Math Portfolio Assignment:
Derivatives of Sine Functions

Bridget Belsher
Derivatives of Sine Functions, Type I
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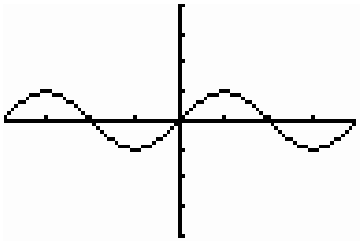
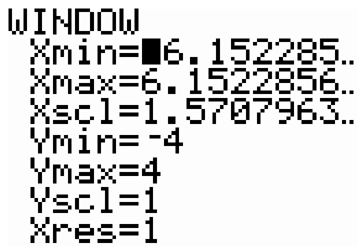
Father Lacombe High School
Mrs. Gabel

I certify this portfolio Assignment is entirely my own work



Above is a graph of the function $f(x) = \sin x$, as seen on GDC



Above displayed is the window setting for the limit -2π ≤ 2π

The above graph, displays the unaltered sin function, in its original form. The functions domain on the graph is -2π ≤ 2π[1]. The result of this is can be seen in the two oscillations with the range. Its behavior can be determined by the behavior of the first oscillation (the oscillation starting at -2π), as the same behavior will be observed in the second oscillation (that starting at 0), due to the repetitive nature of sine functions. From the point (-2π, 0), the slope can be described as increasing with a value of one. However, one can clearly see that at ($(-\frac{3}{2}\pi, 1)$ the function has no slope, as it is a point of **discontinuity**, that is having a slope of zero, as the line is straight. Just after that point, the slope is -1, which causes a constant decrease in the functions y-value and increase in the functions x value, which continues until the point ($-\frac{1}{2}$ π, -1), at which it reaches another point of discontinuity.

The derivative on sin(x) is Cos(x), which is seen in the graph below and further explained as well. Cos(x)'s behavior describes the slope of sine(x). This can be seen, if you compare the to graphs at the point $x = \frac{\pi}{2}$, where the slope of sin(x) is 0 and the Cos(x)'s actual value is 0, being (π/2,0) . The graph for the derivative of Sin(x) can also be found using a GDC, with no knowledge of the behavior of Cos/Sin functions prior to this paper. This can be done by going pressing the [y=] then going [2nd] [catalogue] [log] and then scrolling down to [*n*Deriv]. From here you press [enter] and plug in the necessary so that you end up with it looking something like y= nDeriv [Sin(x), x, x]. The resulting graph is displayed on the next page, and is labeled as f'(x). The seond graph displayed on the next page is labeled f(x) and is of the original function, f(x)=sin(x). The final graph on the next page displays both F'(x) and F(x).

The fact that f(x)=sin(x) derivative is indeed, f'(x)=cos(x) will be proved in the equation on the next page, through the use of limits.

[1] Note that this limit is present throughout all graphs in this paper, and that the said graphs increase and decrease in increments of $\frac{\pi}{2}$.

$f(x) = \sin x$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$ [2]

$$\lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin(x)}{h}$$

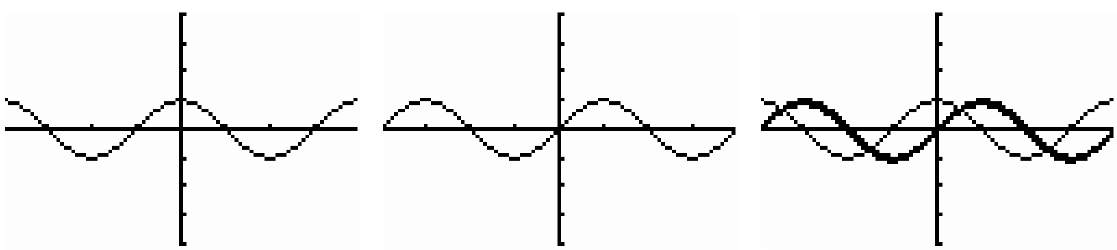
$$\lim_{h\to 0} \frac{\sin x(\cos h - 1) + \sin h \cos x}{h}$$

$$\lim_{h\to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h\to 0} \frac{\sin h \cos x}{h}$$

$\text{Lim} \frac{(\cos h - 1)}{h} = 0$ and $\lim_{h\to 0} \frac{\sin h}{h} = 1$

$$\lim_{h\to 0} \sin x(0) + \cos x(1)$$

$f'(x) = \cos x$



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Basically, when the Sin functions slope is defined as a number, then in the derived function would be directly affected. This can be most clearly explained in the following conjecture. In the derived function Cos(x), the maximum is equivalent to the highest positive slope of Sin(x), in this case being 1 and the minimum of the Cos(x) is equivalent to the greatest negative slope of Sin(x), being -1. The points at which Sin(x) has no slope, the x-value in cos(x) is 0.

