Math Portfolio II
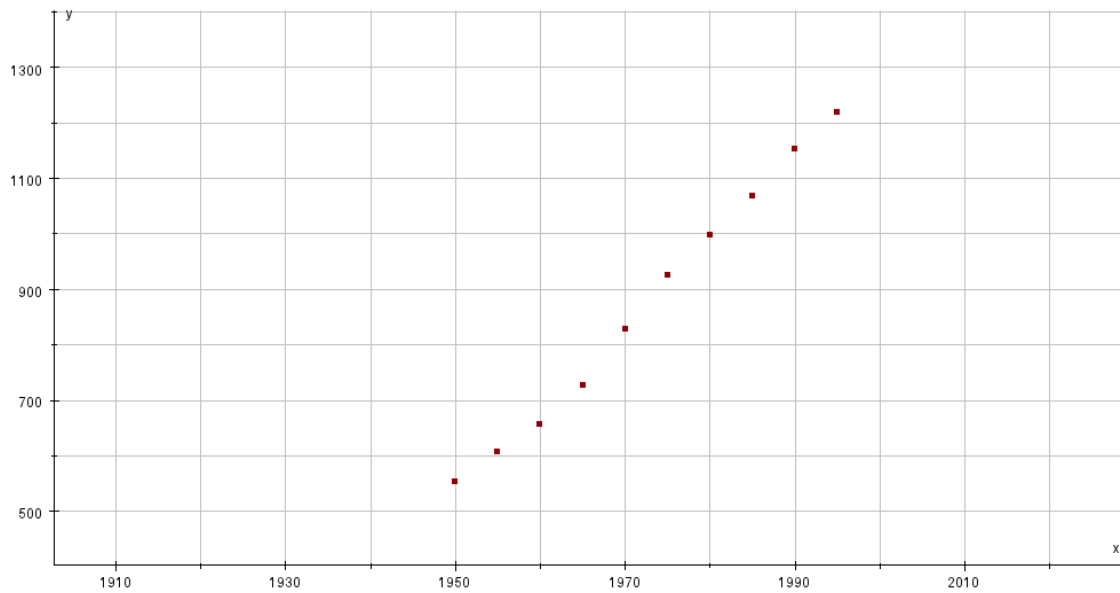
Population Treads In China

By Amber Perng

Aim: In this task, you will investigate different functions that best functions that best model the population of China from 1950 to 1995.

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Population in Millions	554.8	609.0	657.5	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

By plotting the above data points in autograph, we get:

Graph 1: Population of China 1950~1995

Variable
x-axis: the time (t)
y-axis: the population of China (p)

The above graph is plotted according to the population of China from 1955 to 1995. The points show that the population has increased at a constant rate, but had increased a little more from 1765 to 1970 comparing to other years, before it returned back to the constant increase.

To create a model function to fit the behaviour of this graph, I will use **linear, quadratic and** exponential **functions** to find its best fit.

Linear Function: y=ax+b

First, I use two points of (1955, 609) and (1985, 1070) from the given data to find the parameters of the function.
I chose these two points because they are the only two combinations that are all integers, which will be easier to calculate.

  609=1955a+b
1070=1985a+b
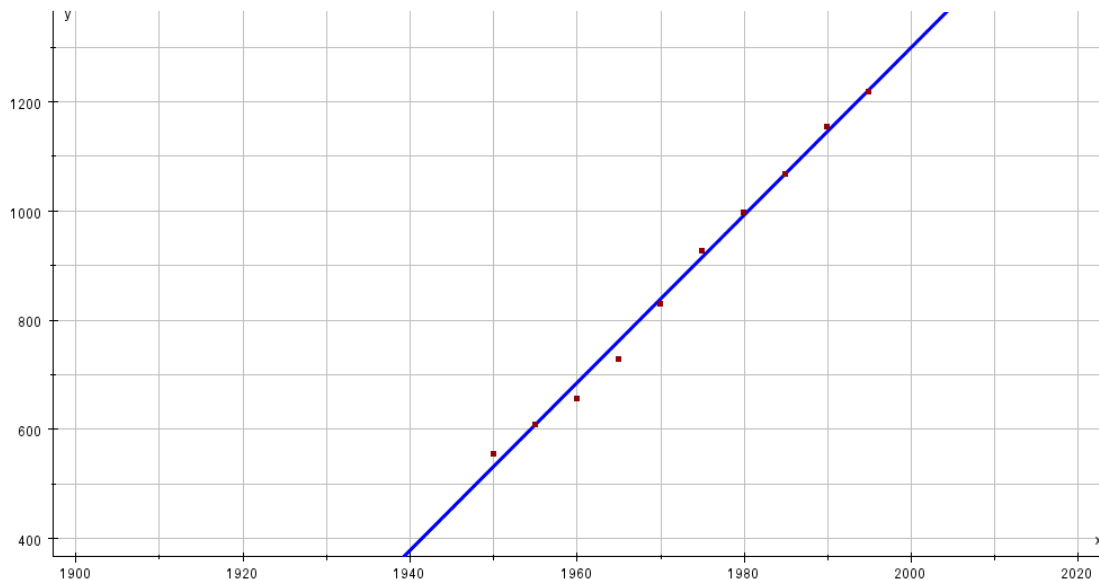
Finding value a and b by using simultaneous equation:

