 1\\9 & 3 & 1\end{bmatrix} x \begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}10\\23\\38\end{bmatrix}$$

$$[A] \qquad [X] \qquad [C]$$

To solve for $\begin{bmatrix}a\\b\\c\end{bmatrix}$ for the first set of three data, we take $[A]^{-1}$ x $[C]$ and get: $\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}1\\10\\-1\end{bmatrix}$ Using that information, a quadratic equation can be created: $1x^2+10x-1$. The model with the actual points is shown by this graph from a graphing calculator:

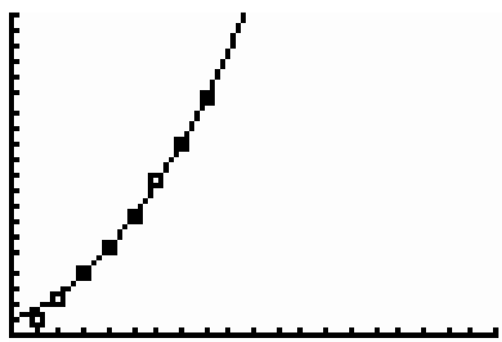


This graph shows that the equation $1x^2 + 10x - 1$ only goes exactly through the first five given data points and gets close to the last three data points only.

Now using the next three data points (4, 55) (5, 74) and (6, 96), we use the same method as the matrix above:

$$\begin{bmatrix} 16 & 4 & 1 \\ 25 & 5 & 1 \\ 36 & 6 & 1 \end{bmatrix} \text{X} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 55 \\ 74 \\ 96 \end{bmatrix}$$

And we get $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1.5 \\ 5.5 \\ 9 \end{bmatrix}$, which makes the quadratic equation $1.5x^2+5.5x+9$. By looking at the graph below, this equation only models the third, fifth, sixth, and seventh data points precisely. The first, second, fourth, and eight data points closely missed.



These model functions each differ and are constrained depending on the points used in the matrices. To clarify what I mean is, the first matrix is created by the first three given data points and so when they are graphed they must go through the first three points, but we don't know how well that equation/function can go through the rest of the data points.

Now we need to use the matrix method again to find a cubic equation that will model the specified data. A cubic equation is: ax^3+bx^2+cx+d, and to solve for it, the matrices will have to be in the dimensions 4 by 4 and a 4 by 1. So the first thing I had to do was to split the data into two different matrices, so I did one group of data the odd x's and one group the even x's.

Odd

1	3	5	7
10	38	74	120

Even

2	4	6	8
23	55	96	149

Now plug each of the x's into the cubic function (ax^3+bx^2+cx+d). Then put each group of data into a 4 by 4 matrix and set the y values into a 4 by 1 matrix.

For Odds:

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 27 & 9 & 3 & 1 \\ 125 & 25 & 5 & 1 \\ 343 & 49 & 7 & 1 \end{bmatrix}}_{[A]} \times \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{[x]} = \underbrace{\begin{bmatrix} 10 \\ 38 \\ 74 \\ 120 \end{bmatrix}}_{[B]}$$

For evens:

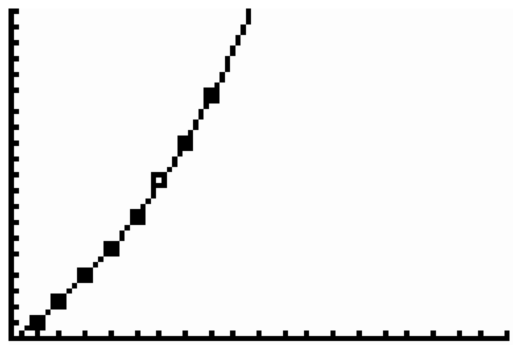
$$\underbrace{\begin{bmatrix} 8 & 4 & 2 & 1 \\ 64 & 16 & 4 & 1 \\ 216 & 36 & 6 & 1 \\ 343 & 49 & 7 & 1 \end{bmatrix}}_{[A]} \times \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{[X]} = \underbrace{\begin{bmatrix} 23 \\ 55 \\ 96 \\ 149 \end{bmatrix}}_{[B]}$$

