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Math Standard Level

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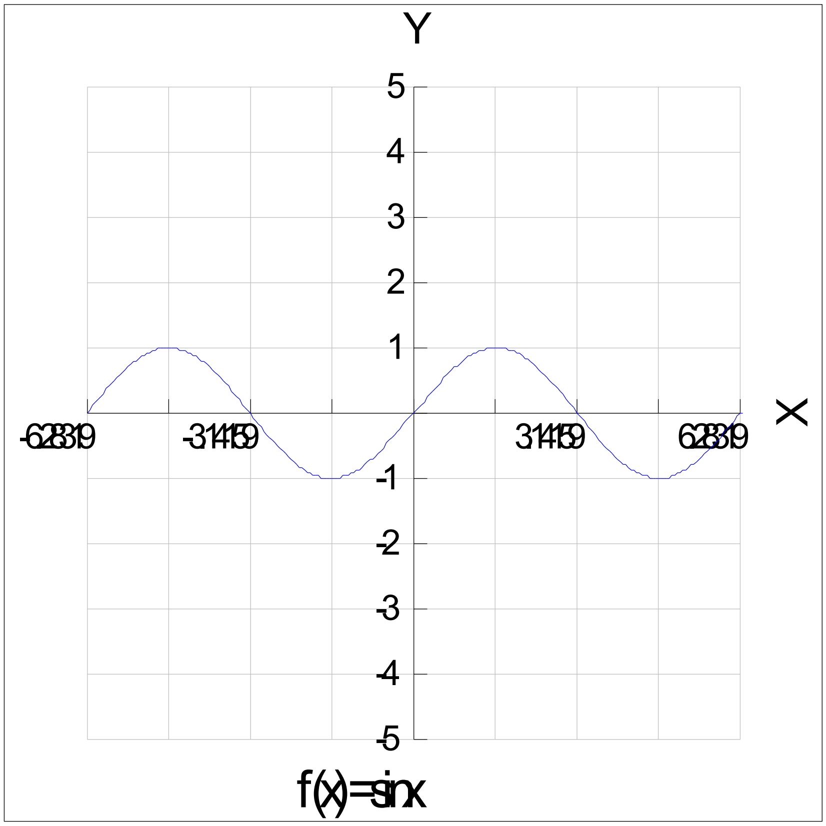
Derivatives of Sine Functions

Method

1. The following information was my investigation to find the derivative of the function $f(x) = \sin x$.

   a) The following is a graph for the function $f(x) = \sin x$ for $-2\pi \le x \le 2\pi$.

TI – 83 Graph Window	
Xmin =	-2π
Xmax =	2π
Xscl =	$\frac{\pi}{2}$
Ymin =	-5
Ymax =	5
Yscl =	1



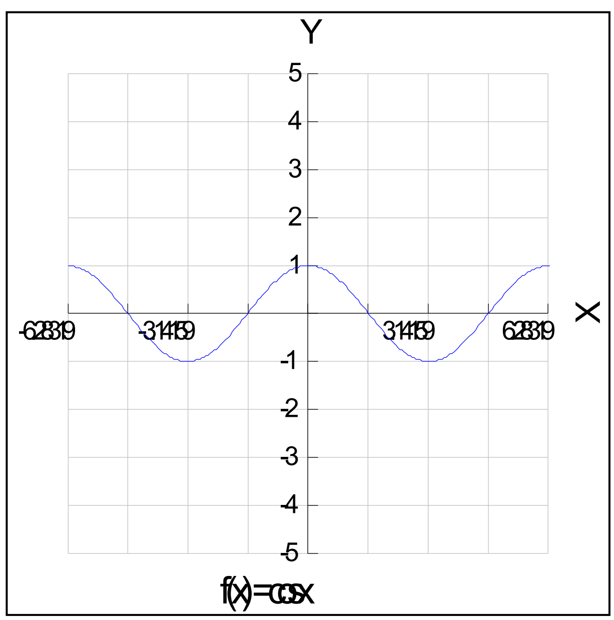
b) The table below describes the behavior of the gradient of $f(x) = \sin x$ for $-2\pi \le x \le 2\pi$.