This is highlighted in the graphs below, where in the Sin(x) function the points at which the slope is zero are colored red. The corresponding points in Cos(x) at which the actual x-value is zero is also highlighted, but in green. In the last graph below, to the right, the two graphs are compared, at the points where the slope of sin(x) =0 and the y value of Cos(x) actually is 0. The same could be done for the maximum and minimum values at which the slope is at its greatest negative and greatest positive slope; however I think at this point the explanation is clear enough for the concept to be understood without additional graphs.

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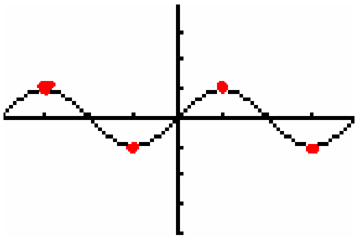
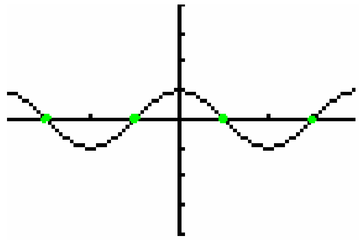
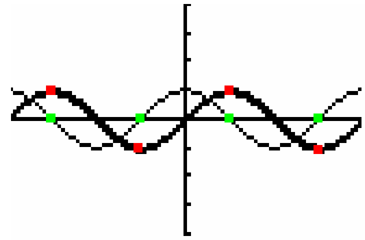
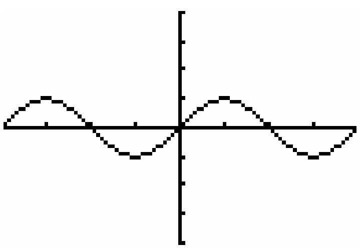
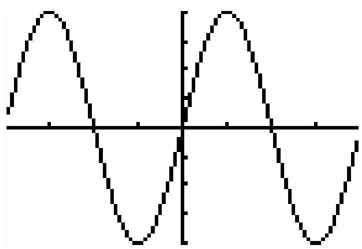
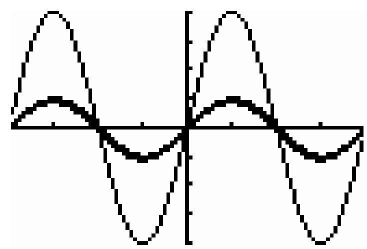
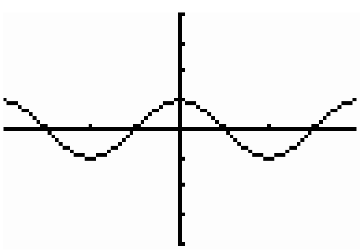
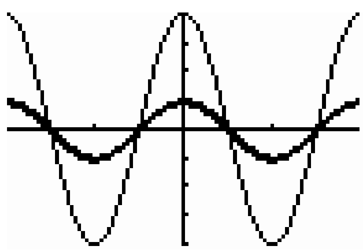
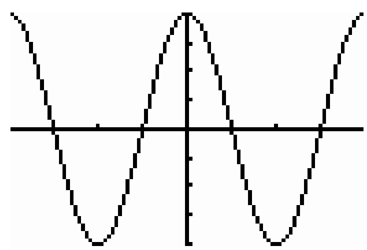
[2] Due to the fact that sin(x +h) = sin(x) multiplied by cos(h) + sin(h) multiplied by cos(x), I have taken the liberaty of substituting in the equation.

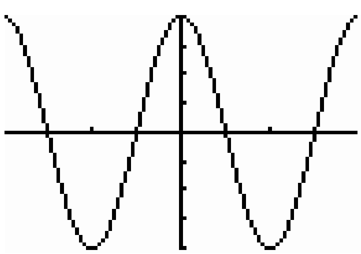
When the sin function is further complicated by being multiplied by a constant, its function changes from y=sin(x) to y=*a*Sin(x). The presence of the *a-value* completely changes the the appearance of the function, as well as that functions derivative, which is y=*a*Cos(x), this change can be seen visually below and will be further explained after the graphs. The change in the function will be displayed in the example y=4Sin(x)

As can be seen in the above graph, the addition of an a value cause the function to vertically stretch according to the value of *a.* This does not cause the y or x intercepts to change, only the absolute maximums and minimums of the function. The same applies to the derivative of y=4sin(x), which is 4Cosx) in comparison with y=cos(x). As is displayed below.