At this time, use the process $[A]^{-1}$ x $[B]$ to solve for [X]. After doing so:

Odd: $[X]=\begin{bmatrix} .0416666667 \\ .625 \\ 10.95833333 \\ -1.625 \end{bmatrix}$ Even: $[X]=\begin{bmatrix} .0625 \\ .375 \\ 12 \\ -3 \end{bmatrix}$

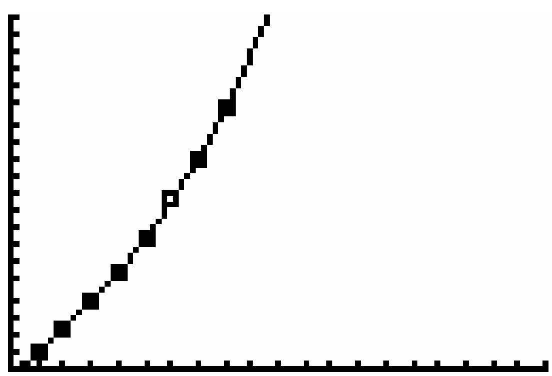
Then put each of the variables into a cubic function format and generate a graph for each equation.

The odd: $f(x)=.0416666667x^3+.625x^2+10.9583333x-1.625$



This graph shows that it touches all the data points, except when x=6. This equation models the given data by almost passing every point.

The even: $f(x)= .0625x^3 +.375x^2 +12x -3$



This graph illustrates that all, but the sixth point is touched by this function. Although this function is different from the odd cubic function, they both go through the same points and not the sixth. The differences of these two graphs are only the different numbers written for the cubic function.

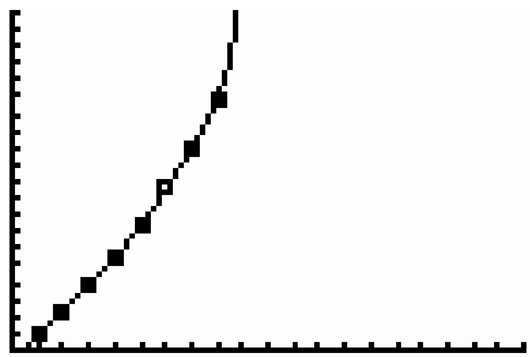
Next, we need to create a polynomial function that passes through all of the particular data. To do this, I decided to create a polynomial function by using matrices. In order to use matrices, we need to make the dimensions the same. Hence, we are given eight precise data points and to solve for a function, we need to make an 8 by 8 matrix and an 8 by 1 matrix. In order to create these dimensions, we need to set up a format for the polynomial function. As a result I set up the first term/part to be ax^7, the second bx^6, and so on so forth. To construct the 8 by 1 matrix, I used the letters of the alphabet a through h. Then, by using the format $ax^7+ bx^6 +cx^5 +dx^4 +ex^3 +fx^2 +gx +h$, I created my 8 by 8 matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ 2187 & 729 & 243 & 81 & 27 & 9 & 3 & 1 \\ 16384 & 4096 & 1024 & 256 & 64 & 16 & 4 & 1 \\ 78125 & 15625 & 3125 & 625 & 125 & 25 & 5 & 1 \\ 279936 & 46656 & 7776 & 1296 & 216 & 36 & 6 & 1 \\ 823543 & 117649 & 16807 & 2401 & 343 & 49 & 7 & 1 \\ 2097152 & 262144 & 32768 & 4096 & 512 & 64 & 8 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{bmatrix} = \begin{bmatrix} 10 \\ 23 \\ 38 \\ 55 \\ 74 \\ 96 \\ 120 \\ 149 \end{bmatrix}$$

[A] [X] [B]

Now take the $[A]^{-1}$ x $[B]$ to solve for [X]. Using a graphing calculator, enter what each of the variables are into a equation and press "GRAPH." The equation is supposed to be:

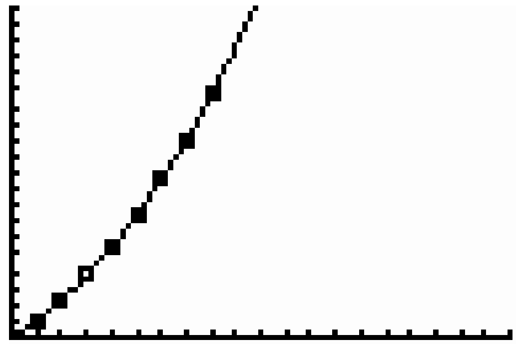
$f(x)=.0025793651x^7 - .0777777777x^6 +.95555552x^5 – 6.152777775x^4 +22.25138888x^3 -43.76944443x^2 +55.79047617x – 18.999999999$



Looking at this graph it shows that this function fits all the points except the sixth. There seems to be a hitch on getting a function that would go through the sixth point throughout this investigation so far. There seems to be no difference between the cubic functions created by matrices and this polynomial function.

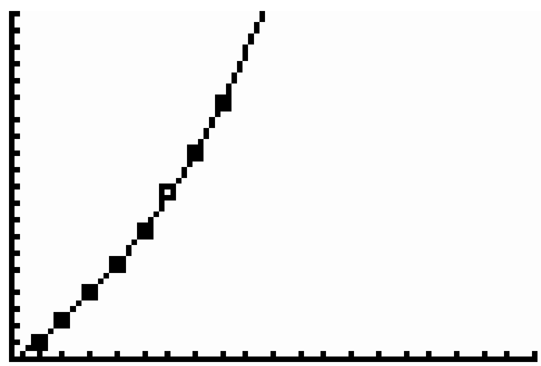
Our last method is using technology to help find another function that fits this data. By using a graphing calculator, enter the data into the calculator's lists. Press the STAT key and choose option 1: Edit from the Edit screen. This will bring up the List screen, and input all the data put the x-values data in list L1 and put your y-values data in list L2. Then, turn on plot 1 from STATPLOTS. Next, I set my windows to fit my data. Now that we have our scatter plot, we need to figure out a equation or two that will model all of the given data points.

To find a quadratic equation, press STAT and go to CALC and go down until QuadReg is seen, then press enter. After doing so, the calculator will give you what a, b, and c is equal too. So the equation is: $1.244047619x^2 + 8.458333333x + .8392857143$. The graph is:



Also by finding the QuadReg, it gives you what r^2 is. And for this equation, the regression is .99982635. This function models the data very well beside the third point.

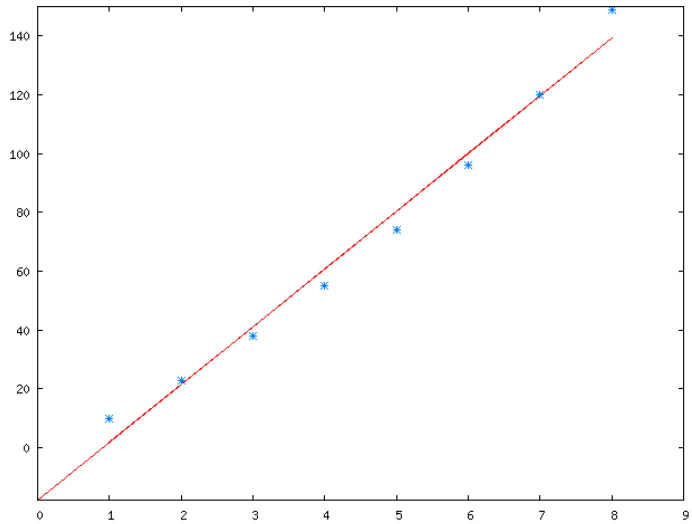
To find a cubic equation, all I did was press CubicReg instead of the QuadReg, and I got the function to be: $.0631313131x^3+.3917748x^2+11.70959596x -2.285714286$. The graph looks like this:



The regression for this function is .99996999372. This regression is greater than the regression from the QuadReg, which indicates the cubic function is stronger and fits the data more closely. This function models the data like the functions created by using matrices because it doesn't go through the sixth point.

