Catherine Wowk

Period 5 Pre-Calculus

Marking Period 1 Portfolio

Fishing Rod

A fishing rod requires guides for the line so that it does not tangle and so that the line casts easily and efficiently. Leo has a fishing rod with overall length 230 cm. The table shown below gives the distance for each of the line guides from the tip of his fishing rod.

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

Variables:

x is the guide number from the tip of the rod (1, 2, 3, 4, 5, 6, 7, 8)

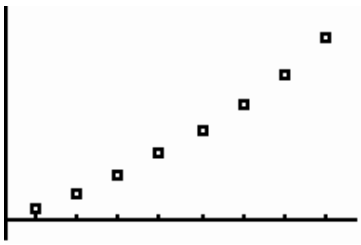
y is the distance from the tip in centimeters (10, 23, 38, 55, 74, 96, 120, 149)

Constraints:

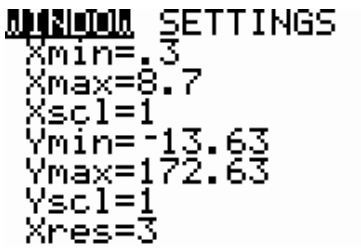
$\mathbf{1 \leq x \leq 8}$

$\mathbf{10 \leq y \leq 149}$

The above table is plotted on the graph below:



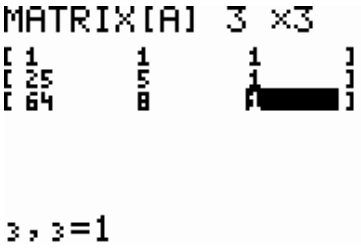
The graph above has a window of the following:



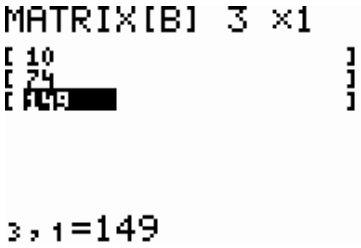
In order to find a quadratic function to model this situation the matrix method was used to find a, b, and c, of the quadratic function. The following three points were chosen from the above table to use in the matrix to find a quadratic function. In order to be sure the line would pass through the first line guide and the last line guide the first and last points were specifically chosen, along with a point in the middle of the two.

(1,10), (5,74), (8,149)

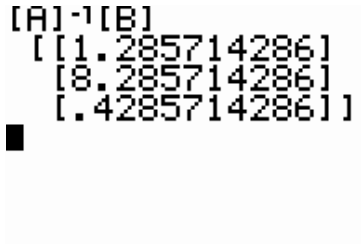
The x variables 1, 5, and 8 were plugged into **$y=ax^2+bx+c$** in order to find Matrix A.



The y variables 10, 74, and 149 were plugged into **$y=ax^2+bx+c$** in order to find Matrix B.



The matrices were solved by multiplying the inverse of Matrix A with Matrix B. The matrix function on the calculator was used to solve the matrices which then gave a, b, and c.



a= 1.285714286

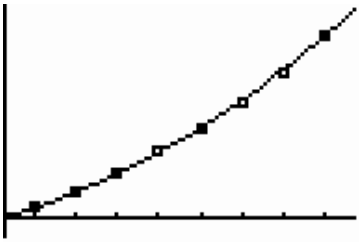
b=8.285714286

c= .4285714286

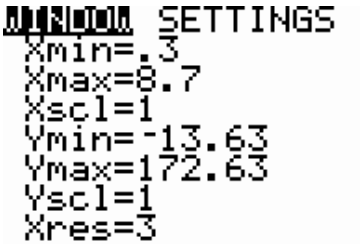
After plugging a, b, and c, into, $y=ax^2+bx+c$, this quadratic function was found:

$y=1.2857x^2+8.2857x+.42847$

The line was graphed on the axes with the original data points. The line goes straight through the data points proving that this quadratic function works for this situation.



The graph above has a window of the following:



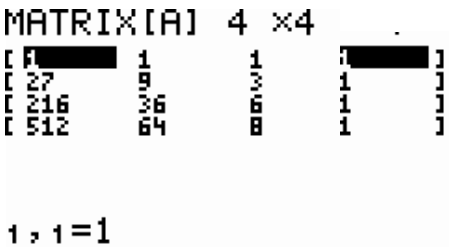
To find the cubic function that will model this situation the matrix method was used. Four points were chosen from the below table which illustrates the distance of the line guides from the tip of the fishing rod.