The result occurs when one uses the *nDeriv* function, as will be displayed below with the example y=4Sinx.

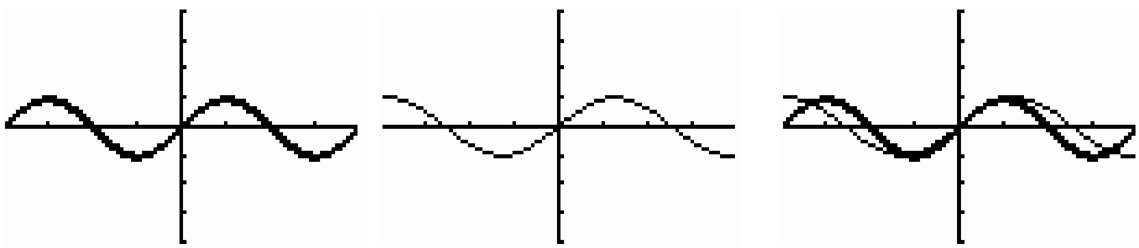


the conjecture made to fit with the change from sin(x) to cos(x) priorly still remains mostly true. That said conjecture was:

"When the Sin functions slope is defined as a number, then in the derived function would be directly affected. This can be most clearly explained in the following conjecture. In the derived function Cos(x), the maximum is equivalent to the highest positive slope of Sin(x), in this case being 1 and the minimum of the Cos(x) is equivalent to the greatest negative slope of Sin(x), being -1. The points at which Sin(x) has no slope, the x-value in cos(x) is 0."

While this conjecture still works in some sense, it also would change due to the presence of the *a-value*. Thus the conjecture for the derivatives of *a*Sin(x) will be defined as: In the derivative of aSin(x), which is aCos(x), the values of the greatest maximum slope and greatest minimum slope change according to the values of *a.* So thus the values of the function *a*Cos(x) would be between the range of *a* and *–a.*

When a Sine function gains a b-value, which is also known as a period, the function will change from f(x)=sin(x) to h(x)=Sin*b*(x) or h(x)=Sin(*b*x). The *b* value's presence causes the function to stretch horizontally. This is displayed in the below graph, with the example $y = \sin \frac{\pi}{4} x$, as compared to sinx.



It is apparent from what is seen on the graphs above that the x-intercepts of Sinx has changed, and because the b value is π/4, we know that the original function ahs been stretched horizontal to a factor of π/4. To further define the impact of the b-value on the function, the derivative will be found using limits, as displayed in the below example, in which the value of the *b* value is 2.

$$h(x) = \sin 2x$$
$$\lim_{h\to 0} \frac{h(x + h) - h(x)}{h}$$
$$\lim_{h\to 0} \frac{\sin(2(x + h)) - \sin(2x)}{h}$$
$$\lim_{h\to 0} \frac{\sin(2x)\cos(2h) + \sin(2h)\cos(2x) - \sin(2x)}{h}$$
$$\lim_{h\to 0} \frac{\sin(2x)(\cos(2h) - 1) + \sin(2h)\cos(2x)}{h}$$
$$\sin(2x) \lim_{h\to 0} \frac{(\cos(2h) - 1)}{h} + \cos(2x) \lim_{h\to 0} \frac{\sin(2h)}{h}$$
$$\sin x(0)(2) + \cos(2x)(1)(2)$$
$$h'(x) = 2\cos(2x)$$

The conjecture when the derivative of Sin(x+c) is Cos(x+c), is the same as in the original conjecture, in which the minimum and maximum slopes of Sinx determine the minimum and maximum values for Cosx. The only change is that the x0intercept change depending on the positive or negative value of c. An example of this can be seen below, in which sin(x + π/4) and Cos (x + π/4) are shown.

$$j(x) = \sin(x + 2)$$

$$\lim_{h\to 0} \frac{j(x + h) - j(x)}{h}$$
$$\text{Lim } \underline{\sin((x + 2)+ h) - \sin(x + 2)}$$

$h \to 0 \qquad h$

$$\lim_{h \to 0} \frac{\sin(x + 2)\cos(h) + \sin(h)\cos(x + 2) - \sin(x + 2)}{h}$$

$$\lim_{h \to 0} \frac{\sin(x + 2)(\cos(h) - 1) + \sin(h)\cos(x + 2)}{h}$$

$$\sin(x + 2) \lim_{h \to 0} \frac{(\cos(h) - 1)}{h} + \cos(x + 2) \lim_{h \to 0} \frac{\sin(h)}{h}$$

$$\sin(x + 2)(0) + \cos(x + 2)(1)$$

$$h'(x) = \cos(x + 2)$$