  1070=1985a+b
− 609=1955a+b
_____

   461=   30a

a =$\frac{161}{30}$ $\doteqdot$ **15.37**
b = 609−1955×15.37=**29439.35**

Which gives the equation of **y=15.37x−29439.35** as below:

Graph 2: Population of China 1950~1995 with linear function

It appears to be rather fit to the coordinates. However, the coordinates above show to contain curves, which linear function cannot do, as it did not go through all the points. Also, if it happens to be linear function, it would mean that China's population would grow infinitely at a constant speed.
Therefore, I have the conclusion that linear function is not the model of the graph.
Quadratic Function: y=ax^2+bx+c

Quadratic function has three parameters, in which I will calculate them out below.

I use (1955, 609), (1985, 1070) and (1960, 657.5) to find the parameters. These sets are chosen because they are the multiples of 5 or even integers, which are easier to calculate.

$609=(1955)^2a+1955b+c$
$657.5=(1960)^2a+1960b+c$
$1070=(1985)^2a+1985b+c$

It would be difficult to calculate if I use the method with linear function. Therefore, I use matrix to find the parameters as below shown:

$$\begin{vmatrix} 609 \\ 657.5 \\ 1070 \end{vmatrix} = \begin{vmatrix} 1955^2 & 1955 & 1 \\ 1960^2 & 1960 & 1 \\ 1985^2 & 1985 & 1 \end{vmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

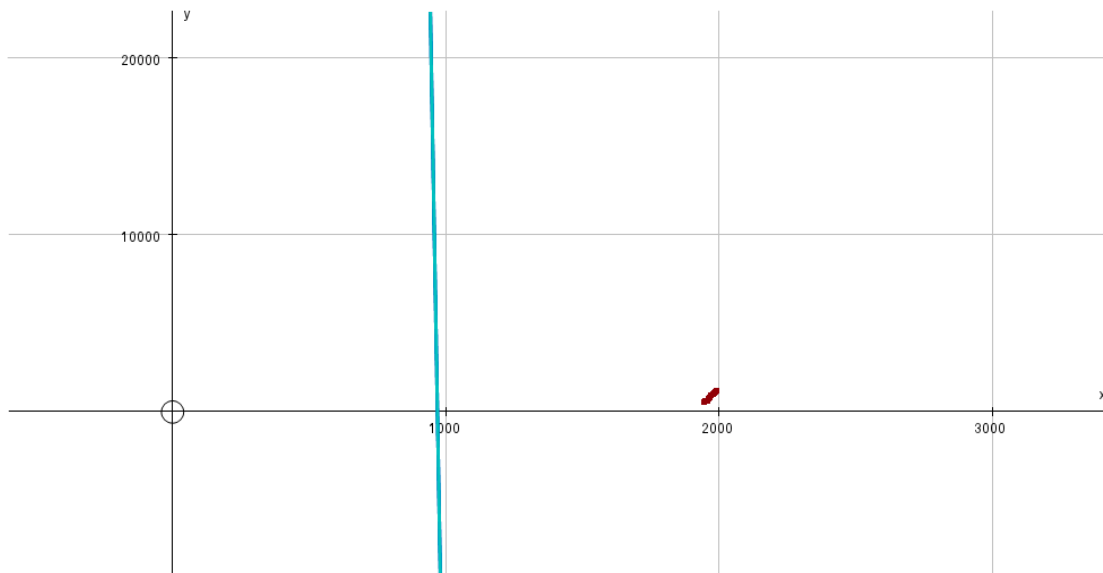In which after I inverse the equation, the parameters will become the main subject:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{vmatrix} 1955^2 & 1955 & 1 \\ 1960^2 & 1960 & 1 \\ 1985^2 & 1985 & 1 \end{vmatrix}^{-1} \begin{vmatrix} 609 \\ 657.5 \\ 1070 \end{vmatrix}$$ using Graphic Digital Calculator

a ≒ 0.23
b ≒ -877.70
c ≒ 850186.83

Which gives the equation of **$y=0.23x^2-877.7x+850186.83$**, as below:

Graph 3: Population of China 1950~1995 with quadratic function

According to the graph, we can see that quadratic function did not fit the points at all. Below is a recalculation by plotting variable x into the equation:

$0.23(1955)^2-877.7(1955)+850186.83=13349.08$ → off by 12740.08

$0.23(1960)^2-877.7(1960)+850186.83= 13462.83$ → off by 12895.33
$0.23(1985)^2-877.7(1985)+850186.83= 14024.08$ → off by 12954.08

From the recalculation, we could see that this equation has an inconstant parameter c, which it varies the value of y when adding itself onto the number, thereby unable to provide an appropriate curve.

I then used Graphic Digital Calculator to revise the equation, below is what the values given by the calculator:

$y=ax^2+bx+c$
a=0.355454545
b=-124.7309394
c=108601.1173

It shows that the values of what I calculated are far from the correct value; this may be another reason of the result of the incorrect graph.

<u>Quadratic Function II: $y=a(x-b)^2+c$</u>

I then decided to try another type of quadratic function. However, this time I tried to find it by using constant controller in Autograph to change the values of parameters in order to fit with the points.

Below is the result after serious of trials and errors:
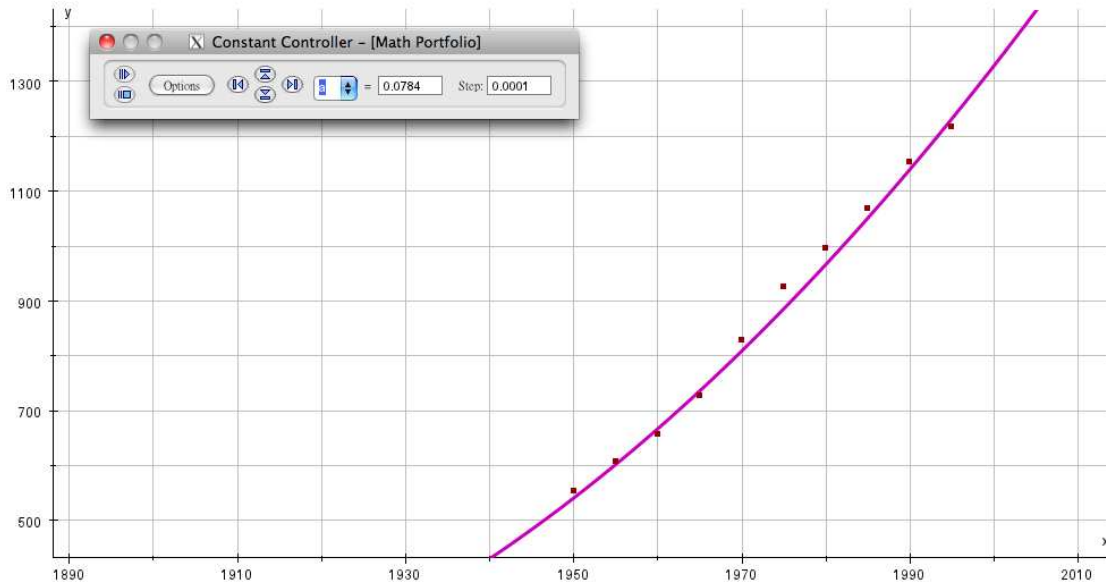
Graph 4: Population in China 1950~1995 with quadratic function II

The values of the parameters are:

a=0.0784
b=1874.37
c=93

Which give the equation of **y=0.0784(x-1874.37)^2+93**

The equation above fits better than the equation **y=0.23x^2-877.7x+850186.83**, however the line of linear function is a better best fit than this one since it misses more points than linear function, such as coordinates from year 1970 to 1990 have all failed to meet the line of the best fit as they tend to be more inclined, compare to other coordinates.

By using constant controller, I was able to visually understand how the changes of the parameters can determine the shape of the graph. Therefore, I have come to an conclusion that quadratic function is not the ideal model for the given data, not only because the given data is shown not to be a perfect curve, thus the quadratic cannot fit onto it completely, but also it means that the population of China will become infinitively large at a constant increase.

Exponential Function: y=ab^x

For exponential function, I will use (1955, 609) and (1985, 1070) to find the parameters of this function, and to see if it will fit with the graph better.