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	23	38	55	74	96	120	149

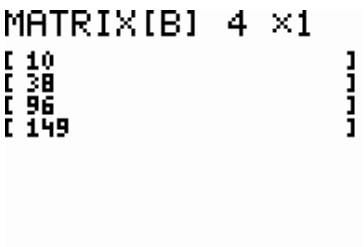
The first and last points for chosen so the graph would be sure the first and last line guides are shown on the graph.

(1,10), (3,38), (6,96), (8,140)

The x variables 1, 3, 6, and 8 were plugged into the equation $\mathbf{y=ax^3+bx^2+cx+d}$ in order to find Matrix A.



The y variables 10, 38, 96, and 149 were plugged into the equation $\mathbf{y=ax^3+bx^2+cx+d}$ to find Matrix B.



To solve the matrices the inverse of Matrix A was multiplied by Matrix B. The matrix function on the calculator was used to solve the matrices which found a, b, c and d of $y=ax^3+bx^2+cx+d$

```
[A]⁻¹[B]
[[.0523809524 ]
 [.5428571429 ]
 [11.14761905 ]
 [-1.742857143]]
```

a= .0523809524

b= .5428571429

c= 11.14761905

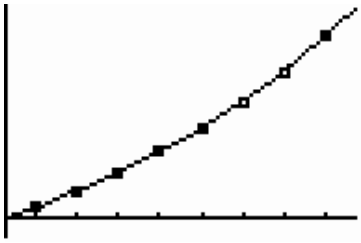
d= -1.742857143

a, b, c, and d were then plugged into $y=ax^3+bx^2+cx+d$ and the cubic function was found.

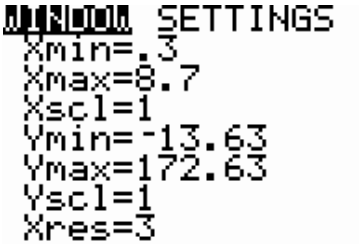
The cubic function below is rounded to the fourth decimal place.

$y=.0524x^3+.5429x^2+11.1476x-1.7429$

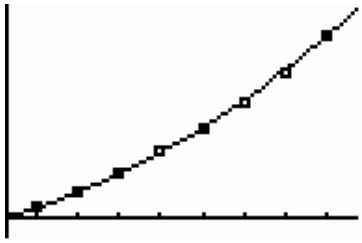
The above function was graphed below with the original data points.

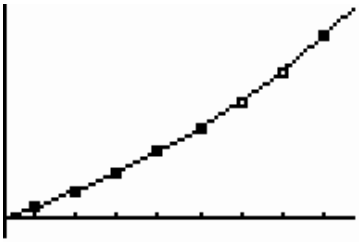


The above graph has the following window:



Below is the graph of both the quadratic function and the cubic function. The quadratic function is on the left while the cubic function is on the right. The points on both graphs are exactly the same. Though they appear to be the same there is a slight difference in the graphs. The quadratic function does not pass through three of the points, though it is very close. The cubic function passes through all but two of the points. The cubic function is more accurate than the quadratic because in order to find the cubic function using the matrix method there were more points than when finding the quadratic function, making the cubic function more precise.





$y=1.2857x^2+8.2857x+.42847$ **$y=.0524x^3+.5429x^2+11.1476x-1.7429$**

To find a polynomial function that passes through every data point, all of the data points were used in the matrix method that will help us find the equation

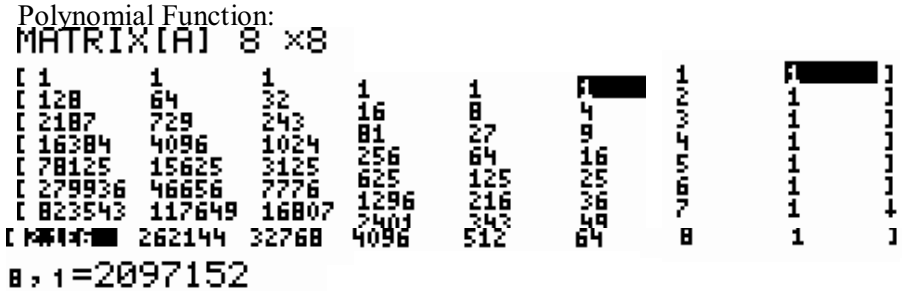
$y=ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h$. The more data points used to find the matrices the more accurate the graph of the polynomial will be. By using all of the data points we will be sure that it will pass through all of them when graphed.

