

Derivative of Sine Functions

Question 1

Investigate the derivative of the function $f(x) = \sin x$.

a) Graph the function $f(x) = \sin x$. For $-2\pi \leq x \leq 2\pi$

$y = f(x)$. Let $f(x) = \sin x$, $-2\pi \leq x \leq 2\pi$

By sketching the graph we get:

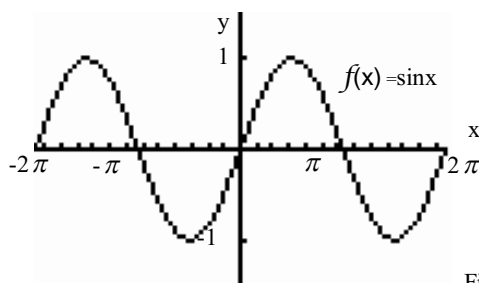


Figure 1 : graph of function $f(x) = \sin x$

Figure 1 reveals the range of the function $f(x) = \sin x$ is $[-1, 1]$.

b) Based on this graph, describe as carefully and fully as you can, the behaviour of the gradient of the function on the given domain.

The gradient of each point on the curve is valued as the gradient of the tangent line of the point. According to the curve in figure 1, the behaviour of the gradient of the function indicates the following characteristics:

The line of the tangent become flatter and flatter as the points move from left to right within -2π to $-\frac{3}{2}\pi$, $-\pi$ to $-\frac{1}{2}\pi$, 0 to $\frac{1}{2}\pi$, π to $\frac{3}{2}\pi$.

The line of the tangent become more and more precipitous as the points move from left to right within $-\frac{3}{2}\pi$ to $-\pi$, $-\frac{1}{2}\pi$ to 0 , $\frac{1}{2}\pi$ to π , $\frac{3}{2}\pi$ to 2π .

In the domain of: $[-2\pi, -\frac{3}{2}\pi[$, $]-\frac{1}{2}\pi, \frac{1}{2}\pi[$, $]\frac{3}{2}\pi, 2\pi]$ the gradient is positive.

The gradient is negative in the domain of: $]-\frac{3}{2}\pi, -\frac{1}{2}\pi[$, $]\frac{1}{2}\pi, \frac{3}{2}\pi[$.

At the point when x equals to $-\frac{3}{2}\pi$, $-\frac{1}{2}\pi$, $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$, the gradient is obviously 0.

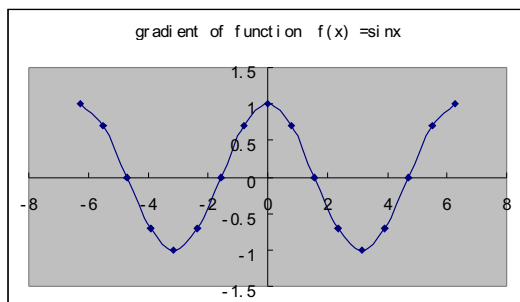
c) Use your Graphics Calculator (GC) to find numerical values of the gradient of the function at every $\pi/4$ unit. Sketch your findings on a graph.

1. The numerical values (in 3 significant figures) of the gradient of the function at every $\pi/4$

unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	1.00	0.707	0	-0.707	-1.00	-0.707	0	0.707	1.00
x	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	0.707	0	-0.707	-1.00	-0.707	0	0.707	1.00	

2. The scatter plot (joined by a smooth curve) for the gradient of $f(x) = \sin x$. at every $\pi/4$ unit is shown below.



Gradient of $f(x) = \sin x$. at every $\pi/4$ unit

d) make a conjecture for the derived function $f'(x)$

conjecture: $f'(x) = \cos x$

e) Use your GC to test your conjecture graphically. Explain your method and your findings.
Modify your conjecture if necessary

1. The graph joined scatter plots with curve of $f'(x) = \cos x$ is shown in figure 2

The derivative of $f(x) = \sin x$ found by calculator with the curve in figure 2 is shown in figure 3

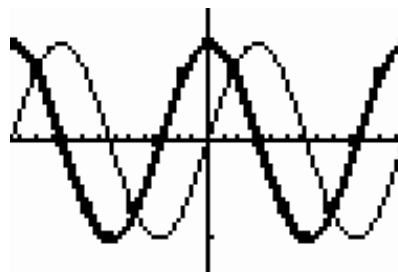
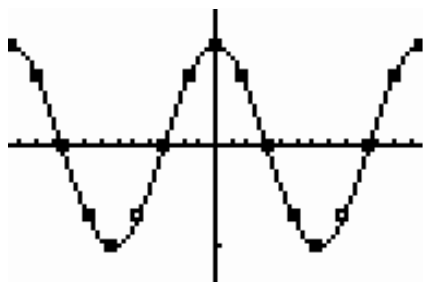


Figure 2 scatter plots and $f'(x) = \cos x$ Figure 3 derivative of sine function found by calculator

Figure 2 reveals that the curve of function $f'(x) = \cos x$ fits the scatter plots very well.

Figure 3 reveals that the curve of the function found by the calculator overlaps exactly on the curve of $f'(x) = \cos x$. Therefore the derivative of $f(x) = \sin x$ is: $f'(x) = \cos x$.