$609 = ab^{1955}$
$1070 = ab^{1985}$

Since they both share the same value of a, therefore by making them into one equation we will have:

$609 \times b^{-1955} = 1070 \times b^{-1985}$

First, divide 1070 from both sides, then divide b^{-1985}, we get:
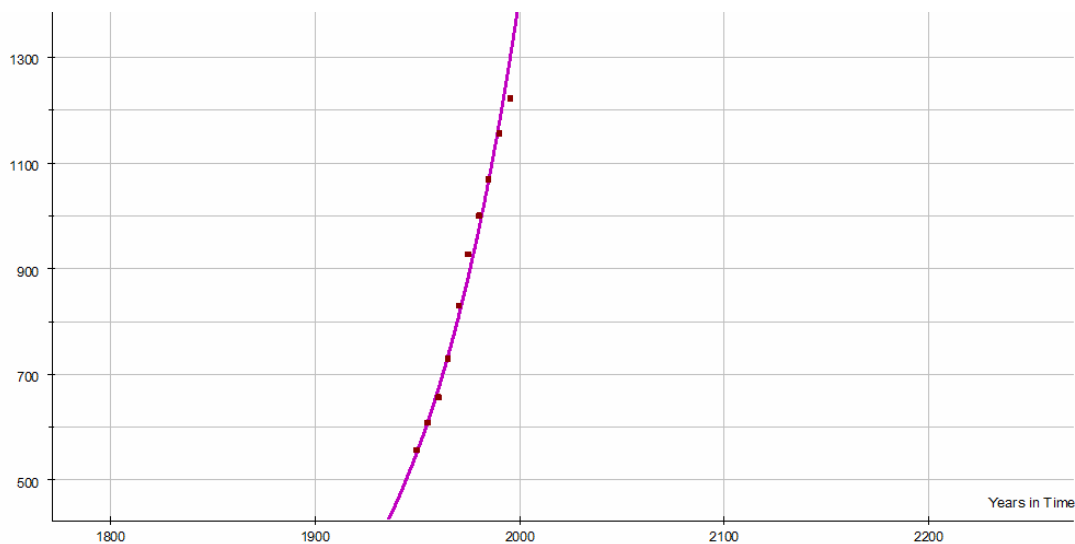
$$\frac{b^{-1985}}{b^{-1955}} = \frac{609}{1070}$$

By simplifying the equation, we get:

$$\frac{1}{b^{30}} = \frac{609}{1070} \qquad b = \sqrt[30]{\frac{1070}{609}} = 1.018964099$$

$609 = a(1.018964099)^{1955}$
$\quad = a(8.93)^{15}$ $\qquad\qquad\qquad\qquad$ $a = 6.82 \times 10^{-14}$

Which we get the equation of $y = 6.82 \times 10^{-14} \times 1.02^{x}$ as below:

Graph 4: Population of China 1950~1995 with exponential function

This model has the closest best-fit compare to other models being produced by other equations; it has managed to fit into most of the points.

However, if it were the model of this graph, it would mean that the population of China is infinitive and grows at a constant rate, which shouldn't happen because from the graph, we could see that the slope of the graph is starting to change in year 1995, which may indicate the decrease of the population.

A researcher suggests that the population, *P* at time *t* can be modeled by:

$P(t) = \frac{K}{1-Le^{-Mt}}$, where K, L and M are parameters.