(1,10), (2,23), (3,38), (4,55), (5,74), (6, 96), (7,120), (8,149)

The x variables 1, 2, 3, 4, 5, 6, 7, and 8 were plugged into the equation

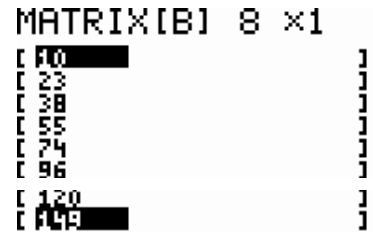
$\mathbf{y=ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h}$ to find Matrix A.

Polynomial Function:



The y variables 10, 23, 38, 55, 74, 96, 120, and 149 were plugged into the equation

$y=ax^7+bx^6+cx^5+dx^4+ex^3+fx^2+gx+h$ to find Matrix B.



To solve the matrices, the inverse of Matrix A must be multiplied by Matrix B. The matrix function on the calculator can be used.

```
[A]⁻¹[B]

[[.0025793651 ]
 [-.0777777777]
 [.9555555552 ]
 [-6.152777775]
 [22.25138888 ]
 [-43.76944443]
 [55.79047617 ]↓
 [-18.99999999]]
```

a=.0025793651

b= -.077777777

c=.9555555552

d=-6.152777775

e=22.25138888

f=-43.76944443

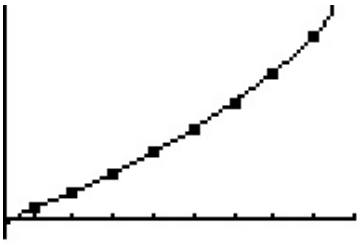
g=55.79047617

h=-18.99999999

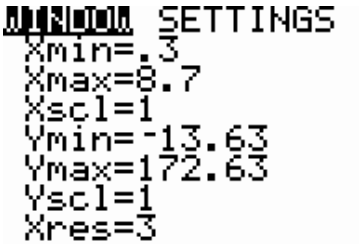
After replacing the variables with the numbers, the polynomial function was found.

$y=.0025793651x^7 -.077777777x^6 +.9555555552x^5 -6.152777775^4 +22.25138888x^3 - 43.76944443x^2 +55.79047617x-18.999999999$

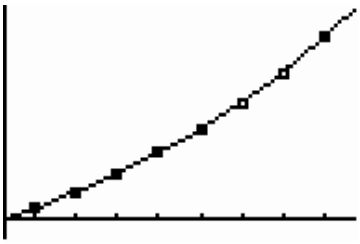
The polynomial function is graphed below with the original data points. The graph passes through each data point.

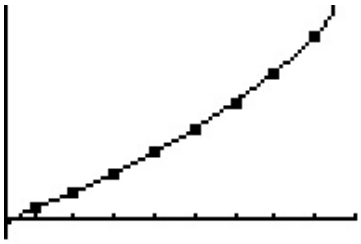


The above graph has the following window:



Shown below is the cubic function and the polynomial function. The cubic function is on the left and the polynomial function is on the right. The polynomial function passes through all the data points while the cubic function only passes through six of them. This shows that the polynomial function is more accurate.



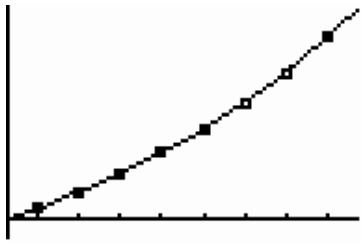


Technology can be used to find another function that fits the data. The Cubic Regression function on the calculator will give the values for a, b, c, and d in a cubic function that fits this model.

```
CubicReg
 y=ax³+bx²+cx+d
 a=.0631313131
 b=.3917748918
 c=11.70959596
 d=-2.285714286
```

The cubic function below was found using technology, and was rounded to the 4th decimal. It is graphed on an axes with the original data points.

$y=.0631x^3+.3918x^2+11.7096x-2.2857$



Compared to the quadratic function, this graph is more accurate because it goes through six points when the quadratic function goes through only five. The first cubic function, also, goes through only six of the data points, showing that the two cubic functions are similar. The polynomial function passes through all eight which proves that the polynomial function best models the situation.

Four functions have been found above that model this situation.

A quadratic function: **$y=1.2857x^2+8.2857x+.42847$**

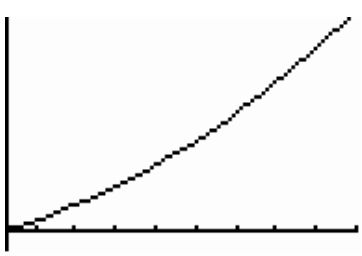
A cubic function: **$y=.0524x^3+.5429x^2+11.1476x-1.7429$**

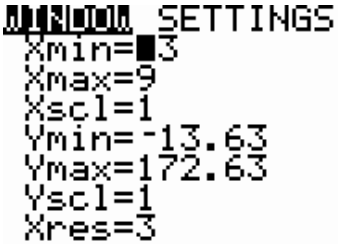
A polynomial function to the seventh power: **$y=.0025793651x^7-.077777777x^6+.9555555552x^5-6.152777775^4+22.25138888x^3-43.76944443x^2+55.79047617x-18.999999999$**

A cubic function found using technology: **$y=.0631x^3+.3918x^2+11.7096x-2.2857$**

Out of these four functions the one that models the situation the best would be the polynomial function to the seventh power. The function was found using all eight of the data points from the original table. The more points used in finding a function the more accurate it will be. When graphed this function was the only one which went through all of the data points on the graph, proving that it is the more precise function out of the four found.

If Leo wished to add a ninth guide to his fishing rod he could do so using the quadratic model **$y=1.2857x^2+8.2857x+.42847$** which makes the graph below with the following window





9 is substituted into x in the equation **$y=1.2857x^2+8.2857x+.42847$**

$y=1.2857(9)^2+8.2857(9)+.42847$

$y=179.14147$

The place to fit the ninth rod would be at point (9, 179.15). Adding a ninth guide would make the rod more efficient.

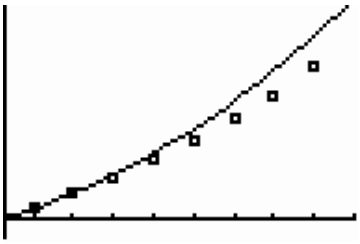
The table below shows the ninth tip compared to the other eight.

Guide number (from tip)	1	2	3	4	5	6	7	8	9
Distance from tip (cm)	10	23	38	55	74	96	120	149	179.15

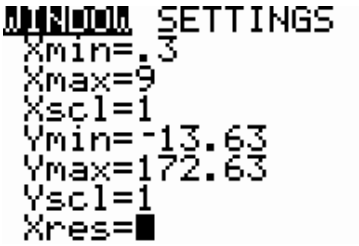
Mark has a fishing rod with an overall length of 300 cm. The table shown below gives the distances for each of the line guides from the tip of Mark's fishing rod.

Guide Number (from tip)	1	2	3	4	5	6	7	8
Distance from tip (cm)	10	22	34	48	64	81	102	124

The quadratic function, **$y=1.2857x^2+8.2857x+.42847$**, from the first situation, is graphed below along with the data points from above.



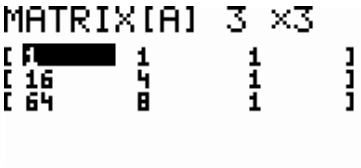
The above graph has a window of the following:



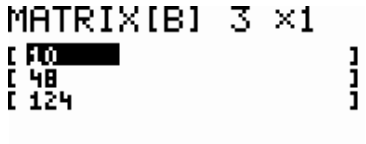
The quadratic line used in the first situation does not fit the data points in the second situation. A new quadratic model should be found using the matrix method.

The points (1,10), (4,48) and (8,124) were used in finding the new quadratic function.

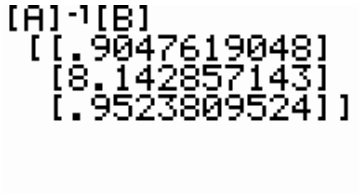
The x values 1, 4, and 8 were plugged into $y=ax^2+bx+c$ to find Matrix A



The y values 10, 48, and 124 were plugged into $y= ax^2 +bx+c$ to find Matrix B



The inverse of Matrix A is multiplied by Matrix B

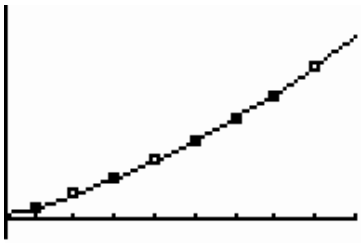


a= .9047619048

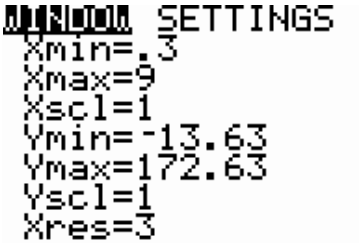
b= 8.142857143

c= .9523809524

The quadratic function found to fit the new situation is **$y=.9048x^2 +8.1429x+.9524$** rounded to the fourth decimal place. Below the new quadratic formula is graphed with the data points from the new table.



The above graph has a window of the following:



Marks rod is longer than Leo's and his guides are closer together. There is a lot of space between the last guide and the tip of the rod in which the line could still get tangled. If he had more guides closer together there would be a lesser chance of the wire getting tangled.