Gradient behavior in given points of $f(x) = \sin x$		
x	Y	**Gradient (+, −, or 0)**
-2π	0	+
$-\frac{3\pi}{2}$	1	0
$-\pi$	0	–
$-\frac{\pi}{2}$	-1	0
0	0	+
$\frac{\pi}{2}$	1	0
π	0	–
$\frac{3\pi}{2}$	-1	0
2π	0	+

c) Because the derivative of a function is the gradient at a given point, my conjecture for $y = f'(x)$ is as follows:

$$f(x) = \sin x$$
$$\therefore f'(x) = \cos x$$

TI – 83 Graph Window	
Xmin =	-2π
Xmax =	2π
Xscl =	$\frac{\pi}{2}$
Ymin =	-5
Ymax =	5
Yscl =	1



nDeriv(sin(x)) at a given X-value	
-2π	1
$-\frac{3\pi}{2}$	0
$-\pi$	-1
$-\frac{\pi}{2}$	0
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

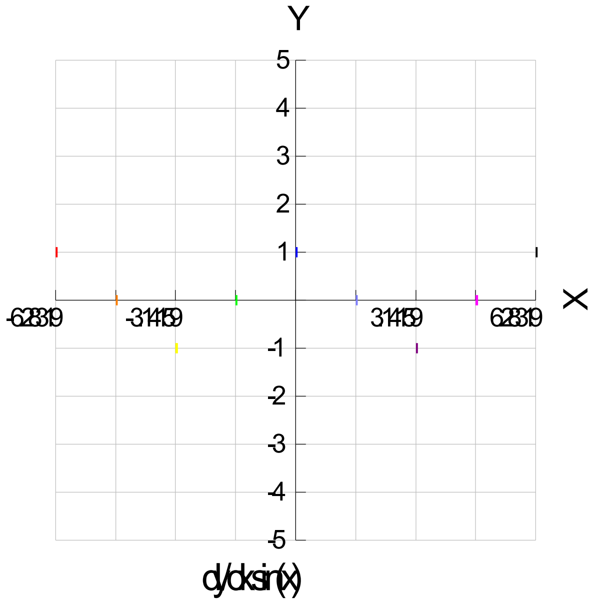
<u>nDeriv of sin$x$ on a TI-83 Calculator</u>

- 2nd, 0 (CATALOG)
- LOG (Alpha N), nDeriv
- For example, to get the derivative of sin$x$ at x = 0, enter:
  nDeriv(sin(X),X,0)

*Relationship of dy/dx of f(x)=*sin*x versus Y-value of f(x)=*cos*x at a given X-value*

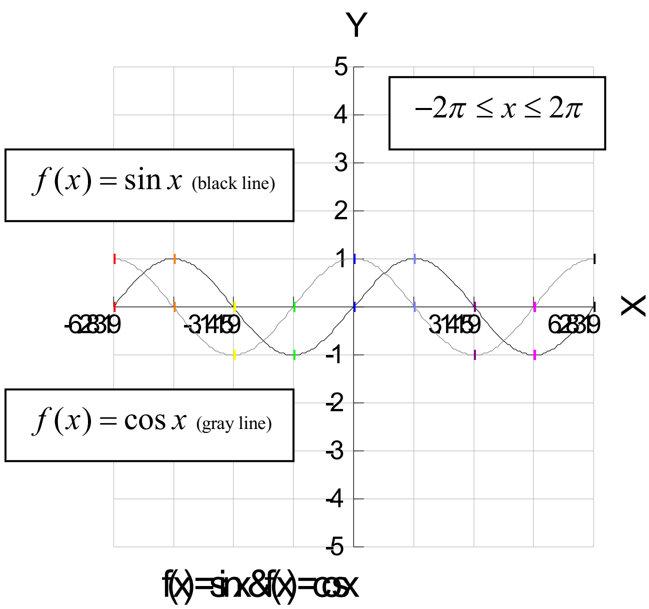
X-Value	dy/dx f(x)=sin$x$	f(x)=cos$x$
-2π	1	1
$-\frac{3\pi}{2}$	0	0
$-\pi$	-1	-1
$-\frac{\pi}{2}$	0	0
0	1	1
$\frac{\pi}{2}$	0	0
π	-1	-1
$\frac{3\pi}{2}$	0	0
2π	1	1

The tables on the preceding page show that the derivative of $f(x) = \sin x$ at the following plotted points $-2\pi \le x \le 2\pi$ in increments of $\frac{\pi}{2}$ starting left to right x $= -2\pi$.



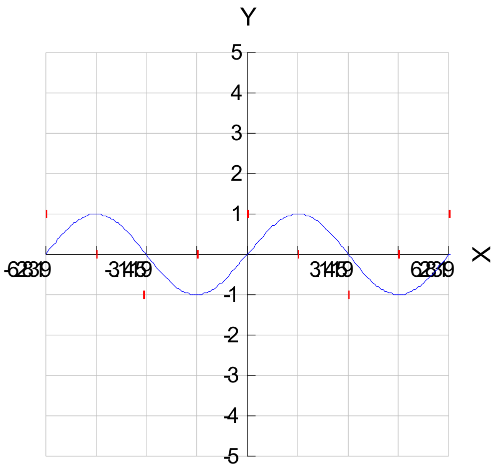
TI – 83 Graph Window	
Xmin =	- 2π
Xmax =	2π
Xscl =	$\frac{\pi}{2}$
Ymin =	-5
Ymax =	5
Yscl =	1

In the following graph, the points are connected to form $y = f'(x)$, $\begin{array}{l} f(x) = \sin x \\ \therefore f'(x) = \cos x \end{array}$.

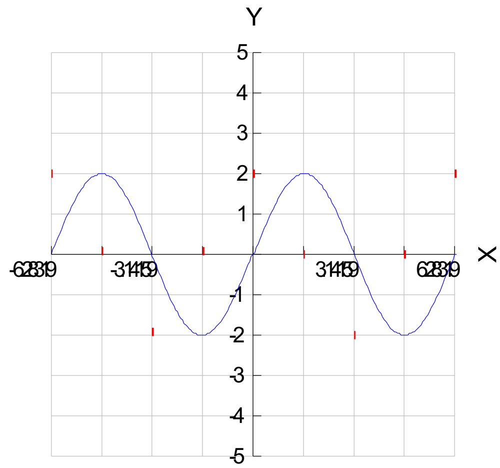


TI – 83 Graph Window	
Xmin =	- 2π
Xmax =	2π
Xscl =	$\frac{\pi}{2}$
Ymin =	-5
Ymax =	5
Yscl =	1

2. The following is my investigation of the derivatives of functions in the form $g(x) = a\sin x$. The red dots in the following graphs below represent the derivative of the function at a particular X-value.

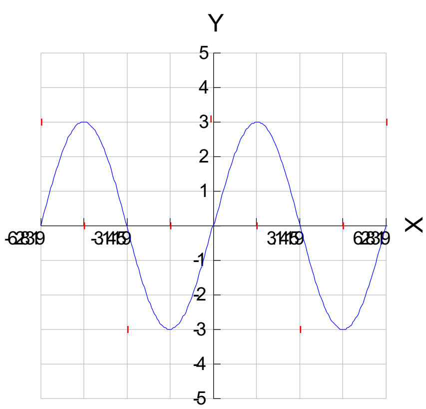


g(x)=1sinx

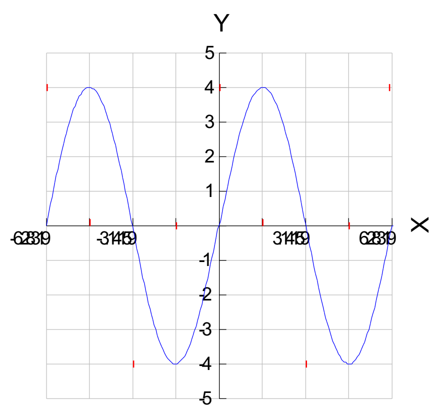


g(x)=2sinx

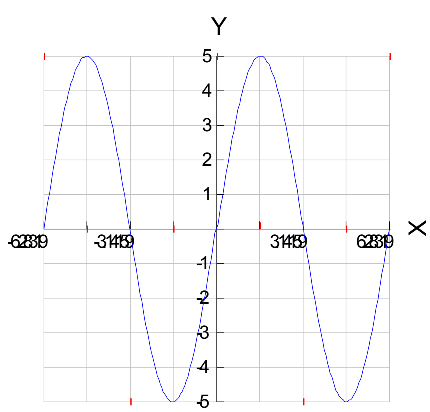
Concluded from the graphs above, my conjecture for the derivative of the function $g(x) = a\sin x$, $-2\pi \le x \le 2\pi$ is that when $1 \le a \le 5$, a determines the function ***amplitude*** or vertical stretch. Consider also the following graphs:



g(x)=3sinx

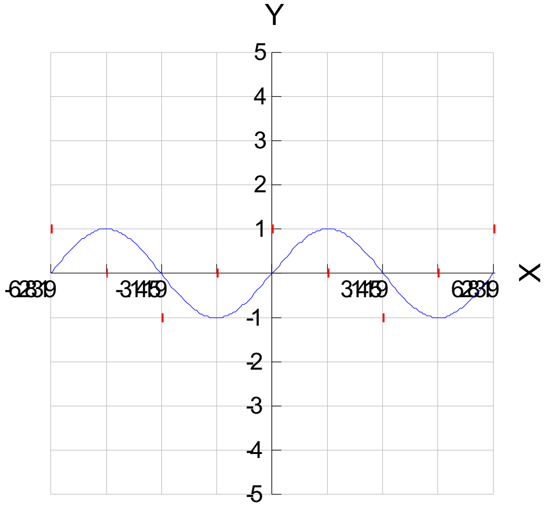


g(x)=4sinx

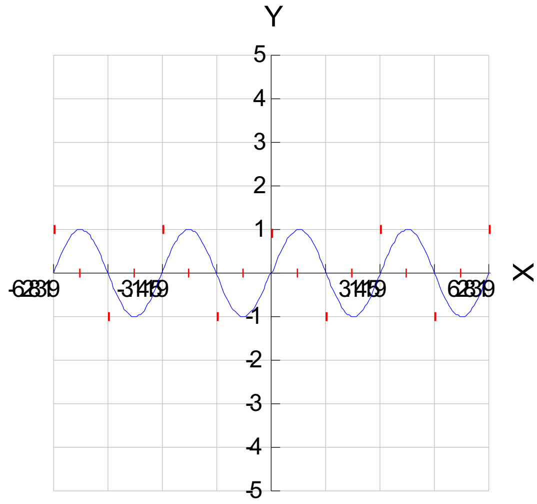


g(x)=5sinx

3. The following is my investigation of the derivatives of functions in the form $h(x) = \sin bx$. The red dots in the following graphs below represent the derivative of the function at a particular X-value.

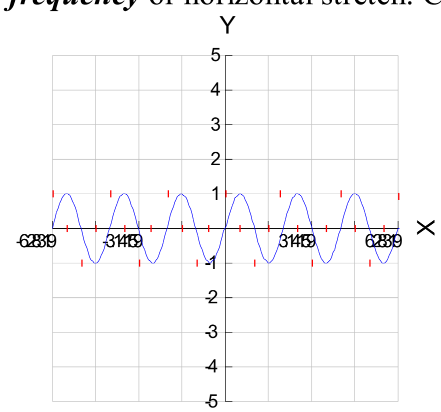


h(x)=sin1x

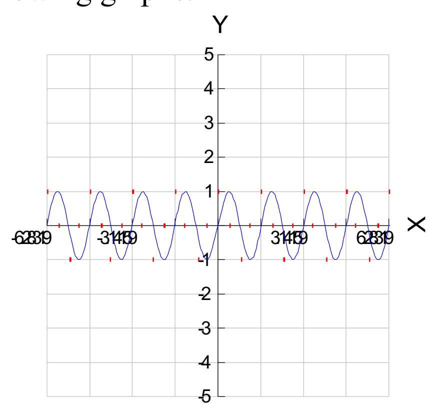


h(x)=sin2x

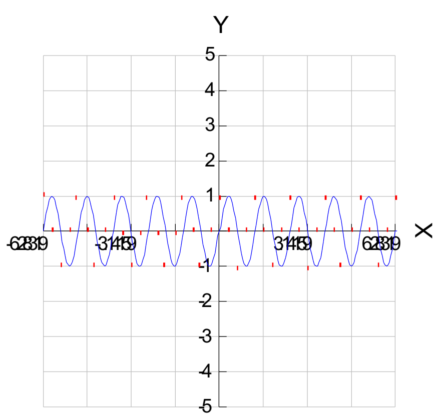
Concluded from the graphs above, my conjecture for the derivative of the function $h(x) = \sin bx$, $-2\pi \le x \le 2\pi$ is that when $1 \le b \le 5$, b determines the function ***frequency*** or horizontal stretch. Consider also the following graphs:



h(x)=sin3x

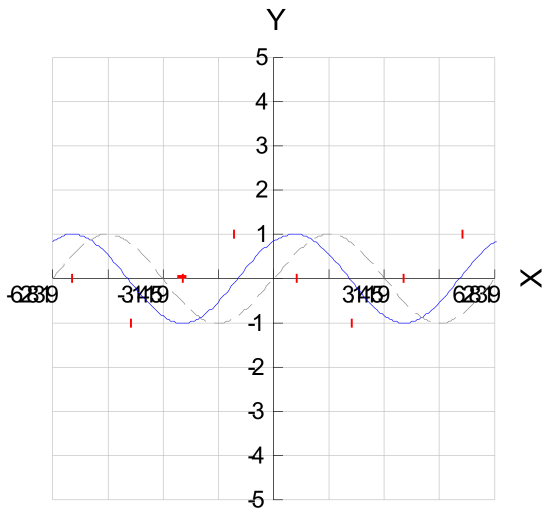


h(x)=sin4x

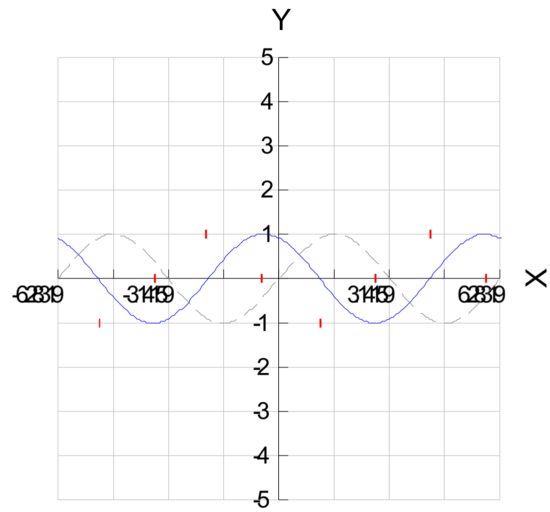


h(x)=sin5x

4. The following is my investigation of the derivatives of functions in the form $j(x) = \sin(x + c)$. The red dots in the following graphs below represent the derivative of the function at a particular X-value. (The gray line is $f(x) = \sin x$.)

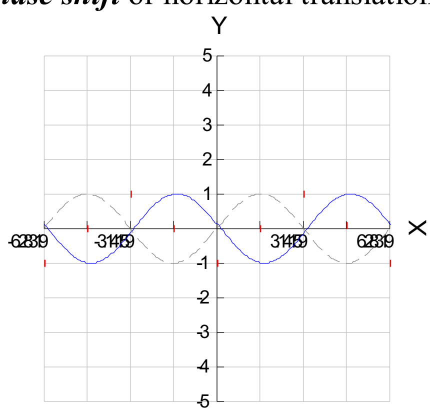


j(x)=sin(x+1)

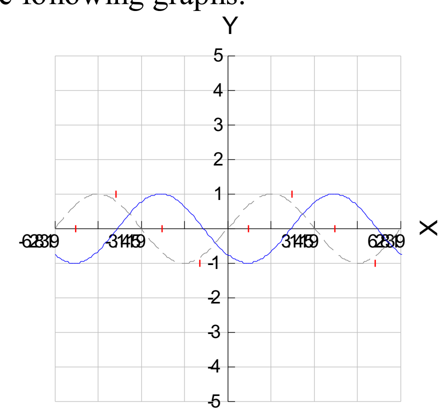


j(x)=sin(x+2)

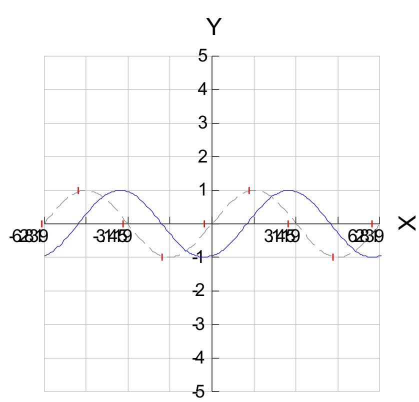
Concluded from the graphs above, my conjecture for the derivative of the function $h(x) = \sin bx$, $-2\pi \le x \le 2\pi$ is that when $1 \le c \le 5$, c determines the function ***phase shift*** or horizontal translation. Consider also the following graphs:



j(x)=sin(x+3)

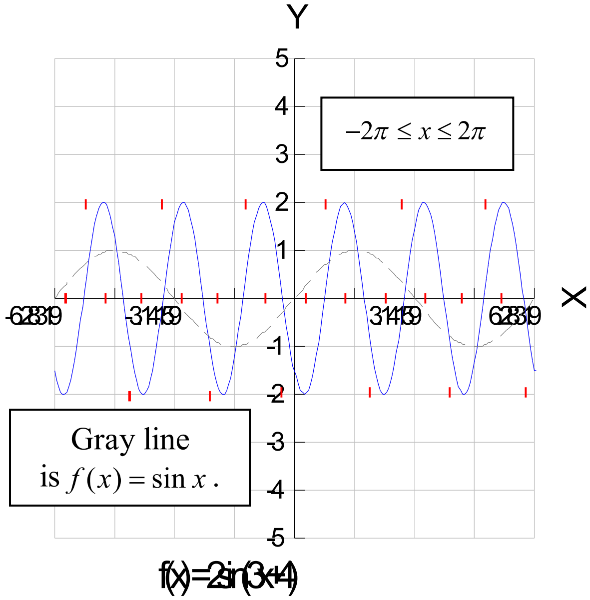


j(x)=sin(x+4)



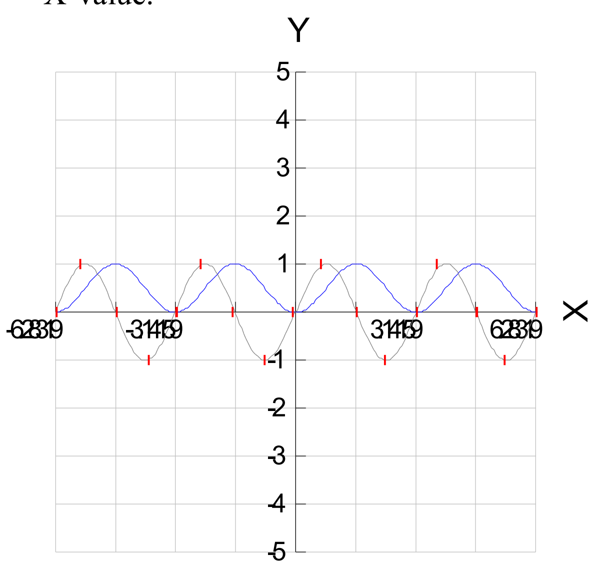
j(x)=sin(x+5)

5. Therefore, based on previous conjectures, the derivative of $k(x) = a \sin b(x + c)$, where *a* (*amplitude*) is 2, *b* (*frequency*) is 3, and *c* (*phase shift*) is 4, is represented as red dots in the following graph:



TI – 83 Graph Window	
Xmin =	-2π
Xmax =	2π
Xscl =	$\frac{\pi}{2}$
Ymin =	-5
Ymax =	5
Yscl =	1

6. Also, based on previous conjectures and the chain rule, the derivative of $m(x) = \sin^2 x$ can be written as $m'(x) = 2 \sin x \cos x$. Consider the following graphs where the red line represents $m(x) = \sin^2 x, -2\pi \le x \le 2\pi$, the gray line represents $m'(x) = 2 \sin x \cos x$, and the red points represent the derivative of $m(x) = \sin^2 x$ at an X-value.



TI – 83 Graph Window	
Xmin =	-2π
Xmax =	2π
Xscl =	$\frac{\pi}{2}$
Ymin =	-5
Ymax =	5
Yscl =	1

Because of the chain rule: $y = (f(x))^n \therefore y' = n(f(x))^{n-1} f'(x)$ and $\begin{matrix} f(x) = \sin x \\ \therefore f'(x) = \cos x \end{matrix}$,

Thus:

$m(x) = \sin^2 x$ **or** $m(x) = (\sin x)^2$

$= 2(\sin x)^1 (\cos x)$

$m'(x) = 2\sin x \cos x$

In conclusion, from my investigation of the Derivative of sine functions, I have discovered the following:

- $\begin{matrix} f(x) = \sin x \\ \therefore f'(x) = \cos x \end{matrix}$
- $k(x) = a \sin b(x + c)$, where the derivative is affected by
  a. a the amplitude
  b. b the frequency
  c. c the phase shift
- The derivative of $m(x) = \sin^2 x$ can be written as $m'(x) = 2\sin x \cos x$.

*In this investigation, there are limitations to graphs given as examples. The values substituted only occurred $1 \le n \le 5$ and, therefore, no negative values can be accounted for in the conjectures. Also, the graphs used only fall under $-2\pi \le x \le 2\pi$ and , and, thus, cannot confirm the conjecture outside of these limits.