Now using LinReg(ax+b) in the calculator, will allow us see the regression and equation of the line. The equation is y=(19.65)x+(-17.81) with the regression .98

And the graph is:



I think the function from the CubicReg best models the given data because the regression is very closer to 1 than the QuadReg. The quadratic equation from the matrix method fails to connect to more than one point. The cubic functions from created by the matrix method models the data just like the CubicReg, but there are so many different ways to split the data into two sets of fours, and my method was just one way. The polynomial function modeled the data quite well, but since the CubicReg and polynomial function both missed one point, the function with the larger regression got to be the better function. So the function, $0631313131x^3+.3917748x^2+11.70959596x -2.285714286$ models the data the most precise.

After creating functions, we need to choose a quadratic model to decide where to set a ninth guide. Using the quadratic function made from the use of technology, I set x=9.

$$1.244047619x^2 + 8.458333333x + .8392857143 \quad (x=9)$$

$$=100.7678571 + 76.07999997 + .8392857143$$

$$=177.7321429$$

$$\approx 178$$

The ninth guide could be placed 170cm from the tip of the fishing rod. By adding this ninth guide, it permits another possible quadratic function by using the last set of 3 data points, since we have already used the first and second set of 3 data points. *See below:*

1	2	3	4	5	6	7	8	9
10	23	38	55	74	96	120	149	178

By adding a ninth guide, it will affect the all the functions created. The cubic equations would not be as precise since there are an odd number of points. The polynomial equation must begin with ax^8 instead of ax^7.

Then the assignment gave us a new set of data to compare with the quadratic model from the first part of this investigation. This time Mark has a fishing rod that is 300cm.

Guide number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	22	34	48	64	81	102	124

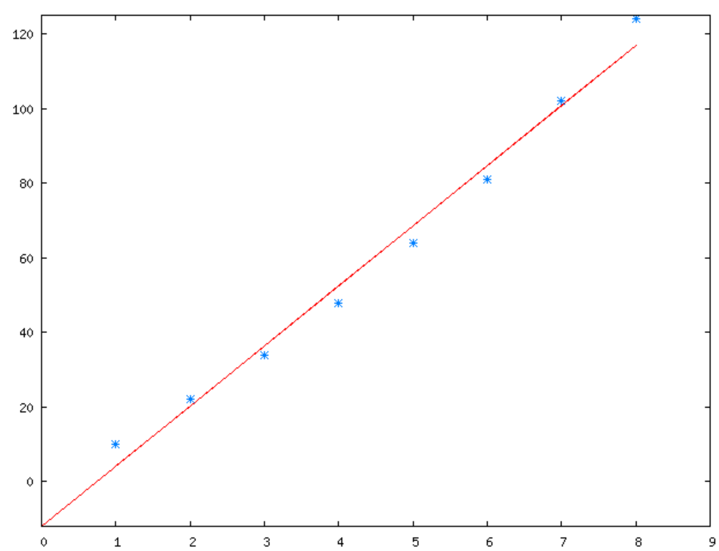
The quadratic model fits this data by only going through the first two points, and misses the rest. The quadratic model from the first part of the investigation doesn't really model this new set of data. To find how to modify the quadratic equation, I used technology to figure out what the QuadReg was and subtracted the first quadratic model by this QuadReg:

$1.244047619x^2 + 8.458333333x + .8392857143$ Original function

$-$ $.9345238095x^2 + 7.720238095x + 2.05351429$ New function

$.3095238095x^2 + .738095243x \; -1.214228547$

So, the quadratic has to modify by $.3095238095x^2 + .738095243x \; -1.214228547$ to fit the data created by the 300cm rod. The regression of the function $9345238095x^2 + 7.720238095x + 2.05351429$, is .9997802355. The LinReg of this data is written y=(16.13)*x+(-11.95) with the regression (.99).



The limitations to this model is that there is can't be an infinite number of guides and rod have only have certain lengths.