2. Method and findings of the derivative of $f(x) = \sin x$:

By joining the scatter plots in figure 2 smoothly we can get a curve which indicates following characteristics:

- symmetrical along x-axis.
- repeat its values every 2π , (so the function is periodic with a period of 2π)
- the maximum is 1 the minimum is -1
- the mean value of the function is zero
- the amplitude of the function is 1

Those are all characteristics of cosine function with the parameter of 1, according to the maximum and minimum values. Therefore the conjecture for the derived function $f'(x)$ is: $f'(x) = \cos x$

Question 2

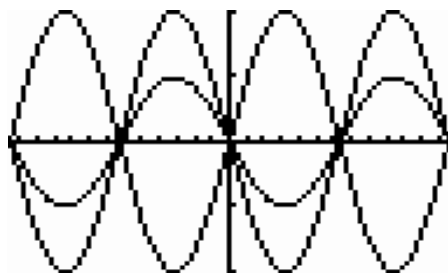
Investigate the derivatives of functions of the form $f(x) = a \sin x$ in similar way

a) Consider several different values of a .

Consider three values as $a = -1, a = 2$ and $a = -2$,

1. When ① $a = -1$ $f(x) = -\sin x$. ② $a = -2$ $f(x) = -2 \sin x$ ③ $a = 2$, $f(x) = 2 \sin x$.

The curves of these functions are shown below:



graph of $f(x) = -\sin x$ $f(x) = -2 \sin x$, $f(x) = 2 \sin x$

2. Description the gradient of the function:

The behaviour of the gradient of $f(x) = 2 \sin x$ is the same as question 1(b)

The behaviour of the gradient of $f(x) = -2 \sin x$ and $f(x) = -\sin x$ are opposite as question 1(b)

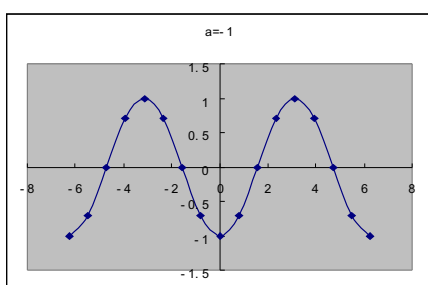
3. Values of gradient of the function at every $\pi/4$ unit, sketch findings on a graph.

- ① when $a = -1$ $f(x) = -\sin x$.

The value of the gradient of $f(x) = -\sin x$ at every $\pi/4$ unit is shown in the table:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	-1.00	-0.707	0	0.707	1.00	0.707	0	-0.707	-1.00
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-0.707	0	0.707	1.00	0.707	0	-0.707	-1.00	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)



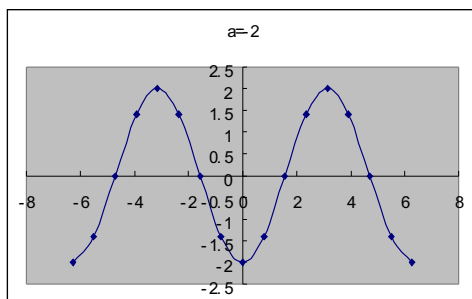
gradient of $f(x) = -\sin x$ at every $\pi/4$ unit

② when $a = -2$ $f(x) = -2 \sin x$

The value of the gradient of $f(x) = -2 \sin x$. At every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	-2.00	-1.41	0	1.41	2.00	1.41	0	-1.41	-2.00
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-1.41	0	1.41	2.00	1.41	0	-1.41	-2.00	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)



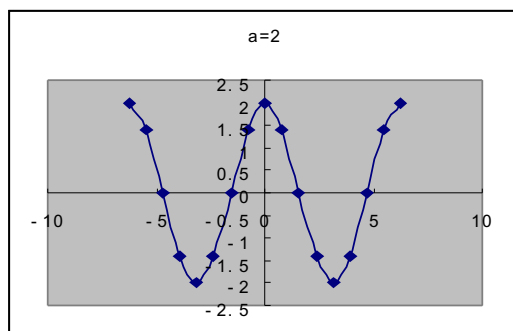
gradient of $f(x) = -2 \sin x$ at every $\pi/4$ unit

③ when $a=2$, $f(x) = 2 \sin x$.

The value of the gradient of $f(x) = 2 \sin x$. At every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	2.00	1.41	0	-1.41	-2.00	-1.41	0	1.41	2
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	1.41	0	-1.41	-2.00	-1.41	0	1.41	2.00	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)



gradient of $f(x) = 2 \sin x$ at every $\pi/4$ unit

b) Make a conjecture of $g'(x)$

$$g'(x) = a \cos x$$

c) Test the conjecture with further examples.

1. Let $a=3$, then $g'(x) = 3 \cos x$.

The value of the gradient of $f(x) = 3 \sin x$. at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	3.00	2.12	0	-2.12	-3.00	-2.12	0	2.12	3
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	2.12	0	-2.12	-3.00	-2.12	0	2.12	3.00	

The scatter plot for the values in the table above with the function $g'(x) = 3 \cos x$. is shown in

figure 4 The derivative of $f(x) = 3\sin x$ found by the calculator with the curve of $g'(x) = 3\cos x$ is shown in figure 5

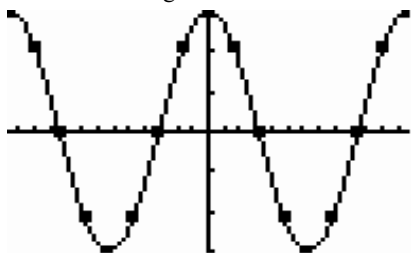


Figure 4 $g'(x) = 3\cos x$.

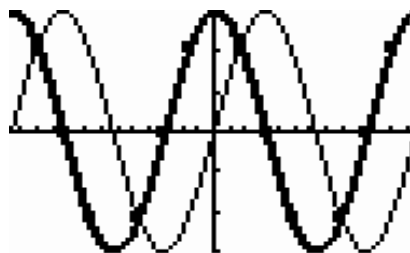


Figure 5 The derivative of $f(x) = 3\sin x$

Figure 4 reveals the curve of $g'(x) = 3\cos x$ fits the scatter plot very well. Figure 5 indicates that the derivative function found by calculator overlaps $g'(x) = 3\cos x$. Perfect, they should be the same function. Therefore $g'(x) = a\cos x$ is the correct conjecture for function $g(x) = a\sin x$.

2. Method and findings of the derivative of $f(x) = a\sin x$:

The curves of conjecture, when $a = -1$, $a = -2$ and $a = 2$, in the graphs indicate the following characteristics:

- symmetrical along x-axis.
- repeat its values every 2π , (so the function is periodic with a period of 2π)
- the maximum value is a , the minimum value is $-a$
- the mean value of the function is zero
- the amplitude of the function is a

According to the characteristics the form of gradient function is similar to a cosine function with parameter of a . Therefore the conjecture of $g'(x)$ is: $g'(x) = a\cos x$.

d) State for what values of a the conjecture holds/

For all values of a the conjecture holds.

The letter a determines the minimum, maximum values and amplitude of sine, cosine function.

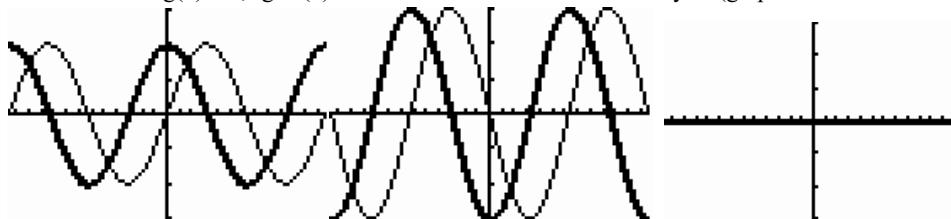
When $a > 0$ the curve will be vertical stretch by a factor of a .

Say $a = 2$ $g(x) = 2\sin x$, $g'(x) = 2\cos x$ (graph is shown below)

When $a < 0$ the curve will be reflection along x-axis and then vertical stretch by a factor of a

Say $a = -3$ $g(x) = -3\sin x$, $g'(x) = -3\cos x$ (graph is shown below)

When $a = 0$ $g(x) = 0$, $g'(x) = 0$ the curve is still exist as $y = 0$. (graph is shown below)



$g(x) = 2\sin x$, $g'(x) = 2\cos x$ $g(x) = -3\sin x$, $g'(x) = -3\cos x$ $g(x) = 0$, $g'(x) = 0$

As a result a can be positive, negative or zero, so for all values of a the conjecture holds.

Question 3

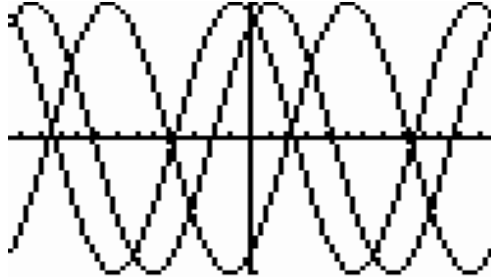
Investigate the derivatives of functions of the form $f(x) = \sin(x+c)$.

a) Consider several different values of c .

1. Consider $c=1$, $c=2$ and $c=-1$

① when $c=1$: $f(x) = \sin(x+1)$ ② when $c=-1$: $f(x) = \sin(x-1)$ ③ when $c=2$: $f(x) = \sin(x+2)$

the graph is shown below:



graph of $f(x) = \sin(x+1)$, $f(x) = \sin(x-1)$,

$f(x) = \sin(x+2)$

2. Description the gradient of the function:

① when $c=1$: $f(x) = \sin(x+1)$

The line of the tangent becomes flatter and flatter as the points move from left to right within -2π to $-\frac{3}{2}\pi - 1$, $-\pi - 1$ to $-\frac{1}{2}\pi - 1$, -1 to $\frac{1}{2}\pi - 1$, $\pi - 1$ to $\frac{3}{2}\pi - 1$ and $2\pi - 1$ to 2π .

The line of the tangent become steeper and steeper as the points move from left to right within $-\frac{3}{2}\pi - 1$ to $-\pi - 1$, $-\frac{1}{2}\pi - 1$ to -1 , $\frac{1}{2}\pi - 1$ to $\pi - 1$, $\frac{3}{2}\pi - 1$ to $2\pi - 1$.

The gradient is positive in the domain of: $[-2\pi, -\frac{3}{2}\pi - 1]$, $[-\frac{1}{2}\pi - 1, \frac{1}{2}\pi - 1]$, $[\frac{3}{2}\pi - 1, 2\pi]$. The gradient is negative in the domain of: $[-\frac{3}{2}\pi - 1, -\frac{1}{2}\pi - 1]$, $[\frac{1}{2}\pi - 1, \frac{3}{2}\pi - 1]$.

② when $c=-1$: $f(x) = \sin(x-1)$

The line of the tangent becomes flatter and flatter as the points move from left to right within $-2\pi + 1$ to $-\frac{3}{2}\pi + 1$, $-\pi + 1$ to $-\frac{1}{2}\pi + 1$, 1 to $\frac{1}{2}\pi + 1$, $\pi + 1$ to $\frac{3}{2}\pi + 1$.

The line of the tangent becomes steeper and steeper as the points move from left to right within -2π to $-2\pi + 1$, $-\frac{3}{2}\pi + 1$ to $-\pi + 1$, $-\frac{1}{2}\pi + 1$ to 1 , $\frac{1}{2}\pi + 1$ to $\pi + 1$, $\frac{3}{2}\pi + 1$ to 2π .

The gradient is positive in the domain of: $[-2\pi, -\frac{3}{2}\pi + 1]$, $[-\frac{1}{2}\pi + 1, \frac{1}{2}\pi + 1]$,

$$\left] \frac{3}{2} \pi + 1, 2\pi \right]$$

The gradient is negative in the domain of: $\left] -\frac{3}{2} \pi + 1, -\frac{1}{2} \pi + 1 \right[$, $\left] \frac{1}{2} \pi + 1, \frac{3}{2} \pi + 1 \right[$.

③ when $c=2$ $f(x)=\sin(x+2)$

The line of the tangent becomes flatter and flatter as the points move from left to right within $-\pi - 2$ to $-\frac{1}{2} \pi - 2$, -2 to $\frac{1}{2} \pi - 2$, $\pi - 2$ to $\frac{3}{2} \pi - 2$, $2\pi - 2$ to $\frac{5}{2} \pi - 2$.

The line of the tangent becomes steeper and steeper as the points move from left to right within -2π to $-\pi - 2$, $-\frac{1}{2} \pi - 2$ to -2 , $\frac{1}{2} \pi - 2$ to $\pi - 2$, $\frac{3}{2} \pi - 2$ to $2\pi - 2$, $\frac{5}{2} \pi - 2$ to 2π .

The gradient is positive in the domain of: $\left] -\frac{1}{2} \pi - 2, \frac{1}{2} \pi - 2 \right[$, $\left] \frac{3}{2} \pi - 2, \frac{5}{2} \pi - 2 \right]$

The gradient is negative in the domain of: $\left] -2\pi, -\frac{1}{2} \pi - 2 \right[$, $\left] \frac{1}{2} \pi - 2, \frac{3}{2} \pi - 2 \right[$

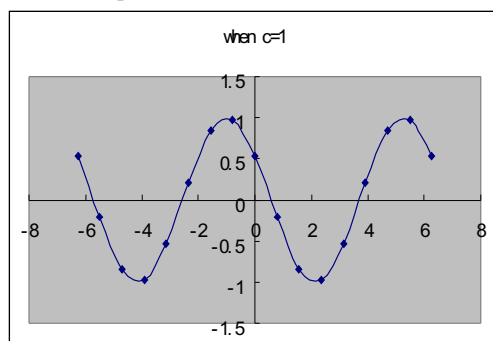
$$\left] \frac{5}{2} \pi - 2, 2\pi \right]$$

3. Values of gradient of the function at every $\pi/4$ unit, sketch findings on a graph.

① The value of the gradient of $f(x)=\sin(x+1)$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	0.540	-0.213	-0.841	-0.977	-0.540	0.213	0.841	0.977	0.540
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-0.213	-0.841	-0.977	-0.540	0.213	0.841	0.977	0.540	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)

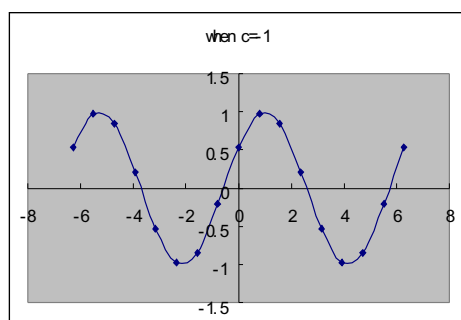


gradient of $f(x)=\sin(x+1)$ at every $\pi/4$ unit

② The value the gradient of $f(x) = \sin(x-1)$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	0.540	0.977	0.841	0.213	-0.540	-0.977	-0.841	-0.213	0.540
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	0.997	0.841	0.213	-0.540	-0.977	-0.841	-0.213	0.540	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)

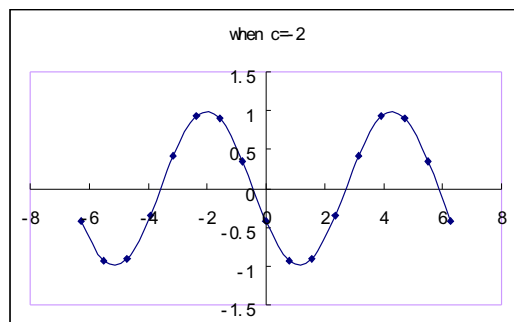


gradient of $f(x) = \sin(x-1)$ at every $\pi/4$

③ The value of the gradient of $f(x) = \sin(x+2)$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	-0.416	-0.937	-0.909	-0.348	0.416	0.937	0.909	0.349	-0.416
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-0.937	-0.909	-0.348	0.416	0.937	0.909	0.349	-0.416	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)



gradient of $f(x) = \sin(x+2)$ at every $\pi/4$

b) Make a conjecture of $g'(x)$

$$g'(x) = \cos(x+c)$$

c) Test the conjecture with further examples.

consider $c = -2$ then $f(x) = \sin(x-2)$, $g'(x) = \cos(x-2)$

The value of the gradient of $f(x) = \sin(x-2)$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	-0.416	0.349	0.909	0.937	0.416	-0.349	-0.909	-0.938	-0.416
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	0.348	0.909	0.937	0.416	-0.364	-0.909	-0.937	-0.416	

The scatter plot for the values in the table above with the function $g'(x) = \cos(x-2)$ is shown in figure 6. The derivative of $f(x) = \sin(x-2)$ found by the calculator with the curve of $g'(x) = \cos(x-2)$ is shown in figure 7:

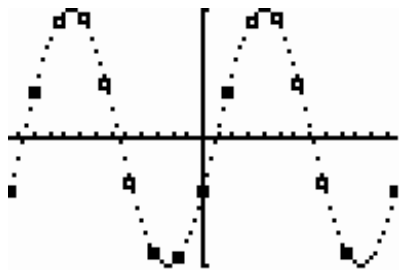


Figure 6 $g'(x) = \cos(x-2)$

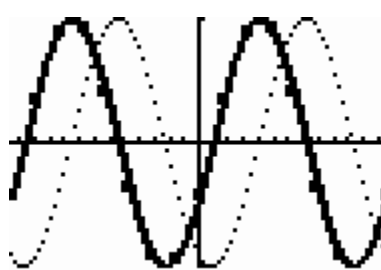


figure 7 derivative of $f(x) = \sin(x-2)$

Figure 6 reveals the curve of $g'(x) = \cos(x-2)$ fits the scatter plot well. Figure 7 reveals that the derivative function found by calculator overlaps $g'(x) = \cos(x-2)$. Perfect.

Therefore $g'(x) = \cos(x+c)$ is the suitable conjecture for function $f(x) = \sin(x+c)$.

2. Method and findings of the derivative of $f(x) = \sin(x+c)$

The curves of conjecture, when $c = -1$, $c = 1$ and $c = 2$, in the graphs indicate the following characteristics:

- symmetrical along the axis of $x-c$.
- repeat its values every 2π (so the function is periodic with a period of 2π)
- the maximum is 1 the minimum is -1
- the mean value of the function is zero
- the amplitude of the function is 1

According to the characteristics the form of function for the gradient is similar to a cosine function with parameter of 1 and horizontal transformed by c units to the left.

Therefore correct conjecture of $g'(x)$ is: $g'(x) = \cos(x+c)$

d) State for what values of c the conjecture holds.

For all values of c the conjecture holds.

Because c won't affect any characteristics of the function.

When $c > 0$ it will cause horizontal translation to the right by c units.

When $c < 0$ it will cause horizontal translation to the left by c units.

When $c = 0$ the curve will be the same as $f(x) = \sin x$, $g'(x) = \cos x$.

So c can be positive, negative or zero it means for any value of c the conjecture holds.

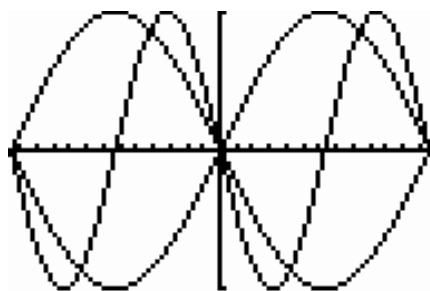
Question 4

Investigate the derivatives of functions of the form $f(x) = \sin bx$

a) Consider several different values of b .

1. Consider $b = 0.5$, $b = -0.5$ and $b = -1$ When $b = -1$ $f(x) = \sin -x$, when $b = 0.5$ $f(x) = \sin 0.5x$

when $b = -0.5$ $f(x) = \sin -0.5x$ the graph is shown below:



graph of $f(x) = \sin -x$ $f(x) = \sin 0.5x$ $f(x) = \sin -0.5x$

① When $b = -1$ $f(x) = \sin -x$

The behaviour of the gradient of the function on the given domain is opposite as question 1 (b).

② when $b = 0.5$ the $f(x) = \sin 0.5x$

The line of the tangent becomes flatter and flatter as the points move from left to right within -2π to $-\pi$, 0 to π .

The line of the tangent becomes more and more precipitous as the points move from left to right within $-\pi$ to 0 , π to 2π .

The gradient is positive in the domain of: $]-\pi, \pi[$

The gradient is negative in the domain of: $[-2\pi, -\pi[$, $]\pi, 2\pi]$

③ when $b = -0.5$ the $f(x) = \sin -0.5x$

The line of the tangent becomes flatter and flatter as the points move from left to right within -2π to $-\pi$, 0 to π .

The line of the tangent becomes more and more precipitous as the points move from left to right within $-\pi$ to 0 , π to 2π .

The gradient is positive in the domain of: $[-2\pi, -\pi[$, $]\pi, 2\pi]$

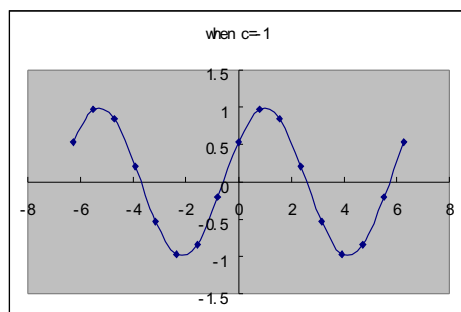
The gradient is negative in the domain of: $]-\pi, \pi[$

3. Values of gradient of the function at every $\pi/4$ unit, sketch findings on a graph.

① The value of the function of $f(x) = \sin -x$. at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	-1.00	-0.707	0	0.707	1.00	0.707	0	-0.707	-1.00
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-0.707	0	0.707	1.00	0.707	0	-0.707	-1.00	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)

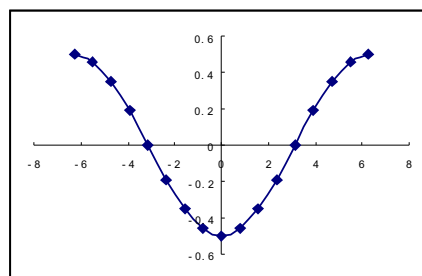


gradient of $f(x) = \sin -x$. at every $\pi/4$ unit

② The value the gradient of $f(x) = \sin 0.5x$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	-0.5	-0.462	-0.354	-0.191	0	0.191	0.354	0.462	0.5
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	0.462	0.354	0.191	0	-0.191	-0.354	-0.462	-0.5	

The scatter plots of values in the table is sketched below: (joined by a smooth curve)

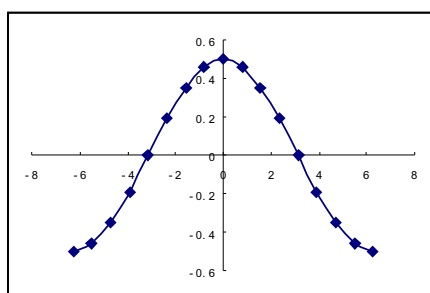


gradient of $f(x) = \sin 0.5x$ at every $\pi/4$ unit

③ The value the gradient of $f(x) = \sin 0.5x$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	0.5	0.462	0.354	0.191	0	-0.191	-0.354	-0.462	-0.5
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-0.462	-0.354	-0.191	0	0.191	0.354	0.462	0.5	

The scatter plots of values in the table are sketched below: (joined by a smooth curve):



gradient of $f(x) = \sin 0.5x$ at every $\pi/4$ unit

b) Make a conjecture of $g'(x)$

$$g'(x) = b \cos bx$$

c) Test the conjecture with further examples.

1. consider $b=2$ then $f(x) = \sin 2x$, $g'(x) = 2 \cos 2x$

The value of the gradient of $f(x) = \sin 2x$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	2.00	0	-2.00	0	2	0	-2	0	2
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	0	-2	0	2	0	-2	0	2	

The scatter plot for the values in the table above with the function $g'(x) = 2 \cos 2x$ is shown in figure 8. The derivative of $f(x) = \sin 2x$ found by the calculator with the curve of $g'(x) = 2 \cos 2x$ is shown in figure 9:

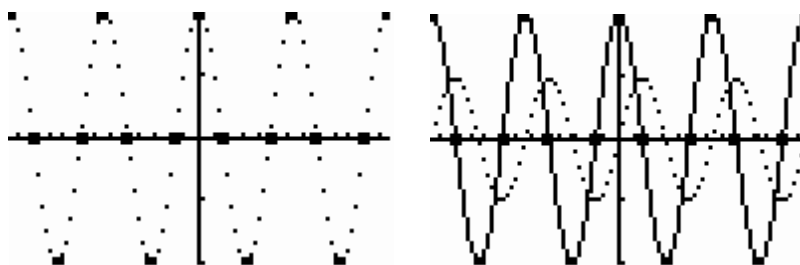


Figure 8 $g'(x) = 2\cos 2x$

figure 9 derivative of $f(x) = \sin 2x$

Figure 8 reveals the curve of $g'(x) = 2\cos 2x$ fits the scatter plot well. Figure 9 reveals that the derivative function found by calculator overlaps $g'(x) = \cos(x-2)$. Perfect.

Therefore $g'(x) = b\cos bx$ is the suitable conjecture for function $f(x) = \sin bx$.

2. Method and findings of the derivative of $f(x) = \sin bx$

The curves of conjecture, when $b = -1$, $b = 0.5$ and $b = -0.5$, in the graphs reveal the following characteristics:

symmetrical along the x-axis.

repeat its values every $\frac{2\pi}{b}$, (so the function is periodic with a period of $\frac{2\pi}{b}$)

the maximum is b the minimum is $-b$

the mean value of the function is zero

the amplitude of the function is b

According to the characteristics the form of function for the gradient is similar to a cosine function

with parameter of b and horizontal stretched by a factor of $\frac{1}{b}$.

Therefore the correct conjecture of $g'(x)$ is: $g'(x) = b\cos bx$.

d) State for what values of b the conjecture holds.

For all values of b the conjecture holds.

Because b determines the period of sine, cosine function.

No matter b is positive or negative; the curve will be horizontal stretched by a factor of $\frac{1}{b}$

When $b = 0$, $f(x) = 0$ the conjecture still exists as: $g'(x) = 0$.

Therefore for all values of b the conjecture holds.

Question 5

Use your results in Question 1 to 4 to make a conjecture for the derivative of $k(x) = a\sin b(x+c)$.

Choose a value for each of a, b, c . Verify your conjecture using the values you have chosen for a, b and c .

a) $k(x) = a\sin b(x+c)$

conjecture: $k'(x) = ab\cos b(x+c)$

b) 1. let $a=-1, b=0.5, c=1$, Then $k(x) = -\sin 0.5(x+1)$, $k'(x) = -0.5\cos 0.5(x+1)$

2. The value of the gradient of $f(x) = \sin 2x$ at every $\pi/4$ unit is shown in the table below:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	0.439	0.314	0.141	-0.0535	-0.240	-0.389	-0.480	-0.497	-0.439
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	-0.314	-0.141	0.0535	0.240	0.389	0.480	0.497	0.439	

c) The scatter plot for the values in the table above with the function $k'(x) = -0.5\cos 0.5(x+1)$ is shown in figure 10. The derivative of $k(x) = a\sin b(x+c)$ found by the calculator with the curve of $k'(x) = -0.5\cos 0.5(x+1)$ is shown in figure 11:

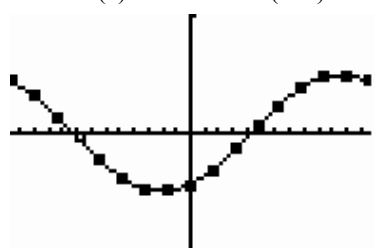


Figure 10 $k'(x) = -0.5\cos 0.5(x+1)$

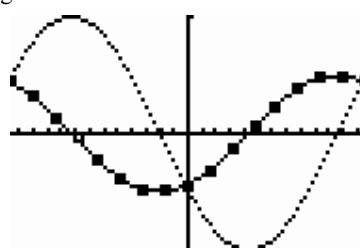


figure 11 derivative of $k(x) = a\sin b(x+c)$

Figure 10 reveals the curve of $k'(x) = -0.5\cos 0.5(x+1)$ fits the scatter plot well. Figure 11 reveals the derivative function found by calculator overlaps $k'(x) = -0.5\cos 0.5(x+1)$ Perfect. Therefore $k'(x) = -0.5\cos 0.5(x+1)$ is the correct conjecture for function $k(x) = a\sin b(x+c)$

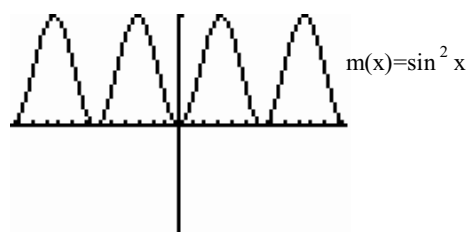
Question 6

Consider $m(x) = \sin^2 x$. Investigate the derivative of this function and show that it can be written as $m'(x) = 2\sin x \cos x$.

a) Graph of the function $m(x) = \sin^2 x$, For $-2\pi \leq x \leq 2\pi$

$$y = m(x), \text{ let } m(x) = \sin^2 x, -2\pi \leq x \leq 2\pi$$

By sketching the graph we get:



From the graph we can get the range of the function $m(x) = \sin^2 x$ is $[0, 1]$.

b) Describe the behaviour of the gradient of the function on the given domain.

According to the graph the gradient of the function reveals these characteristics:

The line of the tangent becomes flatter and flatter as moving from left to right within -2π to $-\frac{7}{4}\pi$, $-\frac{3}{2}\pi$ to $-\frac{5}{4}\pi$, $-\pi$ to $-\frac{3}{4}\pi$, $\frac{1}{2}\pi$ to $-\frac{1}{4}\pi$, 0 to $\frac{1}{4}\pi$, $\frac{1}{2}\pi$ to $\frac{3}{4}\pi$, π to $\frac{5}{4}\pi$, $\frac{3}{2}\pi$ to $\frac{7}{4}\pi$.

The line of the tangent becomes more and more precipitous as moving from left to right within $-\frac{7}{4}\pi$ to $-\frac{3}{2}\pi$, $-\frac{5}{4}\pi$ to $-\pi$, $-\frac{3}{4}\pi$ to $-\frac{1}{2}\pi$, $-\frac{1}{4}\pi$ to 0 , $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$, $\frac{3}{4}\pi$ to π , $\frac{5}{4}\pi$ to $\frac{3}{2}\pi$, $\frac{7}{4}\pi$ to 2π .

In the domain of: $] -2\pi, -\frac{3}{2}\pi]$, $] -\pi, -\frac{1}{2}\pi [$, $] 0, \frac{1}{2}\pi [$, $] \pi, \frac{3}{2}\pi [$ the gradient is positive. In the domain of: $] -\frac{3}{2}\pi, -\pi [$, $] -\frac{1}{2}\pi, 0 [$, $] \frac{1}{2}\pi, \pi [$, $] \frac{3}{2}\pi, 2\pi [$ the gradient is negative

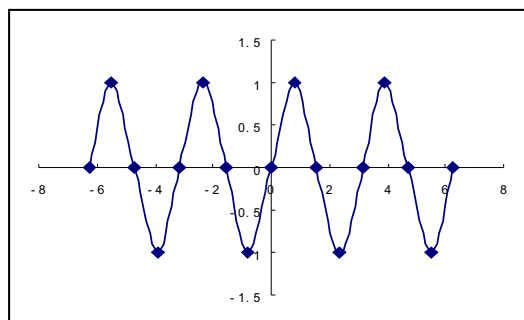
At the point when x equals to $-2\pi, -\frac{3}{2}\pi, -\pi, -\frac{1}{2}\pi, 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$ and 2π , the gradient is obviously 0.

c) Find values of gradient of the function at $\pi/4$ every unit and sketch findings on a graph

The numerical values of the gradient of the function at every $\pi/4$ unit is showed in the table:

X	-2π	$-\frac{7}{4}\pi$	$-\frac{3}{2}\pi$	$-\frac{5}{4}\pi$	$-\pi$	$-\frac{3}{4}\pi$	$-\frac{1}{2}\pi$	$-\frac{1}{4}\pi$	0
$\frac{dy}{dx}$	0	1	0	-1	0	1	0	-1	0
X	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π	
$\frac{dy}{dx}$	1	0	-1	0	1	0	-1	0	

The scatter plot for the gradient of $f(x) = \sin x$ at every $\pi/4$ unit is shown below.



gradient of $f(x) = \sin(x+1)$ at every $\pi/4$

d) Make a conjecture for the derive function $m'(x)$

the conjecture for the derived function is $m'(x) = \sin 2x$.

e) The scatter plot for the values in the table above with the function $m'(x) = \sin 2x$ is shown in figure 12. The derivative of $m(x) = \sin^2 x$ found by the calculator with the curve of $m'(x) = \sin 2x$ is shown in figure 13:

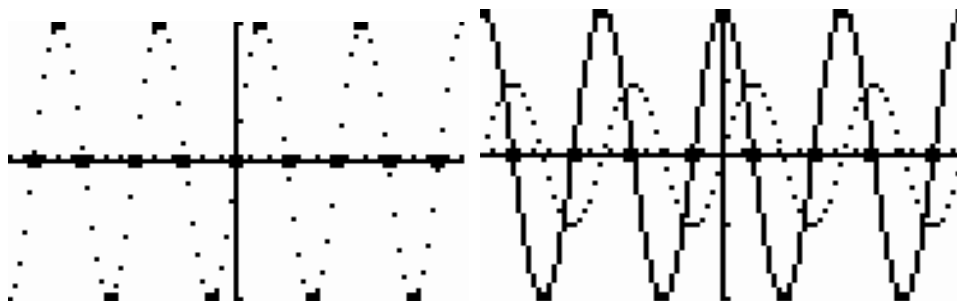


Figure 12 $m'(x) = \sin 2x$

figure 13 derivative of $m(x) = \sin^2 x$

Figure 12 reveals the curve of $m'(x) = \sin 2x$ fits the scatter plot well. Figure 13 reveals that the derivative function found by calculator overlaps $m'(x) = \sin 2x$. Perfect.

Therefore $m'(x) = \sin 2x$ is the correct conjecture for function $m(x) = \sin^2 x$.

2. Method and findings:

The curve in figure 16 reveals the following characteristics:

symmetrical along the axis of $\frac{1}{4}\pi$.

repeat its values every π , (so the function is periodic with a period of π)

the maximum is 1 the minimum is -1

the mean value of the function is zero

the amplitude of the function is 1

According to the characteristics the form of function for the gradient is similar to a cosine function

with parameter of 1 and horizontal stretched by $\frac{1}{2}$ units.

Therefore conjecture of $m'(x)$ is: $m'(x) = \sin 2x$

Based on the double angle formula: $\sin 2x = 2 \sin x \cos x$.

the conjecture $m'(x) = \sin 2x$,

Combine the two equation together we get: $m'(x) = \sin 2x = 2 \sin x \cos x$.

Therefore $m'(x) = \sin 2x$ is the same as $m'(x) = 2 \sin x \cos x$

So the derivative of function $m(x) = \sin^2 x$ is $m'(x) = \sin 2x$ and it can be written as

$m'(x) = 2 \sin x \cos x$.