This is a simple logistic function.
When t=0, $P(0) = \frac{K}{1+Le}$, which K is at its maximum because it is indirectly proportional to parameter t.
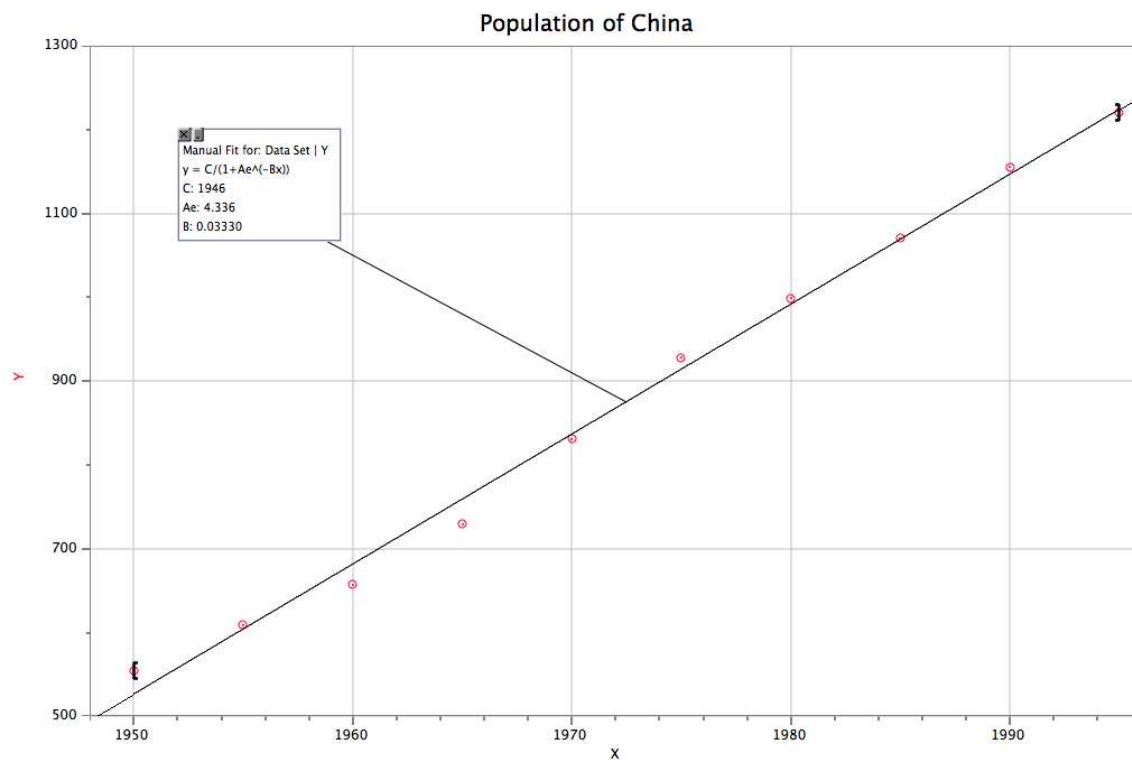Therefore, according to the given data from the above table, we can use GDC calculator to find the parameters:

K≑1946.18
L≑4.34
M≑0.33

By substituting the values of K, L and M into the formula, we have:

$$P(t) = \frac{1946.18}{1 + 4.34e^{-0.33t}}$$

Graph 5: Population of China in logistic function

Here are additional data on population trends in China from the 2008 World Economic Outlook, published by the International Monetary Fund (IMF):

Year	1983	1992	1997	2000	2003	2005	2008

Population in millions	1030.1	1171.7	1236.3	1267.4	1292.3	1307.6	1327.7

In which we have the following graph containing all the points:
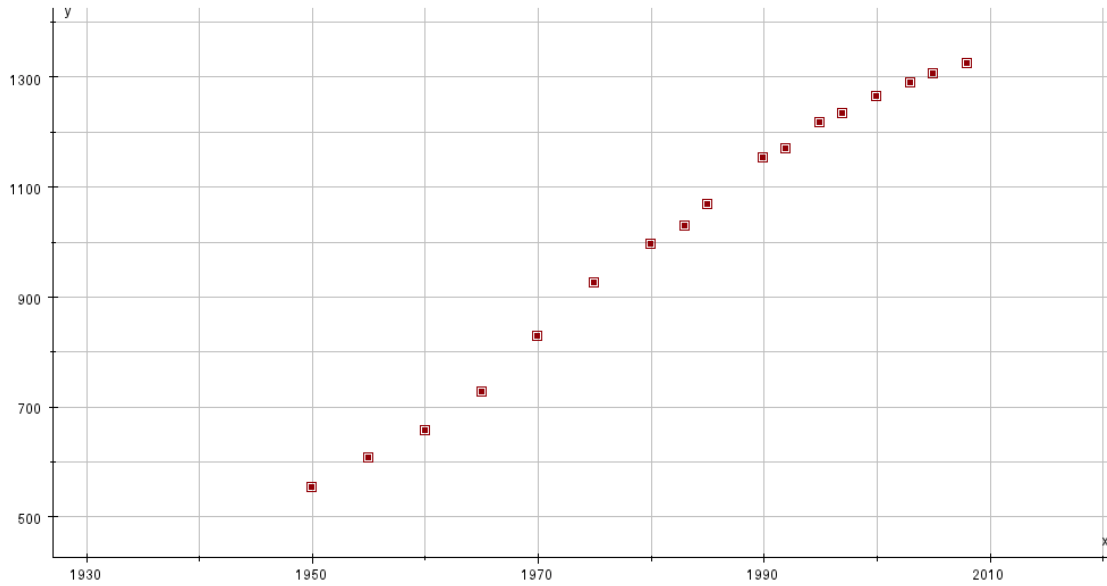
Graph 7: The final population of China graph

I use the function $P(t) = \frac{K}{1 - Le^{-Mt}}$ and find the parameters using Graphic Digital Calculator:

K ≐ 1617.46
L ≐ 1.35
M ≐ 0.399

Which I then plot the equation into the graph: