CROWS DROPPING NUTS

Task:

1. **Introduction:**

Crows love dropping nuts but their beaks are not strong enough to break the nuts open. To crack open the shell they will repeatedly drop the nut on a hard surface until it opens. In this investigation we will determine a possible function that would model the behaviour of the data given below.

2. **Data table given:**

Height of drops (m)	1.7	2.0	2.9	4.1	5.6	6.3	7.0	8.0	10.0	13.9
Number of drops	42.0	21.0	10.3	6.8	5.1	4.8	4.4	4.1	3.7	3.2

3. **Definition of variables as well as constraints:**

- **Variables:** Representing the Number of Drops and Height of Drops,

Let 'h' be the height above ground (in metres) and 'n' be the number of drops favourable outcome (nut crack). Whereby 'h' is the independent variable and 'n' is the dependent variable. Therefore all graphs will be plotted as 'h' (height of drops) and 'n' (number of drops).

Another variable that should also be accounted for is the size of the nut 'S_n'. It was stated that the average number of drops will also be investigated using medium and small nuts. Therefore 'S_n' will be used to illustrate the size of the nuts.

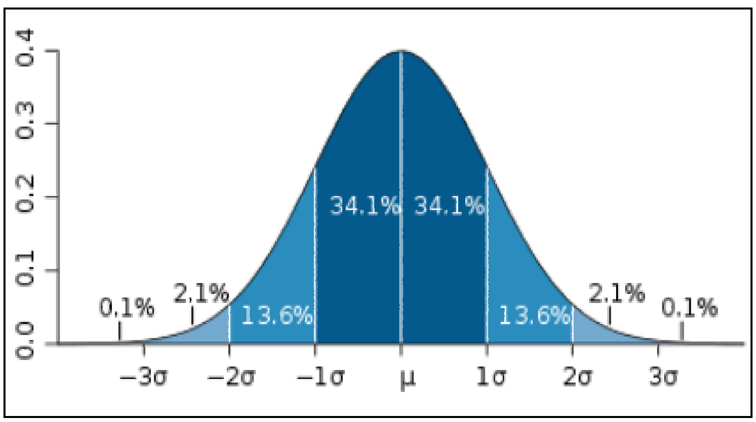
- **Constraints:** Representing the boundary values and types of numbers for h and n,

h is a positive integer such that: $0 < h$. Height is a displacement measure, it tells you the vertical displacement of an object from a 'ground' position. For this data it is assumed that $h = 0$ is the 'hard surface' whereby the nut impacts.

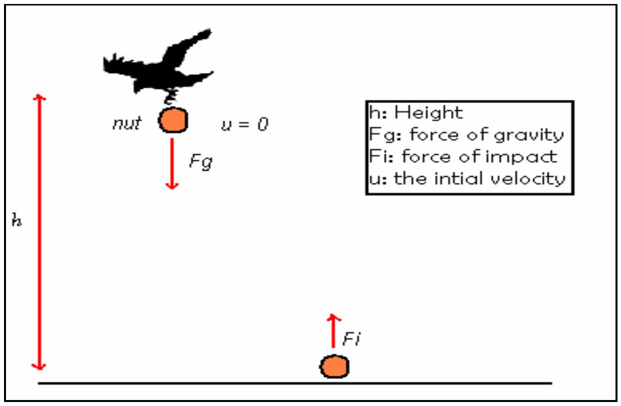
$n \geq 1$ because it must take at least one drop for a nut to successfully crack from a very high height. For one particular trial the number of drops it takes for a nut to break must take an integer value. For example, the nut cannot break on 1.4 drops; it would either break on the first or second attempt. However, the data presented here is an average over a large number of trials, and because of this n can be any positive number, either an integer or fractional. Now another point to note about the value of n is that it represents the 'average' number of drops over a number of trials for different size nuts. This is assumed to imply that n is the mean of the data set for each value of h. However if the data set was not large enough the mean may not be a precise measure, and error bars should be associated with each value of n because of this the functional fit, which may represent the *actual* value of n, $f(h)$ may not lie on the data points given, and this may go to partly explain any variations between the fitted curve and the data points, even if it is only a small effect.

The model given for both *h* and *n* will be the fit for discrete data, whereby there is a variation in the variables. It is noted that the table record containing the average number of drops is only a measure of the height and number of drops required to crack nuts, whereby a number of variable such as size and mass of the nut are not presented. Therefore it is expected these differences from trial to trial would account for variation in any model attempting to fit the data, as these would have significant contributions towards the nut cracking from different heights. As well as it was also not stated if the craw was *stationary* when dropping the nut, therefore in the investigation it was assumed th at the craw dropped the nuts while stationary.

This can also be explained by standard deviation, which can be used to calculate the average or most likely outcome. However, there is always a range of values that can reasonably attribute to the results. Whereby low data values indicate that the points tend to be very close to the same value and the high data points are 'spread out' over a large range of values, this known as standard deviation as shown below.
(Citationhttp://en.wikipedia.org/wiki/Standard_deviation)



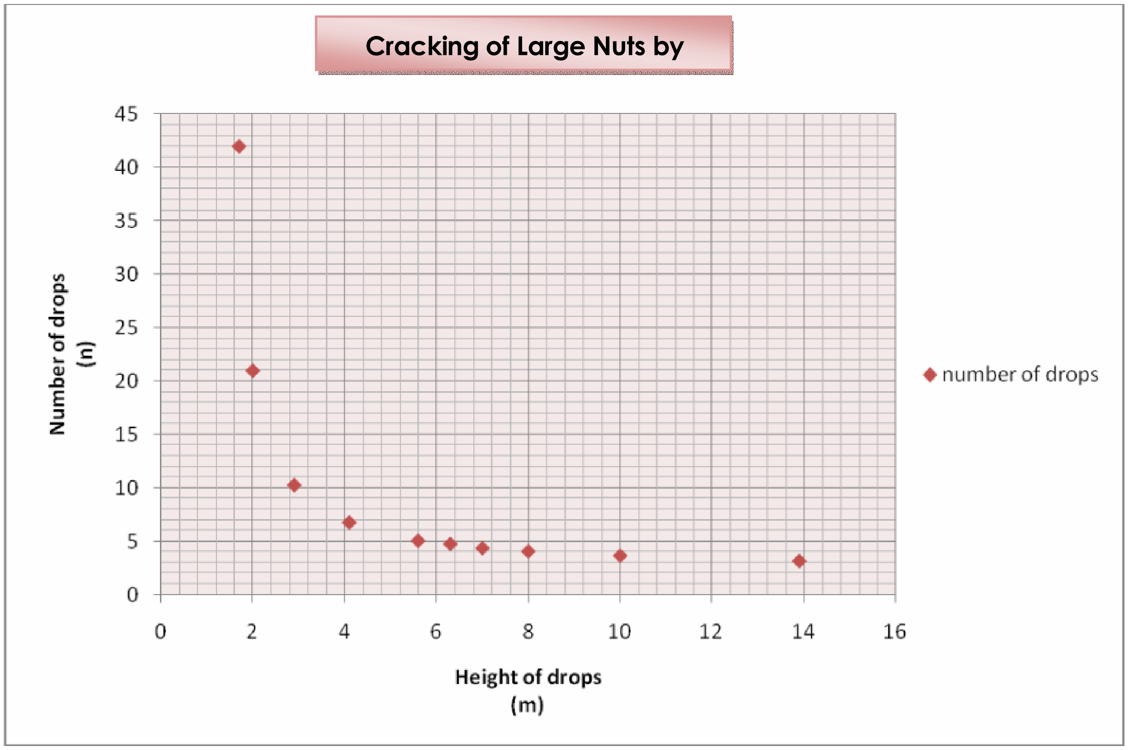
The following diagram is a representation of a crow dropping a nut, illustrating the effect on the nut. If was assumed that the initial velocity is zero and that the crow is stationary when the nut is being dropped, thus the impact of the nut on the ground will purely be dependent on the force of gravity. However the nut will not seize all the impact force when it collides with the ground, thus the nut will rebound as a result.



4. **<u>Graphing the data:</u>**

The graph below is a representation of the number of drops a crow had to drop a 'large nut' from different heights, for the nut to successfully crack.

- Using Microsoft excel the following graph was plotted.



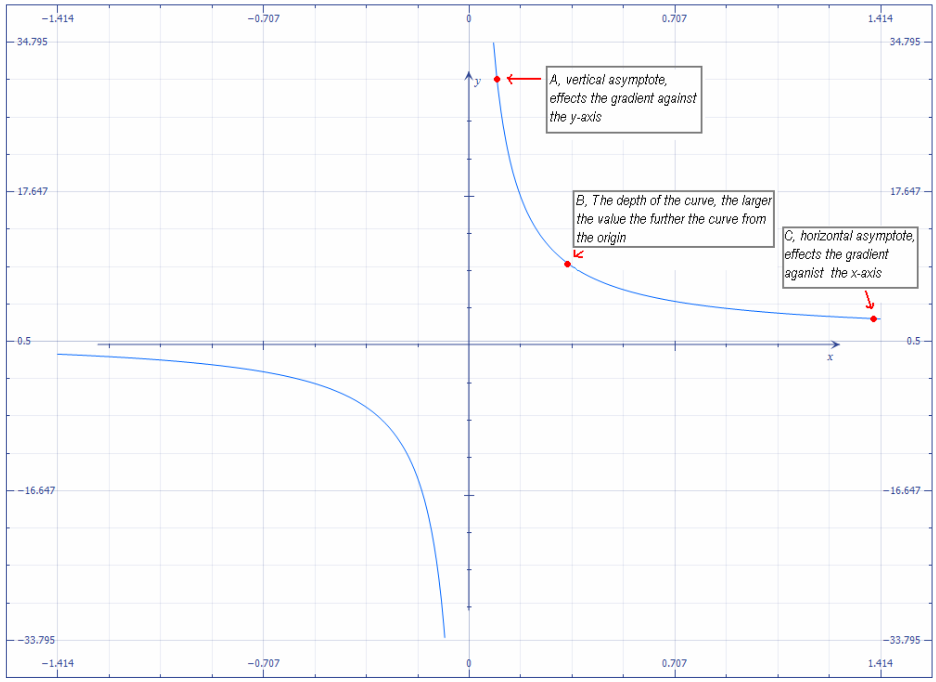
Graph 1, the above graph shows the relation between the height of drops (m) and the number of drops (n)

Parameters:

A parameter is a quantity that defines certain characteristics of a function; in different contexts the term may have different usages.

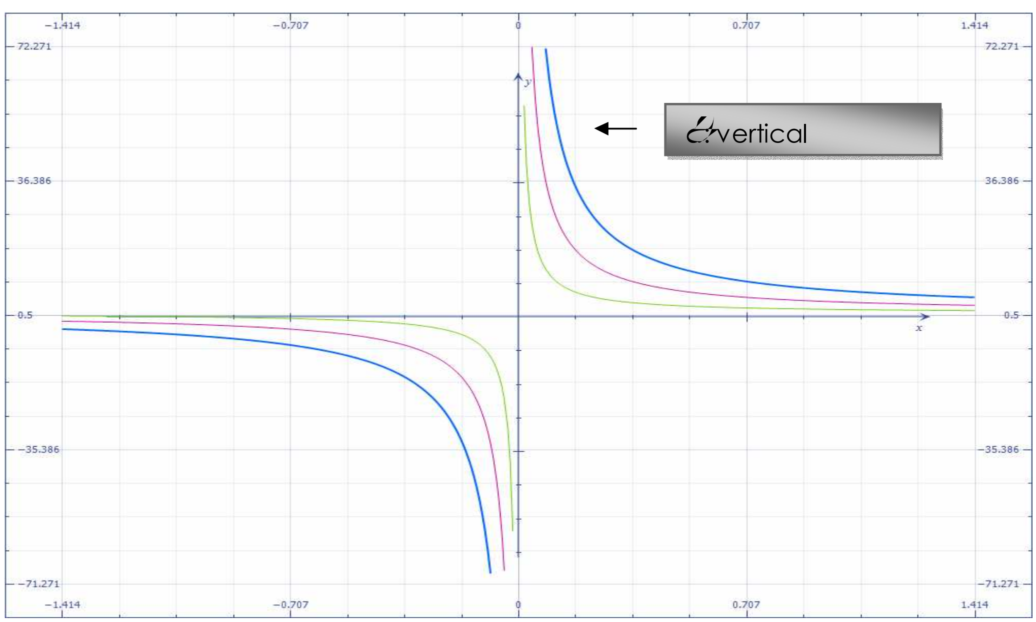
In the following investigation of crows dropping nuts the equations that will be used will consisting of 3 parameters. Whereby they will be labelled *a, b, c.*

Below is an illustration of simple reciprocal function with an explanation of how parameters a, b and c effect the gradient.



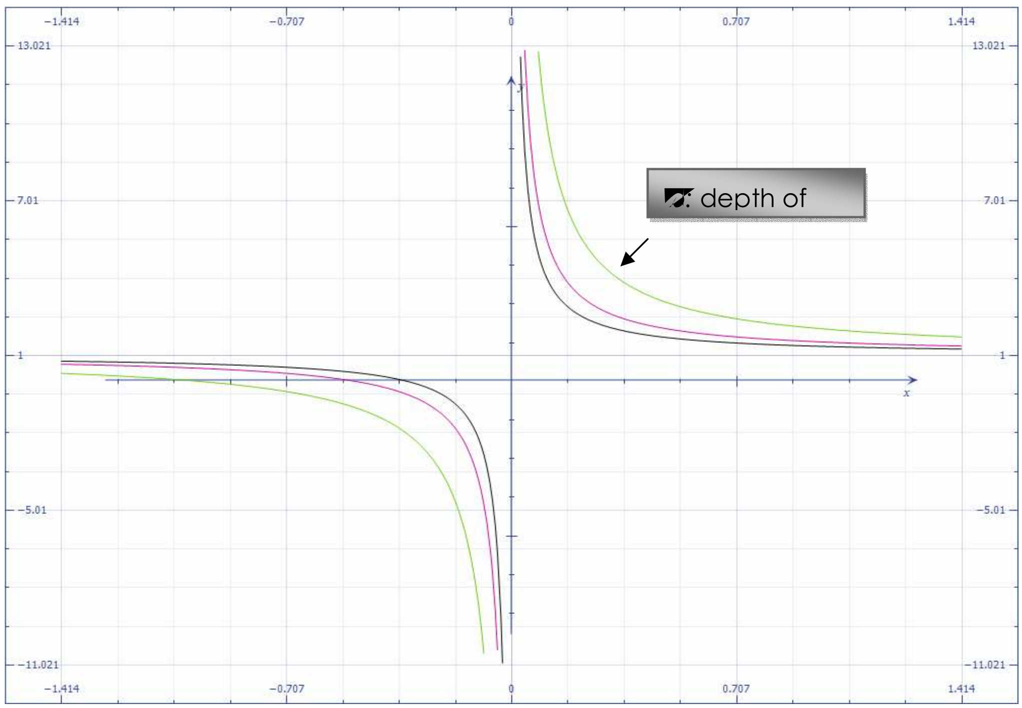
The graph above is an illustration of how parameters *a, b, c* effect the curve though changing the values.

Parameter *a*, is a representation of the vertical asymptote, as apparent in the graph below.



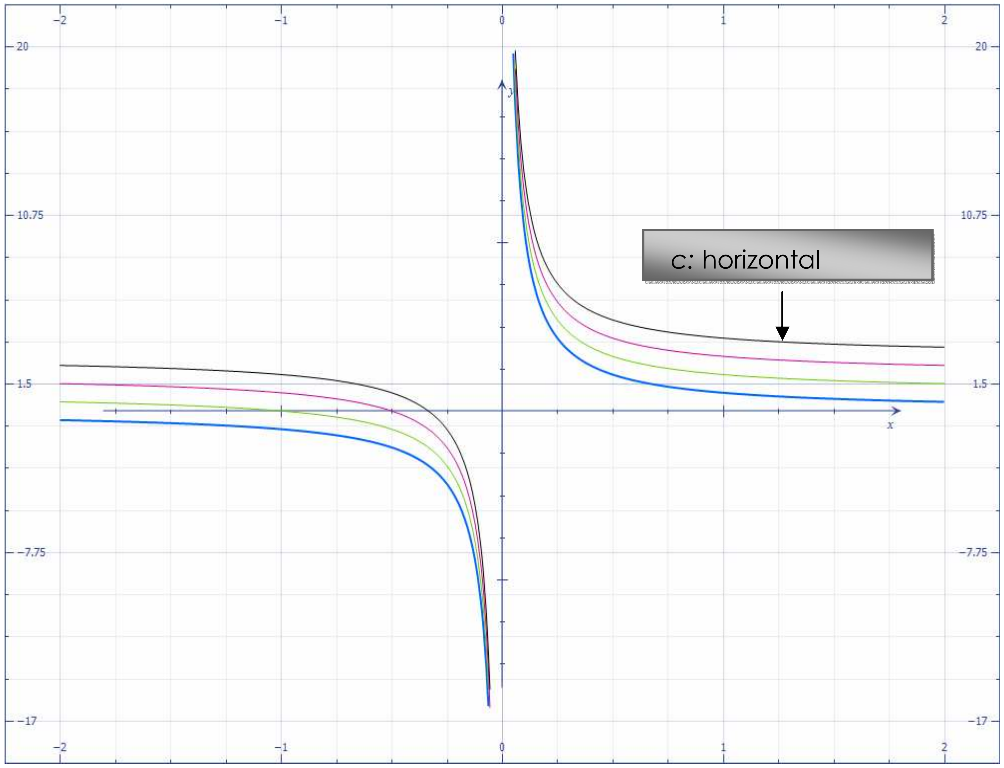
It is clear in the graph the vertical asymptote varies depending on the values of *a*. As the values of the vertical asymptote increases the curve gets further way from the y -axis.

Hence the same was done for parameter *b,* whereby, *b* represents the depth of the curve i.e. how close the curve goes to touch the asymptote in the centre.



It is apparent from the graph that the 'curve depth' varies. As you increase the value of *b* the curve at the origin becomes more or less steeper.

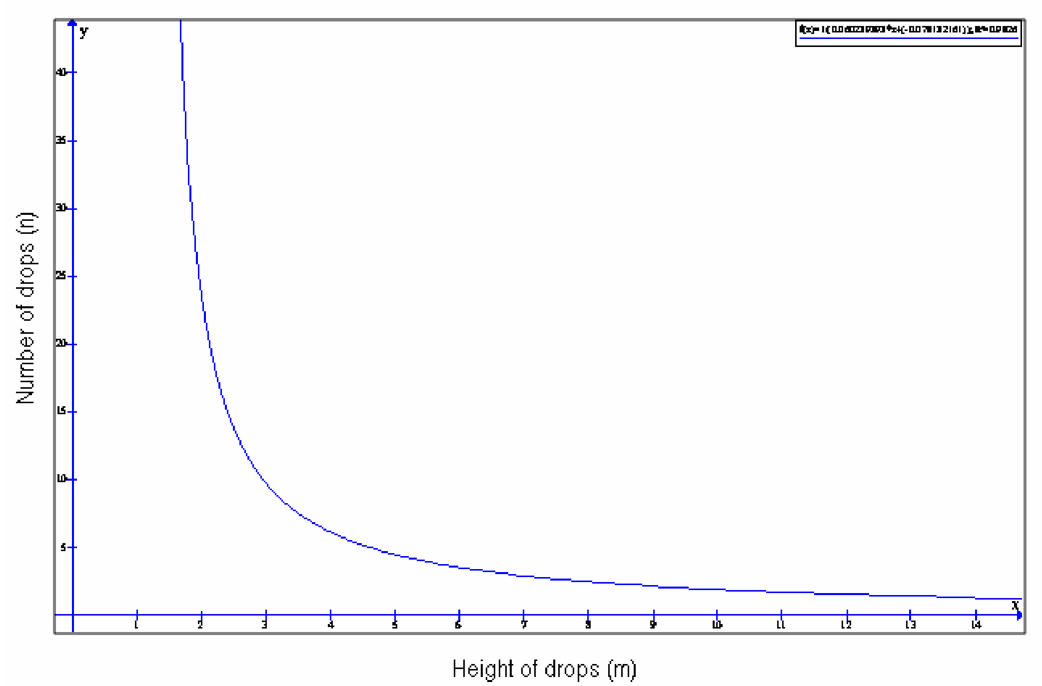
The graph below demonstrates the the horizontal asymptote *c*.



Therefore it is clear that as the values of *c* increase the horizontal asymptote shifts upwards.

I believe that one of the models that best fits the graph is a reciprocal function; it is defined in its simplest form as $f(x) \propto \frac{1}{x}$, as illustrated in the graph below. It is apparent that there is a horizontal and vertical asymptote, whereby the x and y values never reach zero.

- Using 'graph' the following graph was plotted.



I choose this function because it best represents the data points given for the large nuts, even though the function may seem to have slight variation with a few modifications it should have an accurate fit to the data.

After having decided to use a reciprocal function, next we are required to create an equation that best fits the function. However with the lack of detail that we have about the crow dropping the nut, a few physics calculation such as motion and momentum were required.

Using the model in graph 1, the velocity when the nut impacts the group can be calculated. Hence finding the formula for motion under constant acceleration was used,

$$v^2 = u^2 + 2\,as$$

u = initial velocity

v = final velocity

s = displacement

t = total time

a = acceleration

u = (ms^{-1}) = v

s = (m)

t = (s) a = (ms^{-2})

(Citation: http://en.wikipedia.org/wiki/Equations_of_motion)

Therefore using the formula for motion under constant acceleration, the following was derived:

$$v^2 = u^2 + 2\,as$$

It wasn't stated in the data given what the initial velocity of the nut was; hence the following can be stated;

$$u = 0$$

Whereby the displacement would be the height;

$$s = h$$

Hence it is crucial to find the acceleration, in this instants it can be stated that acceleration would depend on gravity, whereby the gravity of the earth is 9.8ms^{-2}.

(Citation: http://en.wikipedia.org/wiki/Earth's_gravity)

Therefore acceleration would equal gravity;

$$\therefore a = g \approx 9.8ms^{-2}$$

Hence by adding all the values in the equation the following was concluded;

$$v^2 = 2\,x\,9.8\,x$$

$$v = \sqrt{19.6}$$

Thus after having calculated the motion of the nut under constant acceleration, therefore it was also important to calculate the momentum of the nut, as this is the quantity that is conserved under an inelastic collision. (Citation http://en.wikipedia.org/wiki/Momentum)

$$p = mv$$

p = momentum p = ($kgms^{-1}$)

m = mass m = (kg)

v = velocity v = (ms^{-1})

Hence knowing the velocity of the nut from the previous equation, the following was deduced;

$$p = m\,\sqrt{19.6\times}$$

Thus it was assumed that the impact of the nut will be proportional to the momentum,

$F_{impact} = F_i \propto p = m\sqrt{19.6 \times}$

However as the nut would bounce off the surface not all the force of the impact with the ground go into deforming the nut (breaking) (Citation http://en.wikipedia.org/wiki/Deformation).

Therefore another force is introduced,

$F_{deformation} = F_d = C_d \times Fi$ where C_d represents the percentage of force that is absorbed by the nut on impact, and is why the notation C_d is chosen as it is the constant of deformation in this model. Hence the range is:$0 < C_d < 1$.

The following is a representation of the formula used to calculate the impact of the nut, whereby, F_d reflects the force of impact. Therefore by substituting the previous expression for momentum ($p = m\,x\sqrt{19.6\,x}$) into the formula for '*deformation of the nut*'. The next step was to calculate the required number of drops for the nut to crack.

$\therefore F_d = C_d \times F_i$ But, $F_i = m\sqrt{19.6 \times}$

Therefore the $F_i = m\sqrt{19.6 \times}$ will be substituted for F_i in the equation below,

$\therefore F_d = C_d \times m\sqrt{19.6 \times}$

It is evident that a nut will break when it is exposed to enough impact force. There will be a threshold were after sufficient impact force the shell cannot contain the energy caused by impact and will fracture to conserve energy.

$\therefore F_{break} = C_b$ where by $C_b = (kgms^{-2})$

For each drop the nut will get weaker (i.e. more deformation to its atomic/molecular lattice). Hence the following equation is a representation of the number of impacts,

$$\therefore F_{nut} = \sum_{1}^{n} nF_{impact} = F_{d1} + F_{d2} + F_{d3} + \cdots + F_{dn}$$

Therefore when, $F_{nut} \geq F_{break}$ then the nut will break.

Whereby; $nF_d \geq C_b$ for simplicity $nF_{impact} = C_b$

But, $F_d = C_d P$ whereby the $C_d P$ is an illustration of how much of the impact force went into the deformation of the nut with relation to the momentum.

Therefore; $p = m\,x\sqrt{19.6\,x}$

Thus applying all the equation above, the following steps were processed;

$n \times m \times C_d \times \sqrt{19.6 \times} = C_b\ (impact)$,

Therefore the equation of deformation $F_{deformation} = F_d = C_d \times Fi$ was used to calculate the 'impact' divided by the, 'impact force that went into deforming the nut with relation to the momentum as shown below.

$$\bar{n} = \frac{C_b}{C_d \times m \times \sqrt{19.6 \times}}$$

Whereby, $\bar{n}$ = average of n

5. **Range:**

Therefore the range can be estimated through the above calculations;

$0 < C_d \leq 1$ thus it is apparent that 'the amount of impact force that goes into deformation' must be greater than zero however it can equal to or be less than 1 depending on the velocity of the nut when it collides with the ground.

$0 < C_b$ Whereas the 'impact (amount of drops) must be greater than zero for the nut to start to weaken.

$0 < C \leq 3$ However *C* in this case is an illustration of the amount of drops that would cause the nut to crack, the nut must be dropped from a height greater than zero, whereas if it is dropped from a really high height it is possible to assume that the nut will crack in the first few drops.

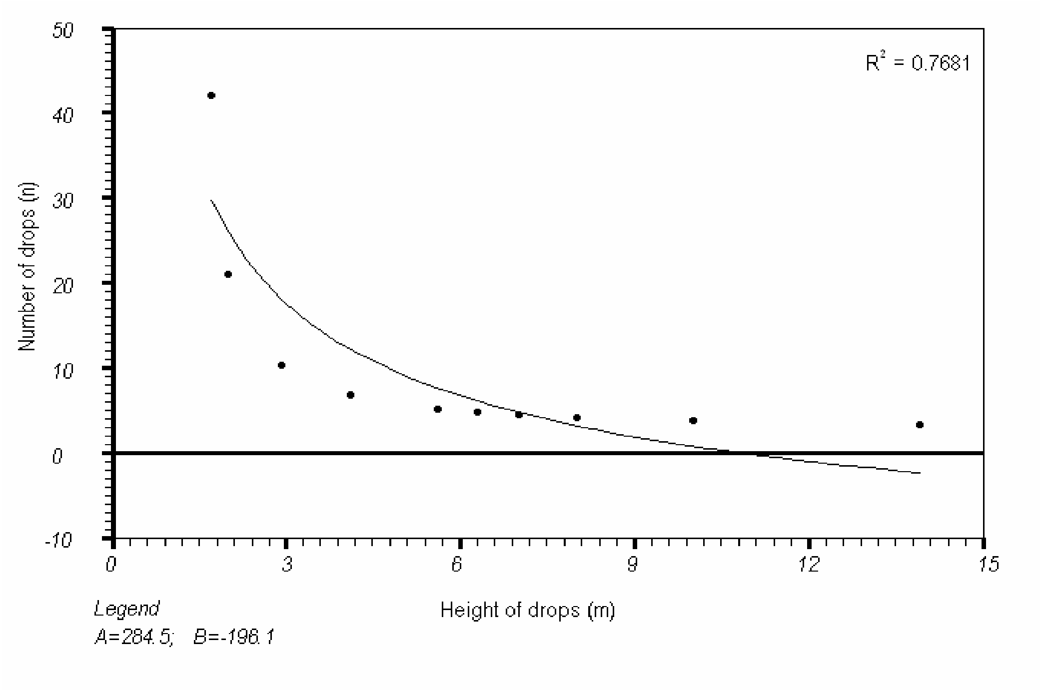
Therefore the equation $\bar{n} = \frac{C_b}{C_d \times m \times \sqrt{19.6 \times}}$ can be substituted for the following reciprocal function.

$$y = \frac{a}{bx} + c$$

Whereby; $y = \frac{a}{\sqrt{19.6x}} + c$

Therefore the following diagram is a plot of the actual data with comparison to the model chosen using the above explanations;

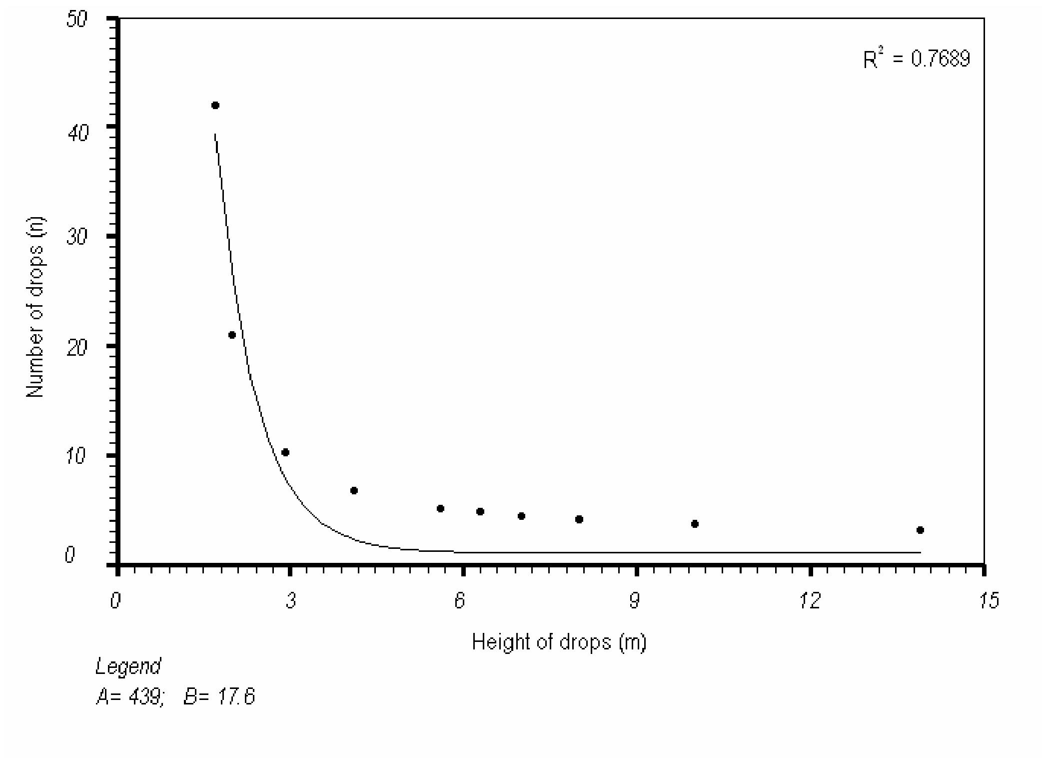
- Using 'lab fit' the following graph was plotted.



It is apparent that the model does not explain the data very well, because it implies that *n* can be negative. Therefore there were modifications to the graph to prevent *n* from being a negative value.

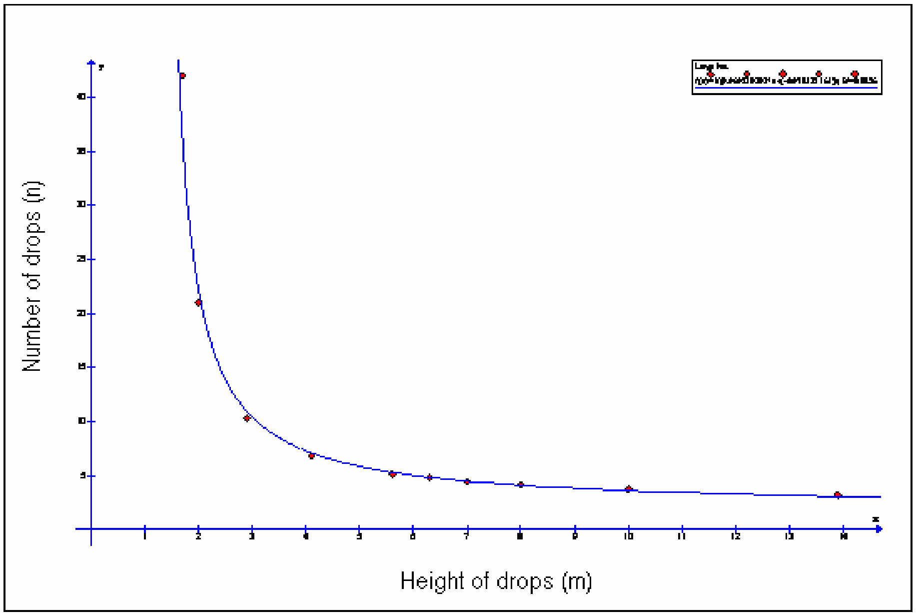
Whereby; $y = \frac{a}{\sqrt{Bx}} + 1$ was the second attempt to fit the model to the data, however this time we forced the asymptote to be one. Thus the following was obtained;

- Using 'lab fit' the following graph was plotted.



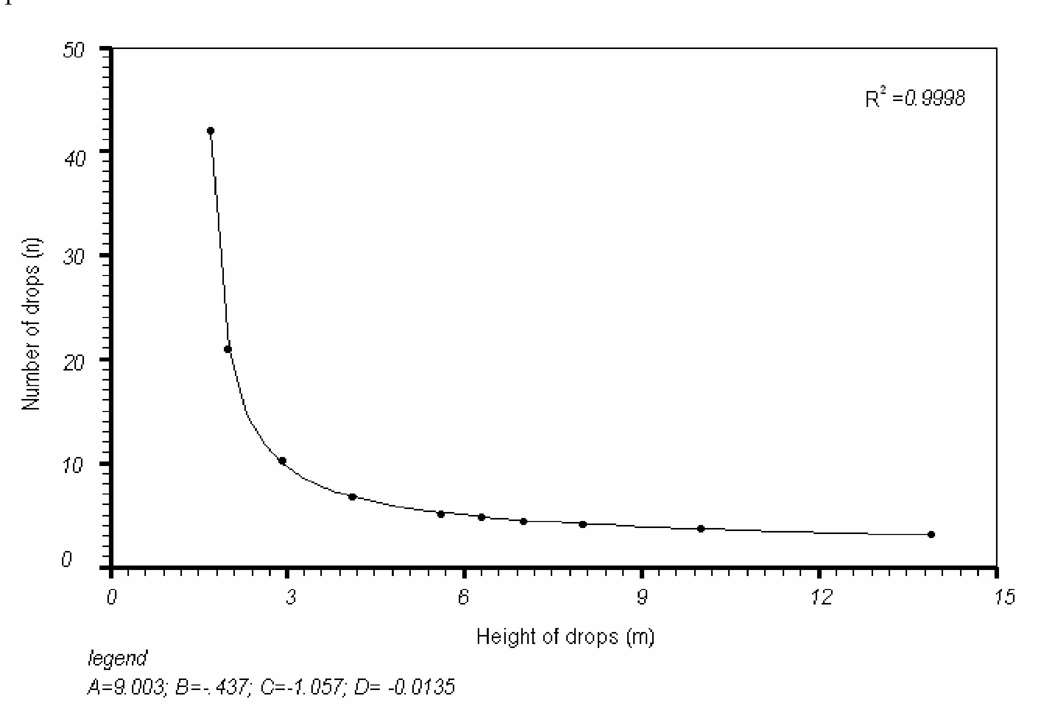
However even though n is no longer a negative value the model does not accurately fit the data, as the curve does not lie on the points, hence some of the approximations made in the model were not valid. However the model did show that there is a reciprocal relation between n and h.

However after a few more modification the following results were deduced;



Whereby, we needed to change all the values of the parameters to allow our model to fit the function. Thus it is apparent that we obtained a much better fit in comparison to the first two attempts.

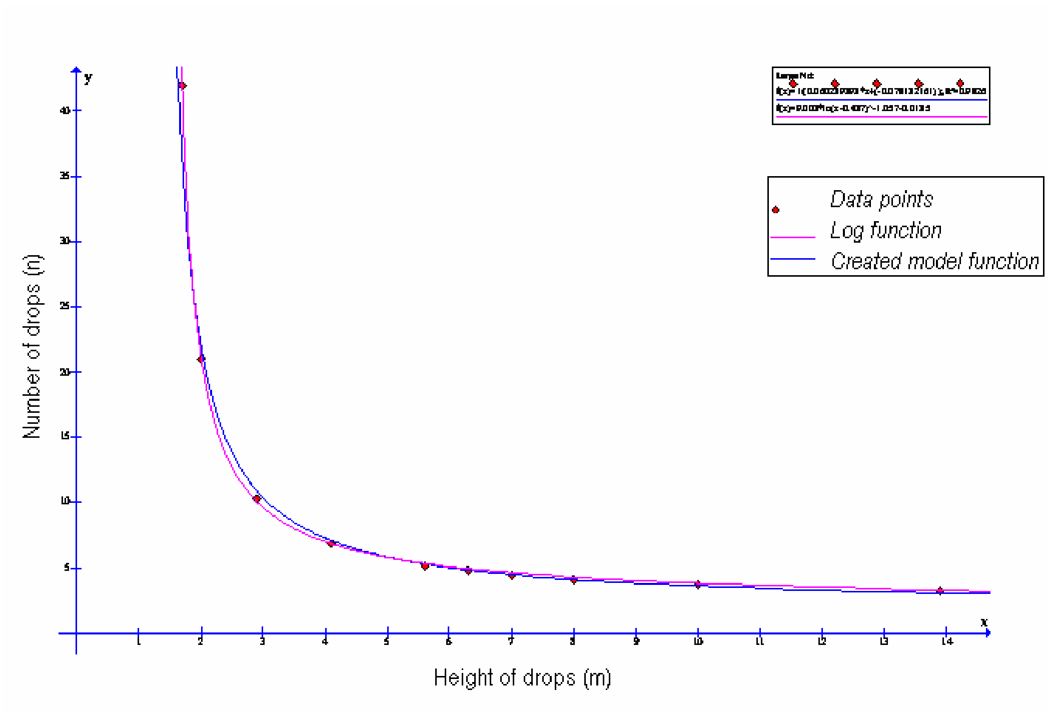
Therefore to test the accuracy of our equation another function using technology was plotted. The function below as an illustration of a log function that contains 4 parameter.



The equation for the function is $A\log(x+B)^{c} + Dx$

It is clearly demonstrated that the equation using technology has a highly accurate fit with comparison to the data.

Below is the comparison of my model function and the function found using technology,



It is clearly demonstrated that the log function has a greater fit then the created reciprocal function. Whereby, the log function has a better curve in comparison to the reciprocal function. It appears that the reciprocal function almost misses the 1st and the 3rd data points, whereas the log function clearly crosses both points. Thus even though the reciprocal function has a great fit the log function is more accurate. However it would be expected that the log function has a better fit because it has 4 parameters where as the reciprocal function has 3.

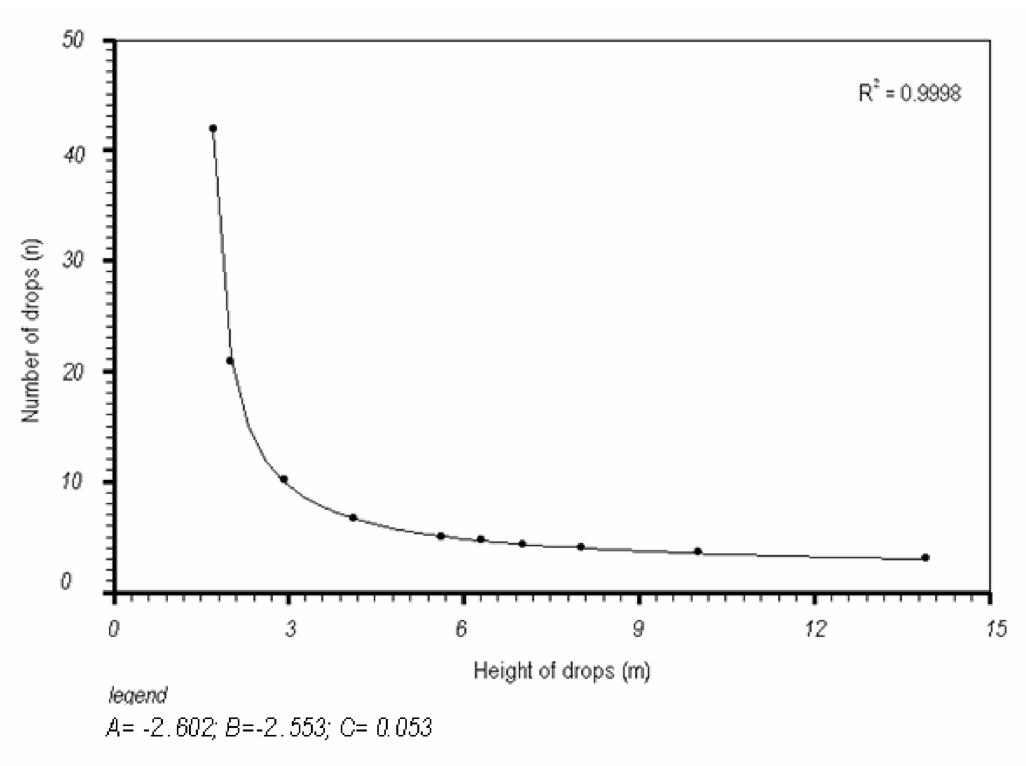
However another function was investigated to test if a better fit can be obtained, the next model chosen is a reciprocal power function this allows us to vary the power of x in the denominator. This changes the gradient of the curve. The equation chosen is stated below,

$$n = \frac{1}{A + B \times x^{C}}$$

Whereby, the parameter c is a power of x.

Therefore the following graph plots the relation of *h* and *n* in relation to the large nut.

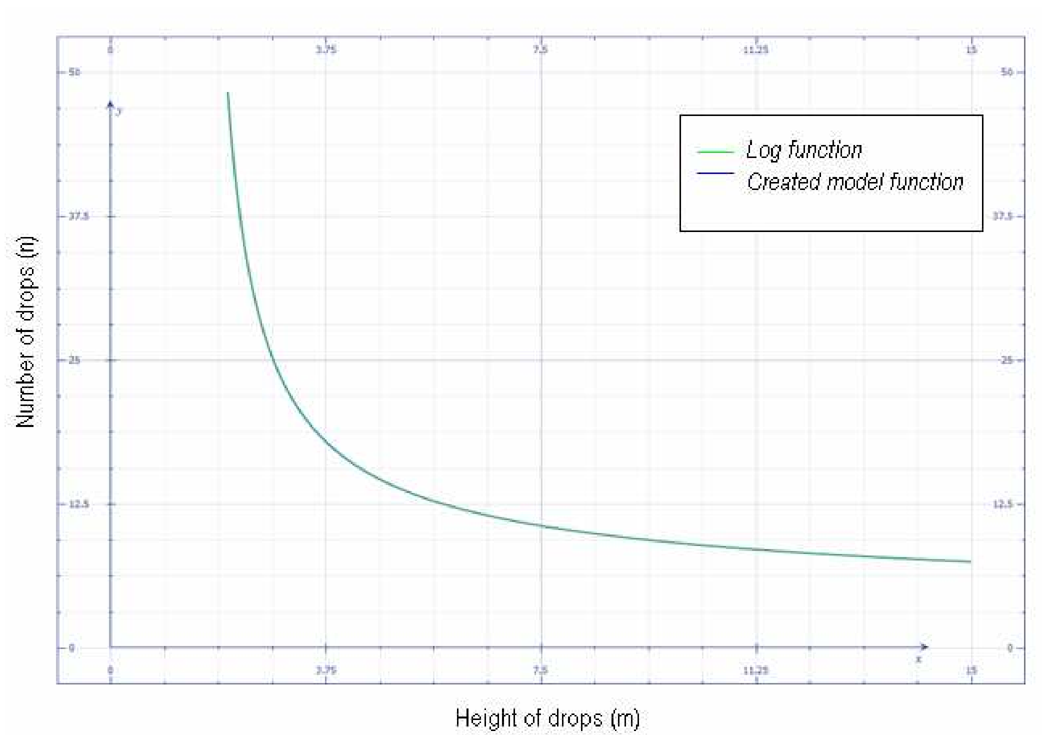
- Using 'lab fit' the following graph was plotted.



Therefore it is apparent from the graph that a reciprocal power function has a more accurate fit in comparison to the first model $y = \frac{a}{bx} + c$, whereby the formula for the reciprocal power function is;

$n = \frac{1}{A+B\times x^C}$ as stated earlier.

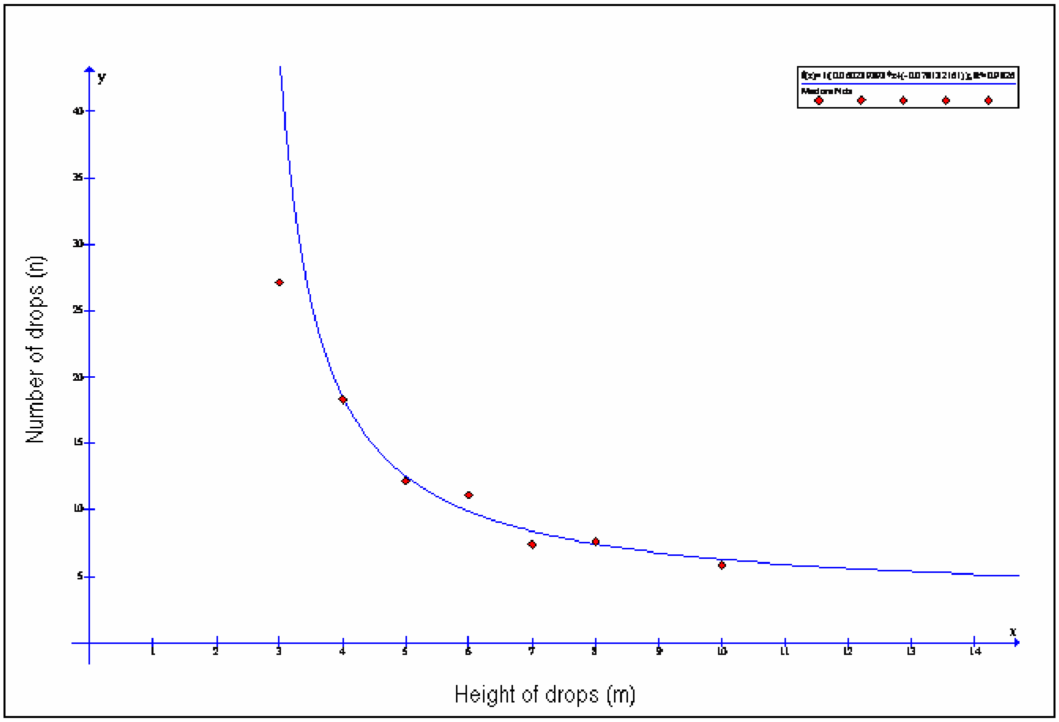
Hence by subtitling in the values for graph it was apparent that the function above fitted the data very accurately. Below is an illustration of the log function in comparison to the model reciprocal power function.



It appears from the above results that both equations have near exact fits, thus it can be assumed that the reciprocal power function is equally as good as the log function both obtaining an accuracy of 0.9998. Even though the log function had 4 parameters in comparison to the reciprocal power function which only had 3.

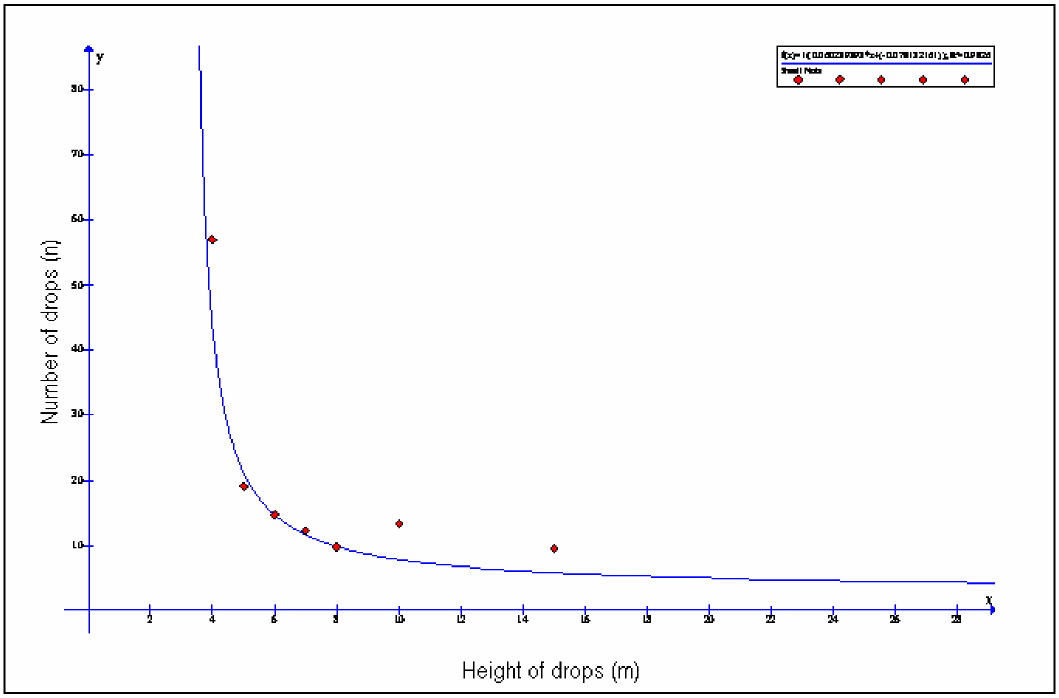
Below we will investigate how well the 1st model (reciprocal function) fits the data of different nuts.

Firstly the medium nut will be investigated;



It appears that the equation fits most of the data, but due to the limitations of the equation it completely misses the first point. However it's apparent that the equation fit the data very well however the slight variation of the middle data points above and below the curve can be assumed to be inaccuracy due to the recording of the initial data.

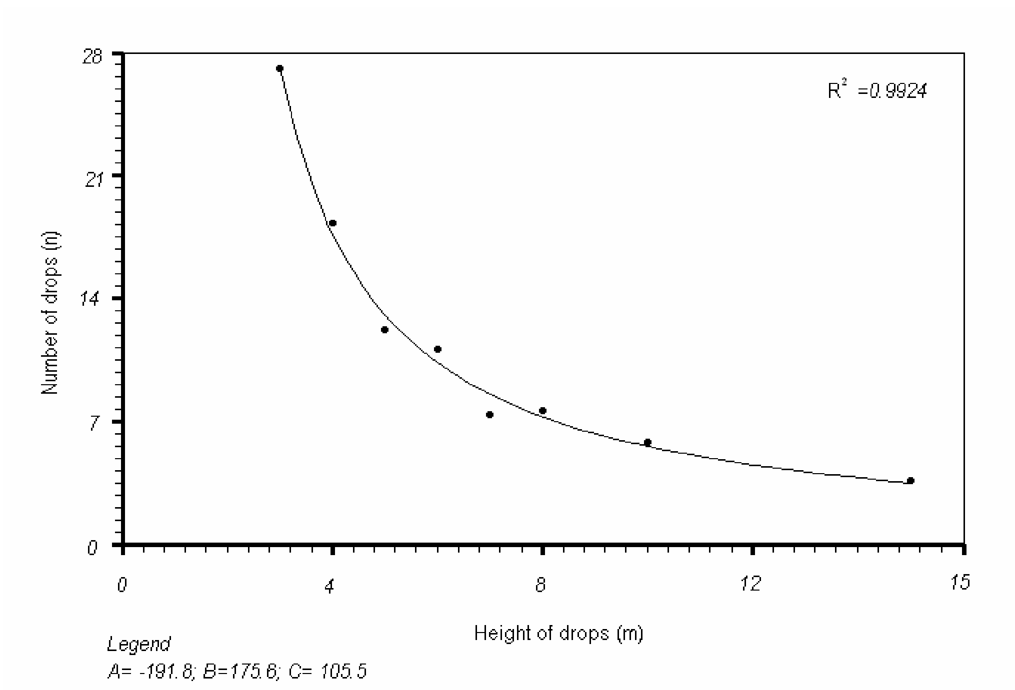
Again the equation was used, however this time for small nuts,



Again it appears that the equation had an accurate fit to the data, however it is clearly demonstrated that it completely misses the last two data points. It can be assumed that there might have been some inaccuracy in data collection, whereby it is demonstrated that the small nuts at greater height took a greater number of drops.

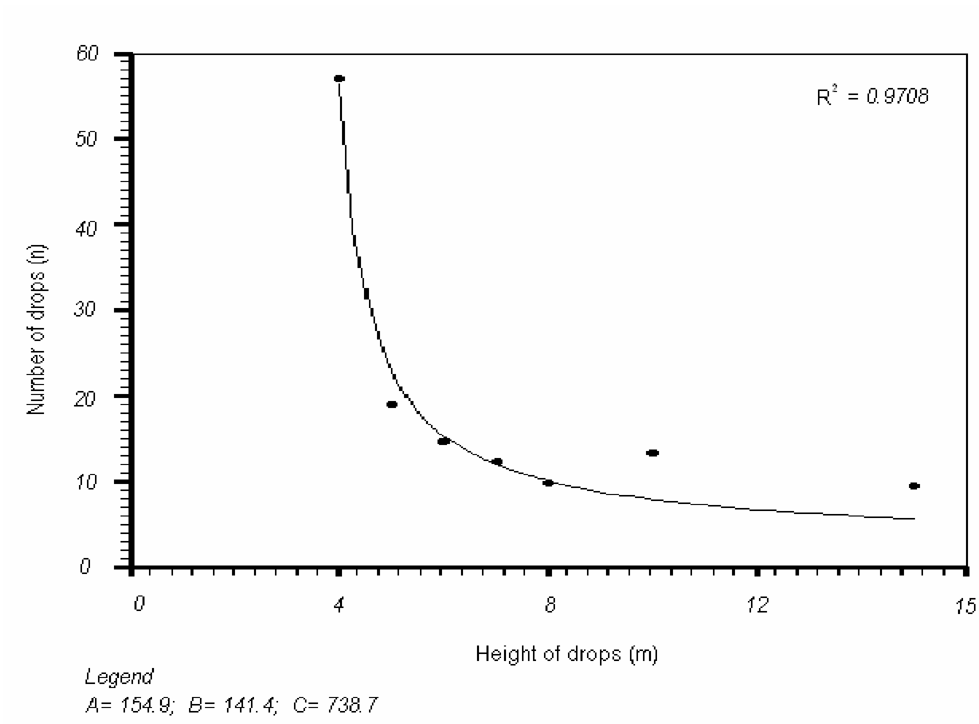
Therefore equation two was used (reciprocal power function) and was fitted to the medium nut and the following was concluded.

- The graph below is a representation of the medium nut using 'lab fit';



It is apparent that the reciprocal power function had a much more accurate fit in comparison to equation 1. Whereby it goes between the values obtaining the best possible fit.

Equation two was again used, however this time to test for small nuts. The graph below is an illustration of equation two



It appears that the reciprocal power function (equation 2) has very similar results to the reciprocal function (equation1).

6. Discussion of limitations:

- It appeared that the reciprocal power function obtained a very accurate fit in all size nuts, however the data presented in both the medium and small nut varied, we can safely assume that the errors that appeared in the data when plotted we possibly due to the variation in the exact nut sizes as well as the initial velocity when the nut was dropped.
- It can be assumed through the results above that so long as the data being collected maintains an accurate curve with minimal variation; the reciprocal power function will fit the data quiet accurately. However if there was variation within the data as found in the small nut, hence the equation will not correctly fit the data.
- However in the first equation (reciprocal function) it was proven that the model was not an accurate fit to the data. Whereby the curve of the data resulted in having a negative value. Hence the equation needed to be modified for a better fit

- Even though after modifying the data and forcing a horizontal asymptote to better fit the data points it appeared that the model still did not fit the data accurately, and hence the equation needed to be changed.
- However if all the parameters were modified as shown in the graphs above it appeared that the equation fits the data quiet well. However this equation has many limitations. If the curve of a particular data (other than the ones investigated) had a steeper slope thus the equation would not obtain an accurate fit. Moreso if there was more data calculated, thus it would be safe to assume that the equation after a certain amount of time would completely miss the data, because it apparent that the curve is constantly descending, thus after a specific period of time the curve will cross into the negative value. Whereby the equation has reached its utmost limit.

- However as for this investigation it can be concluded that a reciprocal function is not as accurate as a reciprocal power function.