Mathematics Coursework 2009: Crows Dropping Nuts

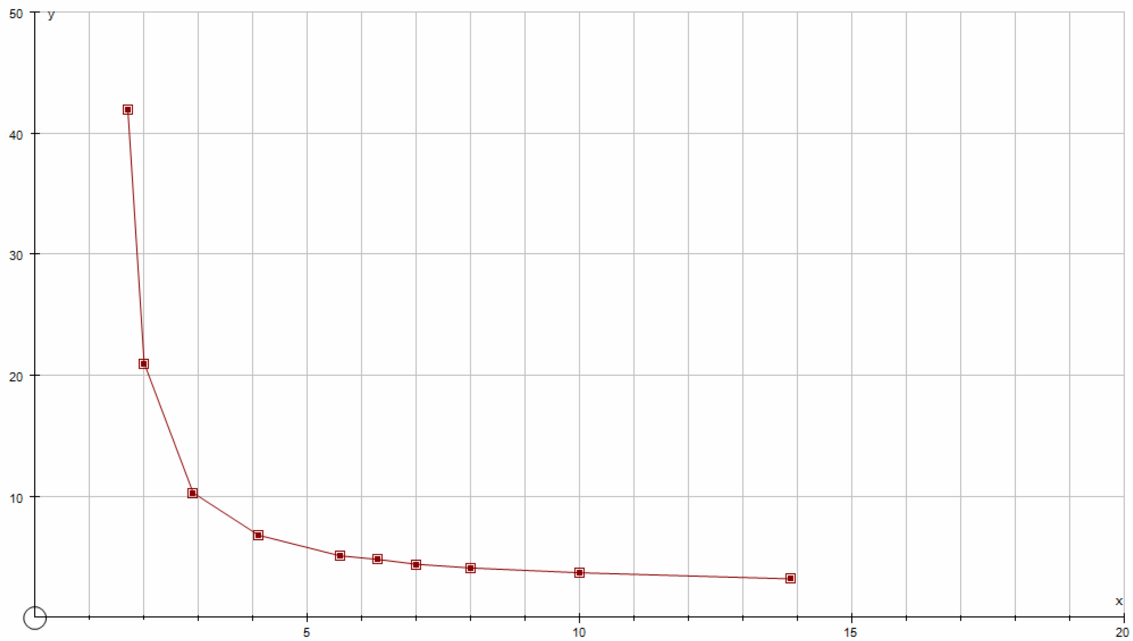
Standard Level

Introduction
The following table shows the average number of drops it takes to break open a large nut from different heights.

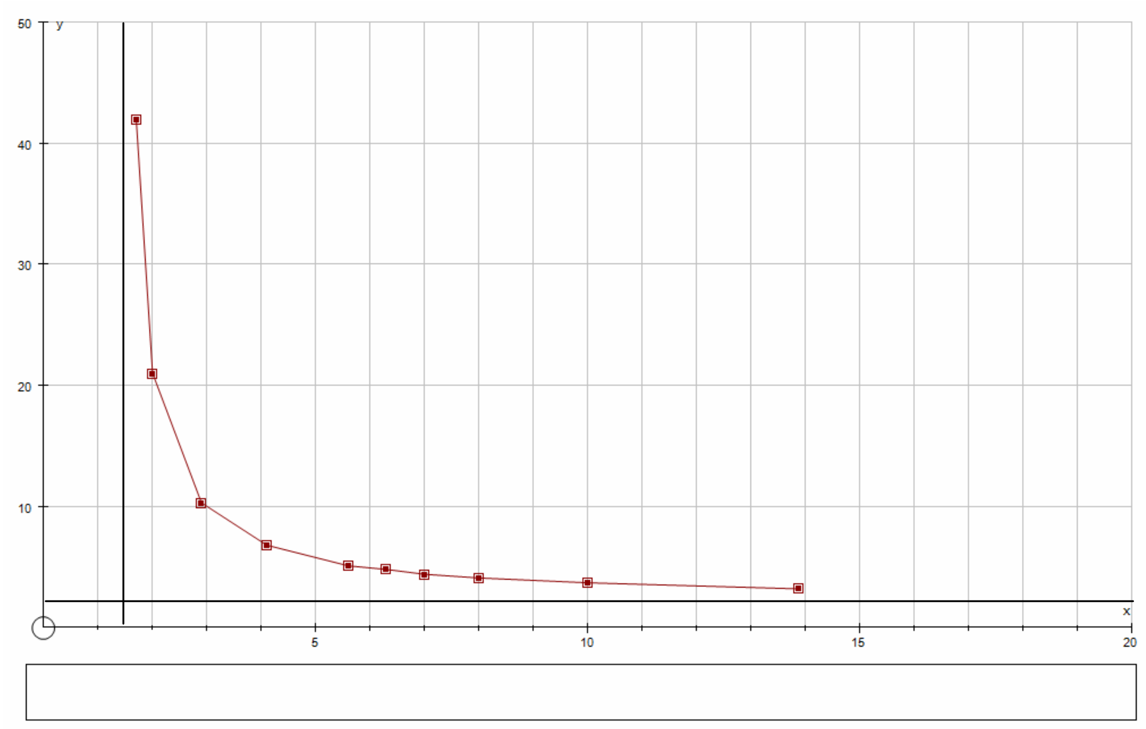
Large Nuts

Height of drop in meters *(□)*	1.7	2.0	2.9	4.1	5.6	6.3	7.0	8.0	10.0	13.9
Number of drops *(□)*	42.0	21.0	10.3	6.8	5.1	4.8	4.4	4.1	3.7	3.2

The following line graph, portraying the above table, shows us the number of drops by the height for each drop.

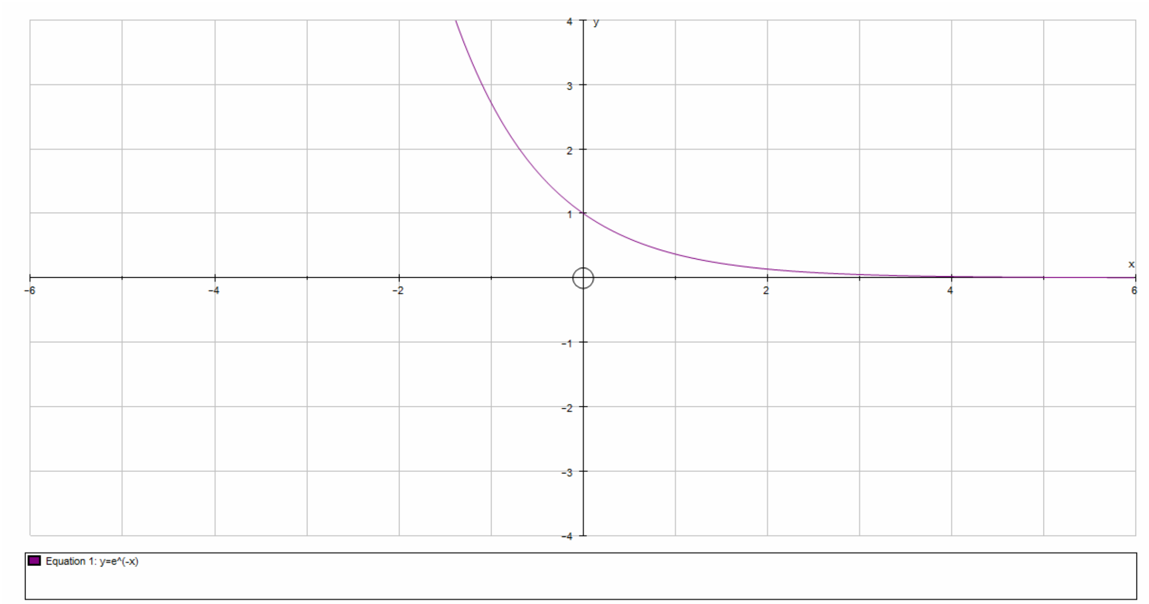


Variables:
In this graph, there many different variables that we need to perceive. One variable is how big the nut is. How large the nut is will invariably affect the number of times that the nut is dropped until it cracks open. By stating that the nuts are different sizes, it will help us recognize the nature of the model. Another variable is the fact that the number of drops are described as an average, because it is impossible (for example) to drop something 10.3 times. However, the reason that it has been recorded as an average is because it gives us more lucid data, which can be transcribed as a graph more easily. An additional variable is how the height that the nut is dropped affects the number of times the nut is dropped, and how it is put into an average.

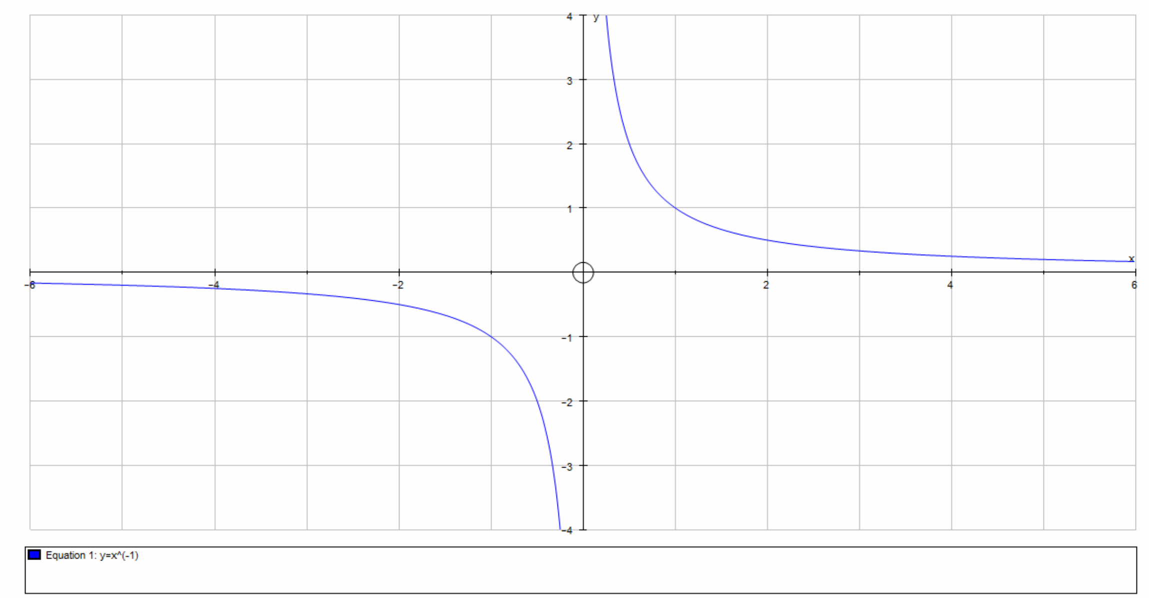


Parameters:
A parameter that can be stated is that in the above graph, there are asymptotes on both the □ and □ axes. This means that this graph isn't exact, where there can be an unlimited number of possibilities for the values on the axes. In the graph, when the height of the drops ***(□−□□□□)*** is too low, the number of drops ***(□−□□□□)*** is too high. And in reality, this would be impossible, so we must propose parameters for this graph. This means that the graph cannot touch the axes, because the model would not work. For example, if the height of the drops was 0 meters, then there would have to be □ number of drops, which is physically impossible.

One type of function that can model the behavior of the graph can be the inverse exponential $y=e^{-x}$.

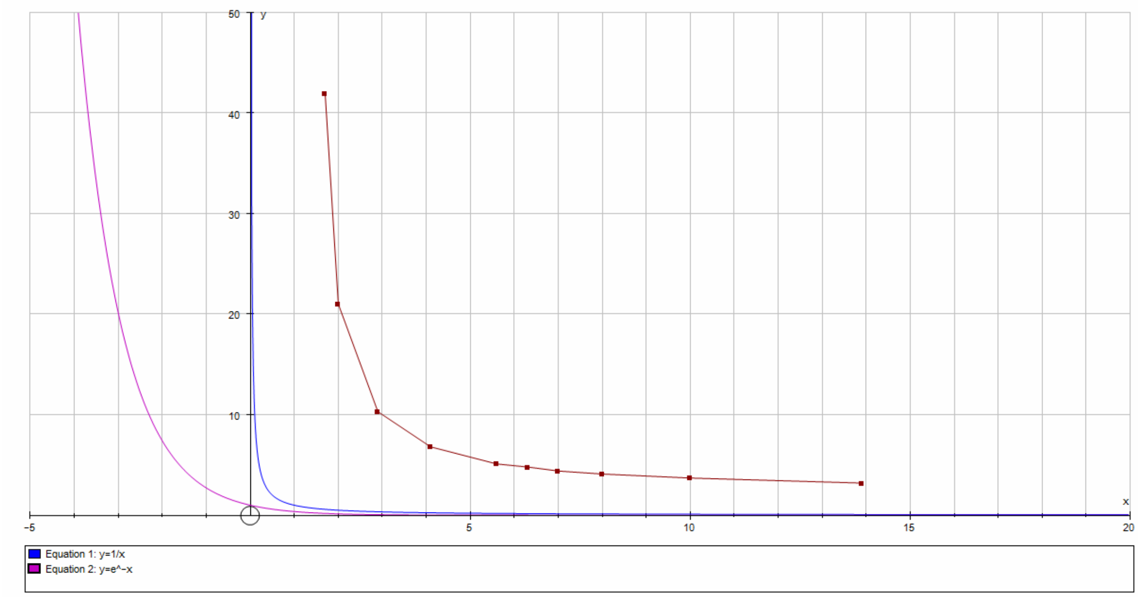


Another type of function that can model the behavior of the graph is $y=x^{-1}$.



By just observing these two graphs, the inverse exponential looks as if it can be manipulated in such a way that it can represent the original ‘large nut’ graph.

When looking at all three functions on one graph, we see the following:



In order to find the equation of the inverse exponential graph □=‚□-−□., we will use simultaneous equations. And for □=‚□-−□□., we will use the autograph system via trial and error.

□=‚□-−□.

To create a model for the inverse exponential graph, we can use simultaneous equations. We need to create two different constants, and use information from the table to ascertain the equation. So, by using □=□□‚□-−□□.+□□ and finding □ and □ (the two constants), we should be able to form a fairly accurate equation. (We should note that all values will be rounded up to 3 decimal places depending on the value).

We should take the first and last sets of data from the table, and substitute them as ‚□‚□□.. Then substitute those values into □=□□‚□-−□□.+□□.
So ‚□.□□‚□□□□. is plugged into □=□□‚□-−□□.+□□ to form the first equation, □□=□□‚□-−□□.□□.+□□

We should do the same with the last set of data, ‚□□.□□‚□□.□□.. So after plugging that into □=□□‚□-−□□.+□□, we get the second equation, □.□□=□□‚□-−□□□□.□□.+□□.

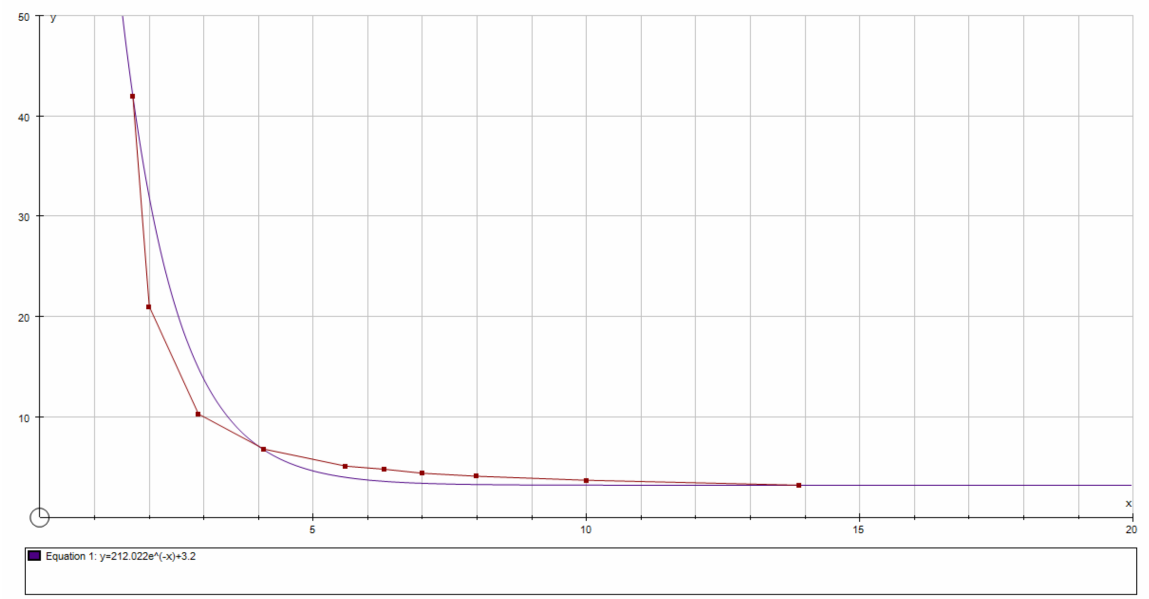
We should subtract the first equation from the second equation in order to find □ and □. So,
‚□□=□□‚□-−□□.□□.+□□.−‚□.□□=□□‚□-−□□□□.□□.+□□..
‚□□−□□.□□=□□‚□-−□□.□□.−□□‚□-−□□□□.□□.+□□−□□.
□□.□□=□□□□.□□□□□□□
□=□□□□□□.□□□□□□□

Now we should substitute the new value of □ into either the first or second equation.

□.□□=,□□□.□□□□□□□□□-−□□□□.□□.+□□
□=□□.□□

Therefore, □=□□□□□□.□□□□□□□,□-−□□.+□□.□□

The below graph consists of my model □=□□□□□□.□□□□□□□,□-−□□.+□□.□□ versus the original ‘large nut’ graph



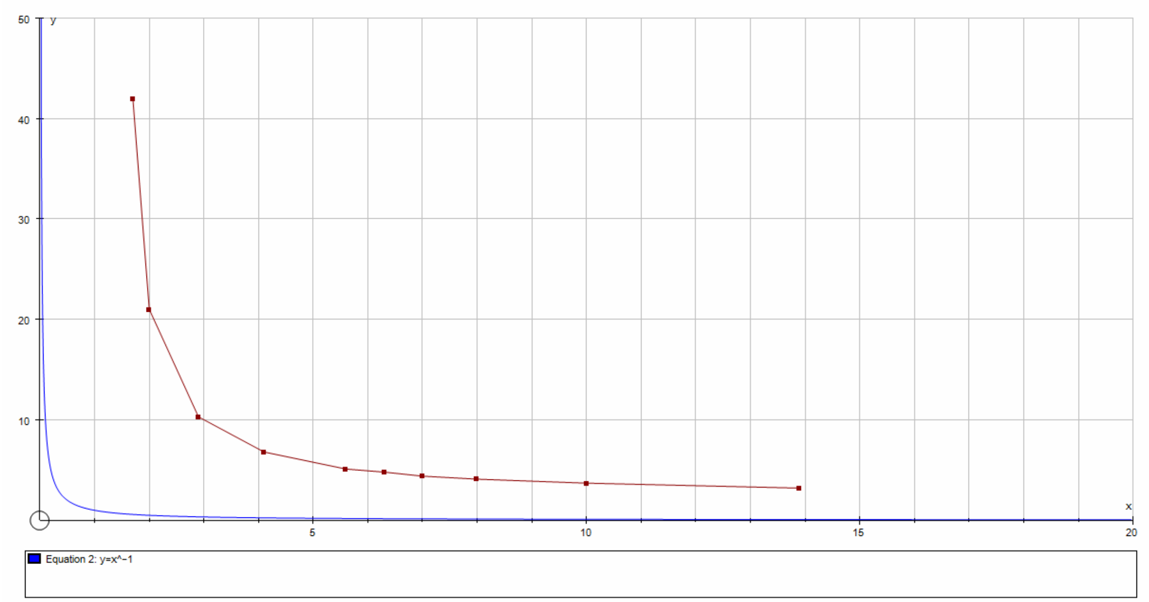
My model does not have as steep a descent of that of the original ‘large nut’ graph and the curve of my graph drops to just below that of the original’s, but then equalizes with the original at the point ,□□.□□,□□.□□.. In my model, the asymptote matches that of the original’s exactly on the □−□□□□, but not on the □−□□□□. My model crosses on 3 points of that on the original; ,□.□□,□□□□., ,□.□□,□.□□., and ,□□.□,□.□.. Though there are some similarities, the results are not particularly satisfying. So, we will use the trial and error method with □=,□-−□. which will result in a more accurate answer, because since we will be using more parameters, we will be able to shape the equation more closely to the original ‘large nut’ graph.

□=,□-−□.

By using the method of trial and error, we will estimate an approximately accurate model to the original 'large nut' graph. We will manipulate the function by using the following format:

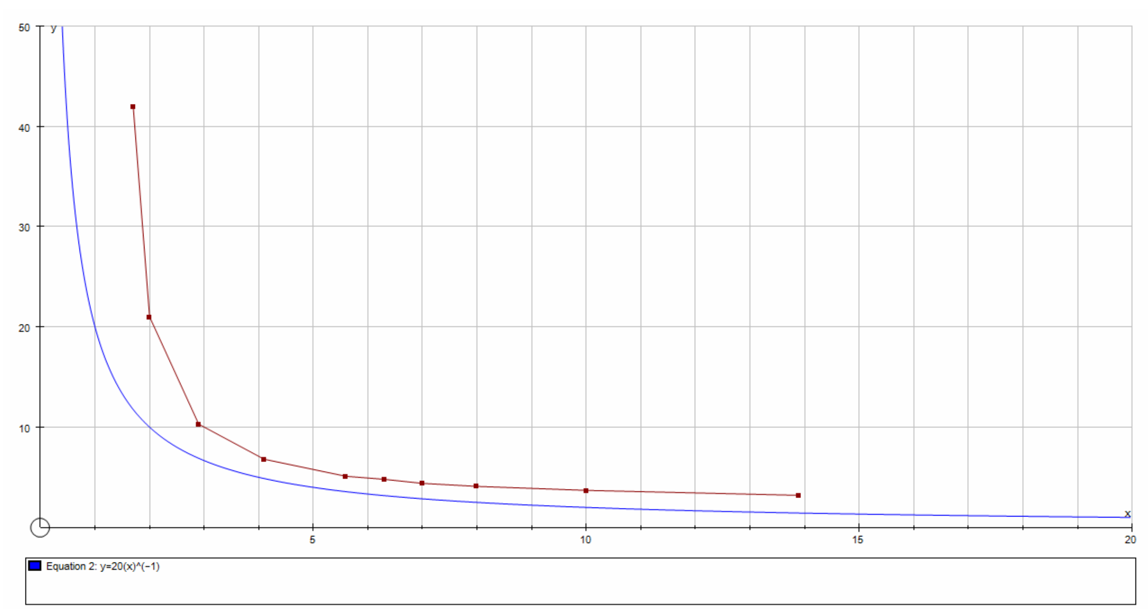
□=,□(□+□)-−□.+□

□=,□-−□.



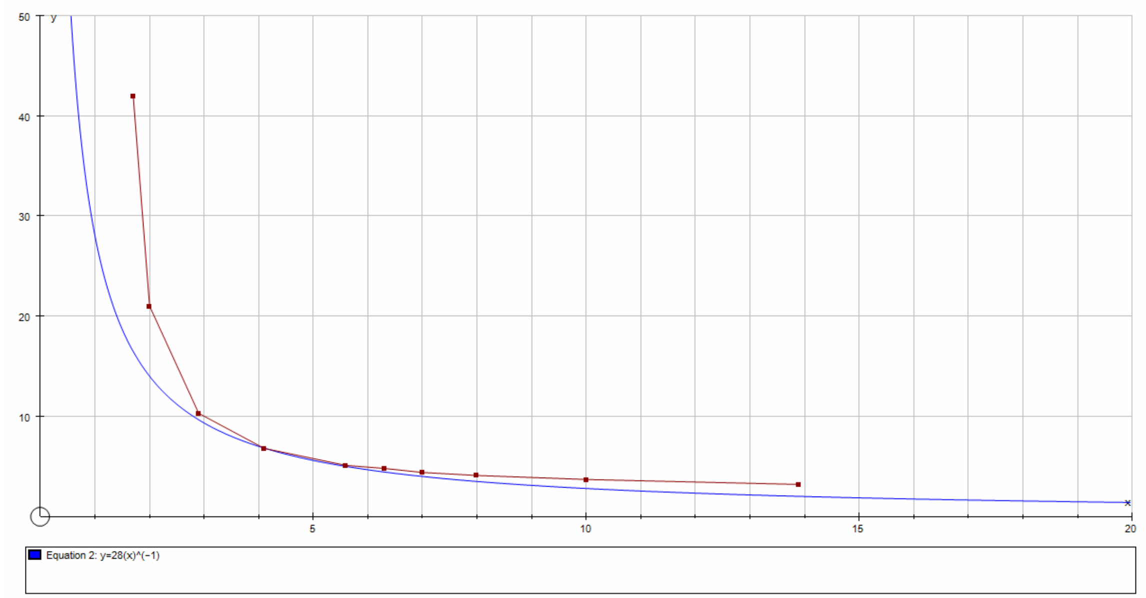
We will increase the value of □ in **□=,□(□+□)-−□.+□**

□□=□□□□(,□-−□□.)



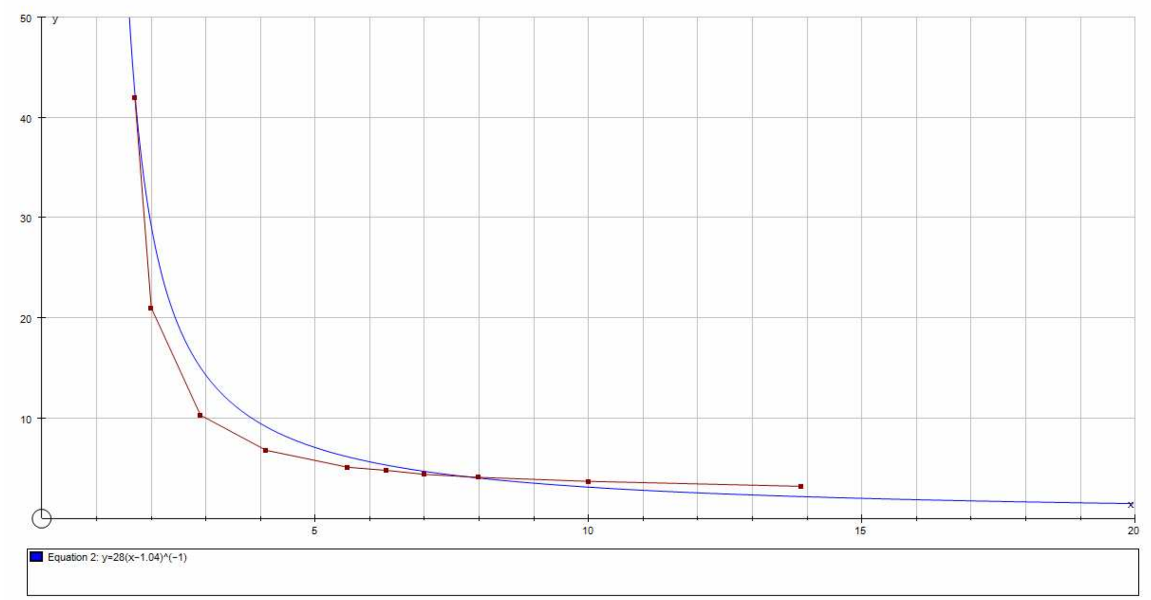
We will increase the value of □ in □=,□(□+□)-−□.+□

□=□□□□(,□-−□□.)



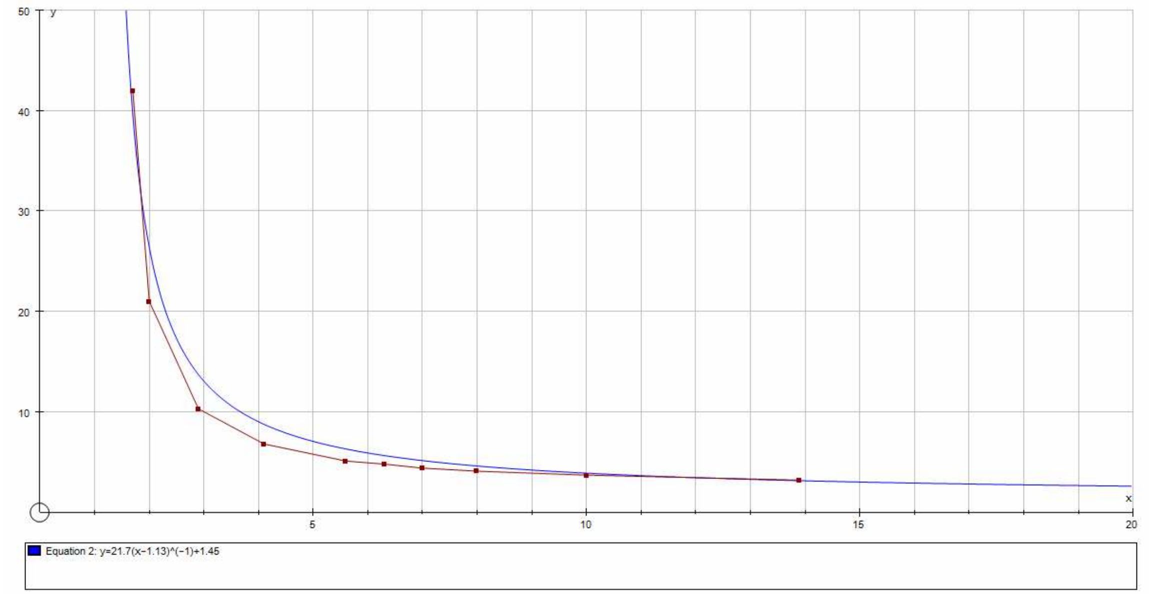
We will increase the value of □, and decrease the value of □ in □=,□(□+□)-−□.+□

□=□□(,□−□.□□)-−□□.



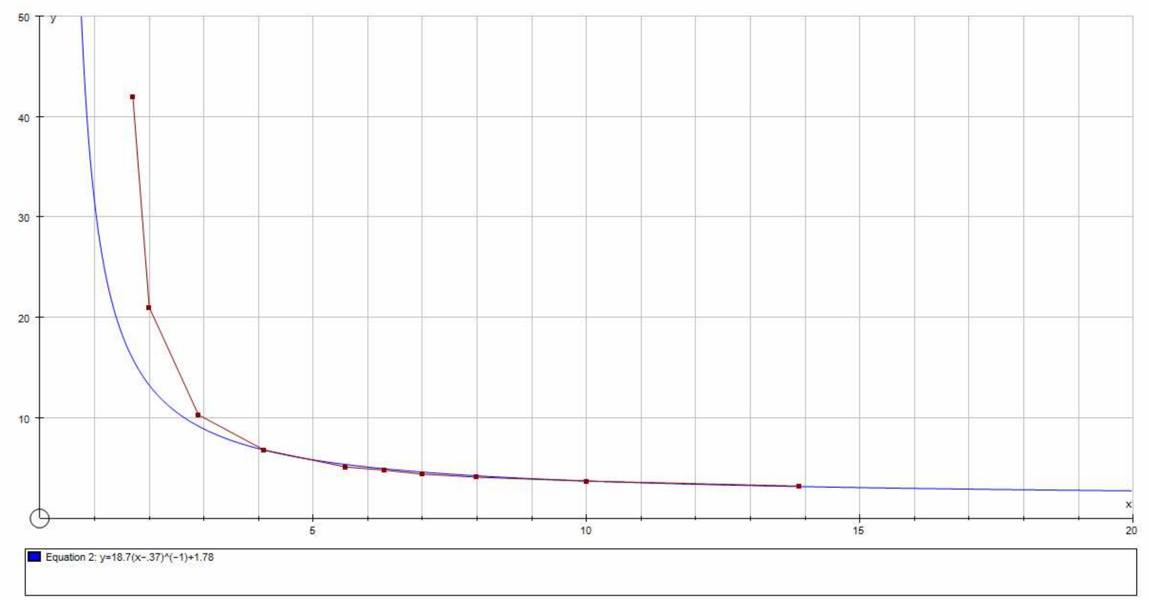
We will decrease the values of □ and □ and increase the value of □ in □=,□(□+□)-−□.+□

□=□□.□(,□−□.□□)-−□□.+□□.□□□□



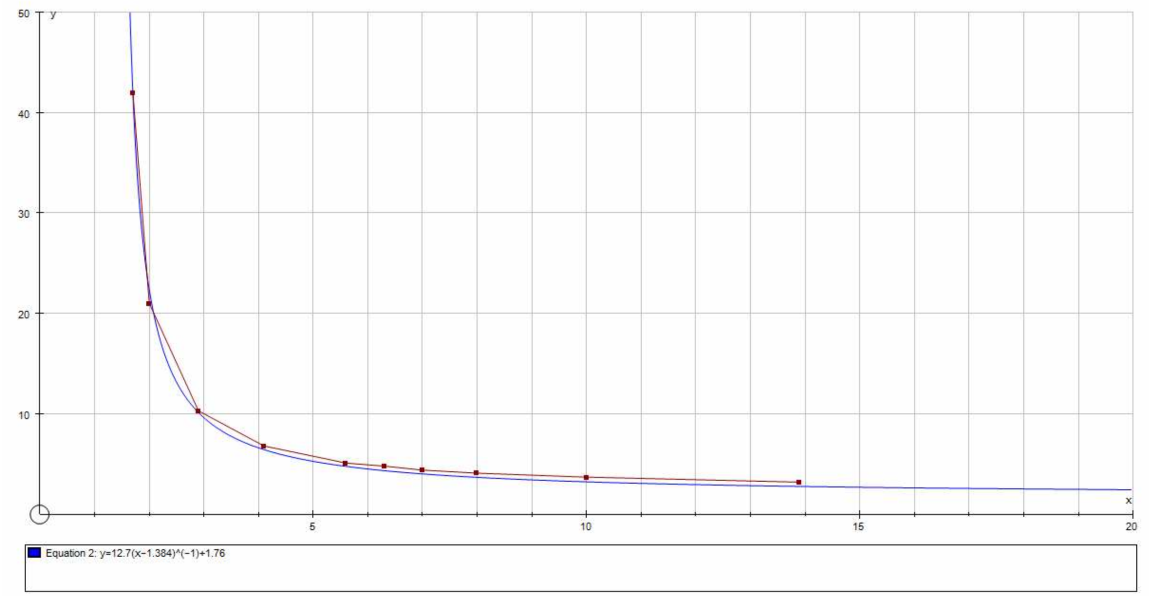
We will decrease the value of □ and increase the values of □ and □ in □=,□(□+□)-−□.+□

□=□□.□(,□−.□□)-−□□.+□□.□□□□



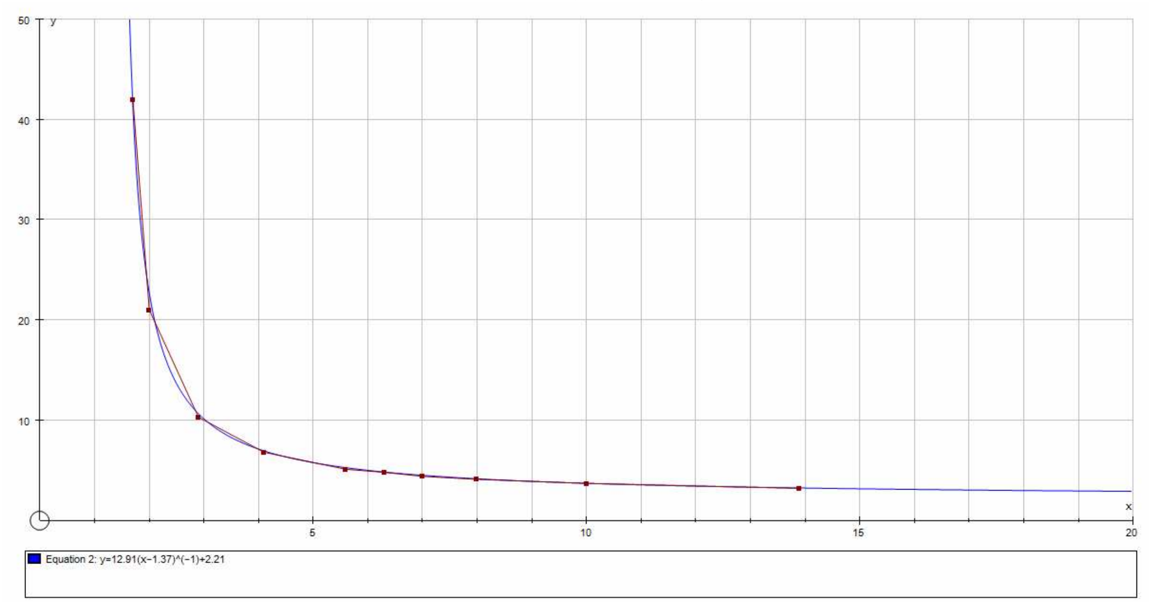
We will decrease the values of □ and □ increase the value of □ in □=,□(□+□)-−□.+□

□=□□.□(,□−□.□□□)-−□□.+□□.□□□□



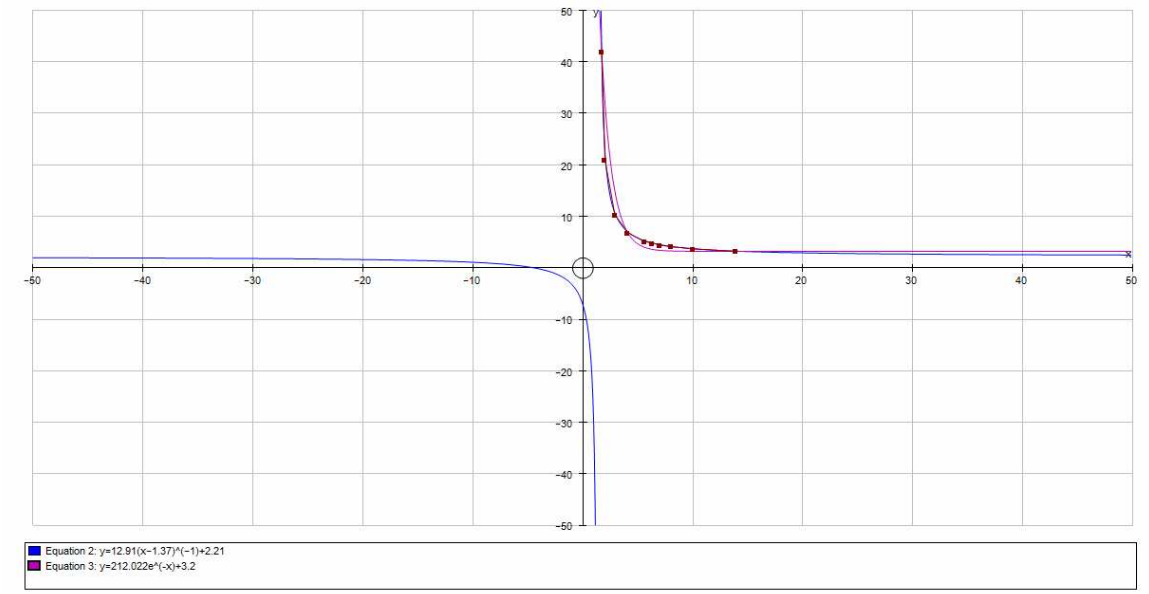
We will increase the values of □ and □ decrease the value of □ in □=,□(□+□)-−□.+□

The Final Result: □=□□.□□(,□−□.□□)-−□□.+□□.□□□□



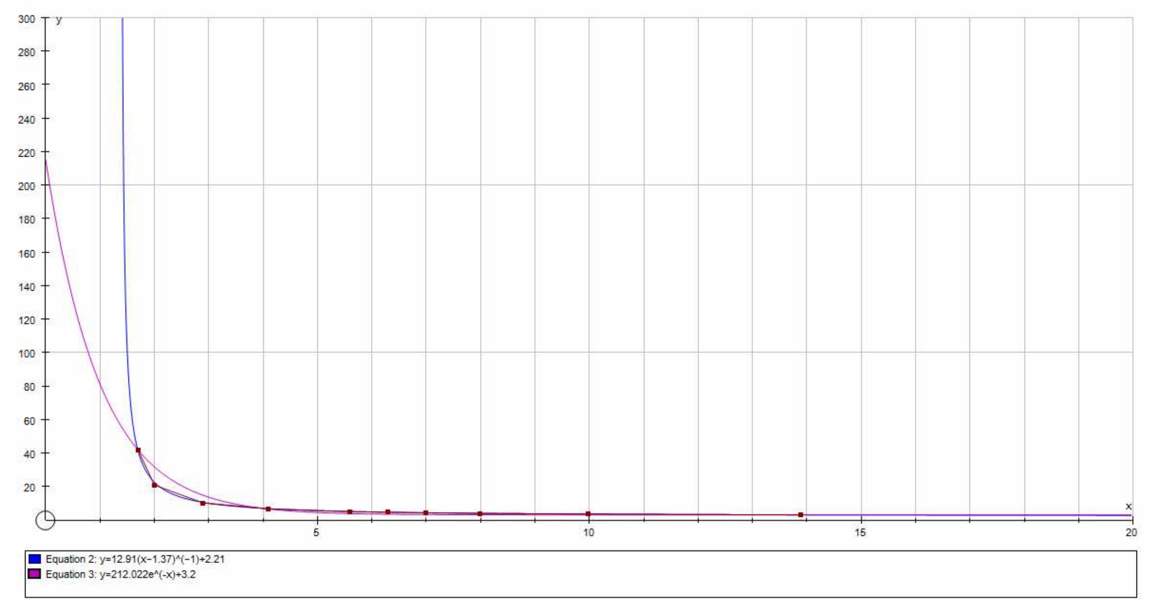
In the function, we can see that the shape is very similar to that of the original. Though, the function □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ doesn't run through any point as accurately as □=□□□□□□.□□□□□□,□-−□□.+□□.□□, the properties of the graph (area under the curve) will be more similar.

In order to get truly determine the more accurate function, we need to compare the graphs to each other. The following graph displays □=□□□□□□.□□□□□□,□-−□□.+□□.□□ as the purple line and □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ as the blue line.



The biggest difference between the graphs is that □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ has a reflected graph across, onto the negative axis. And as we can see on the above graph, the original 'large nut' graph function isn't reflected. This is because there cannot be a negative height or a negative number of drops.

If we take a close up for the graph (only the positive axis), we will see the following:



One difference is that the inverse exponential □=□□□□□□.□□□□□□,□-−□□.+□□.□□ is not quite as steep as the original 'large nut' graph, however □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ is as steep.

We can also see here that □=□□□□□□.□□□□□□,□-−□□.+□□.□□ crosses the □=□□□□□□□□ at approximately (□□,□□□□□□), which shows a clear limitation to the graph. On the other hand, □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ shows that it has 2 asymptotes – 1 on each axis. This thereby, portrays the original 'large nut' graph with more accuracy.

And, □=,□-−□. has a more similar area under the curve than □=,□-−□..

Area under the curve of the original 'large nut' graph:
74.91

Area under the curve of □=□□.□□(,□−□.□□)-−□□.+□□.□□□□
73.913

Area under the curve of □=□□□□□□.□□□□□□,□-−□□.+□□.□□
77.773

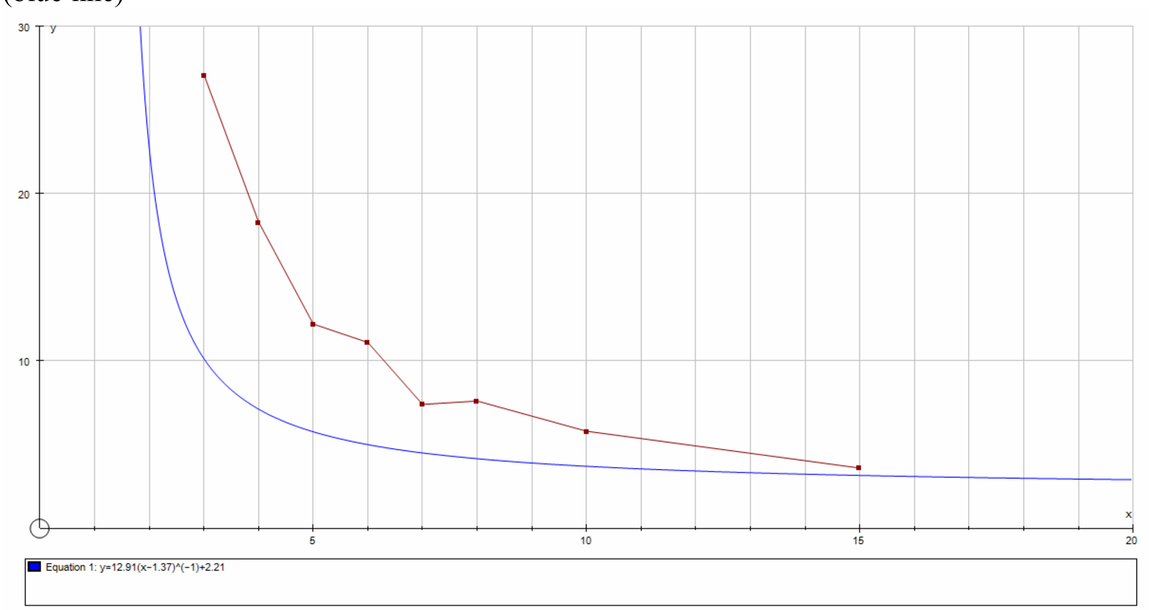
And because □=,□-−□.'s area under the curve is closer to the original's, it is more similar.

Now, let us apply the function that is most similar to our original graph □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ to other models of different sized nuts.

Medium Nuts

Height of drop in meters (□)	1.5	2.0	3.0	4.0	5.0	6.0	7.0	8.0	10.0	15.0
Number of drops (□)	-	-	27.1	18.3	12.2	11.1	7.4	7.6	5.8	3.6

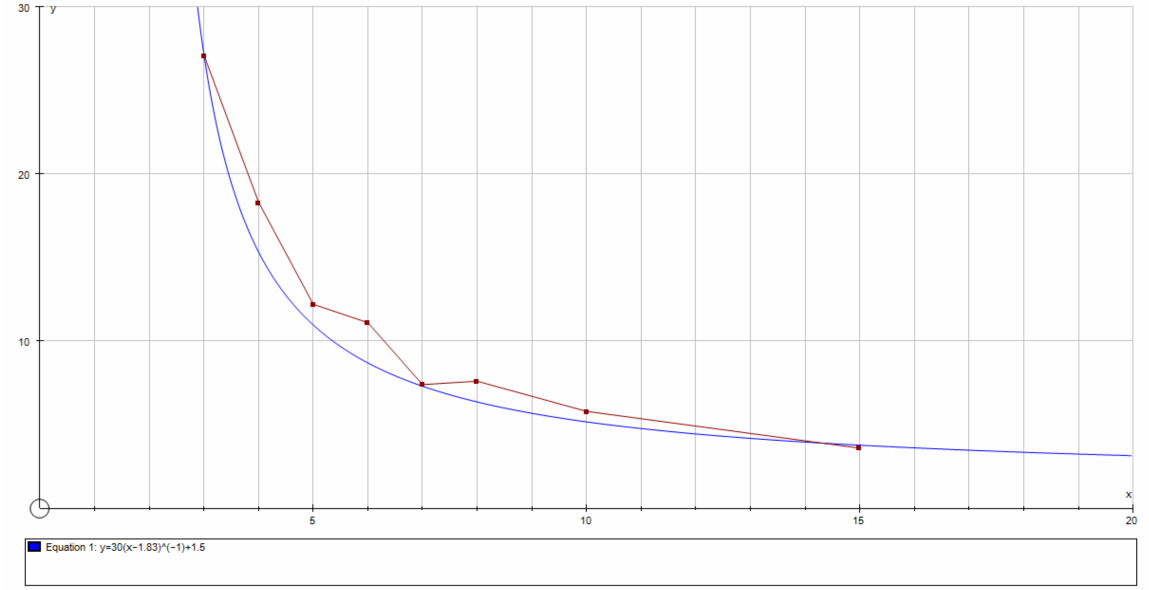
Graph showing the medium nuts (red line) versus □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ (blue line)



We will manipulate the graph (via the trial and error method) by manipulating the function in the □=,□(□+□)-−□.+□ format.

We need to increase the value of □ and decrease the values of □ and □ in □=,□(□+□)-−□.+□

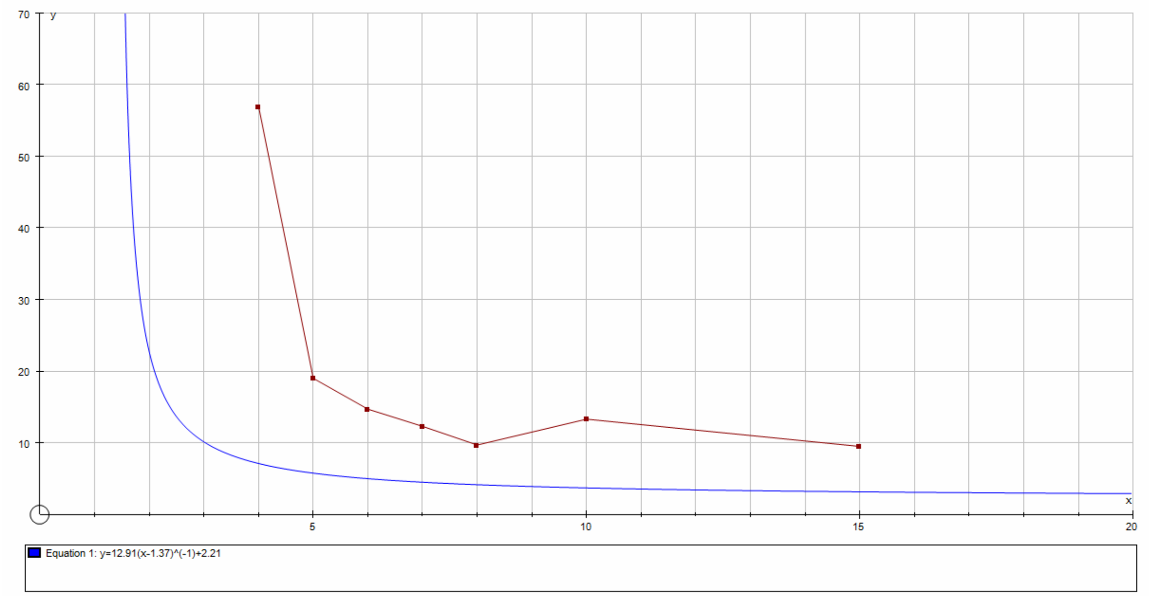
□=□□(,□−□.□□)-−□□.+□□.□□



Small Nuts

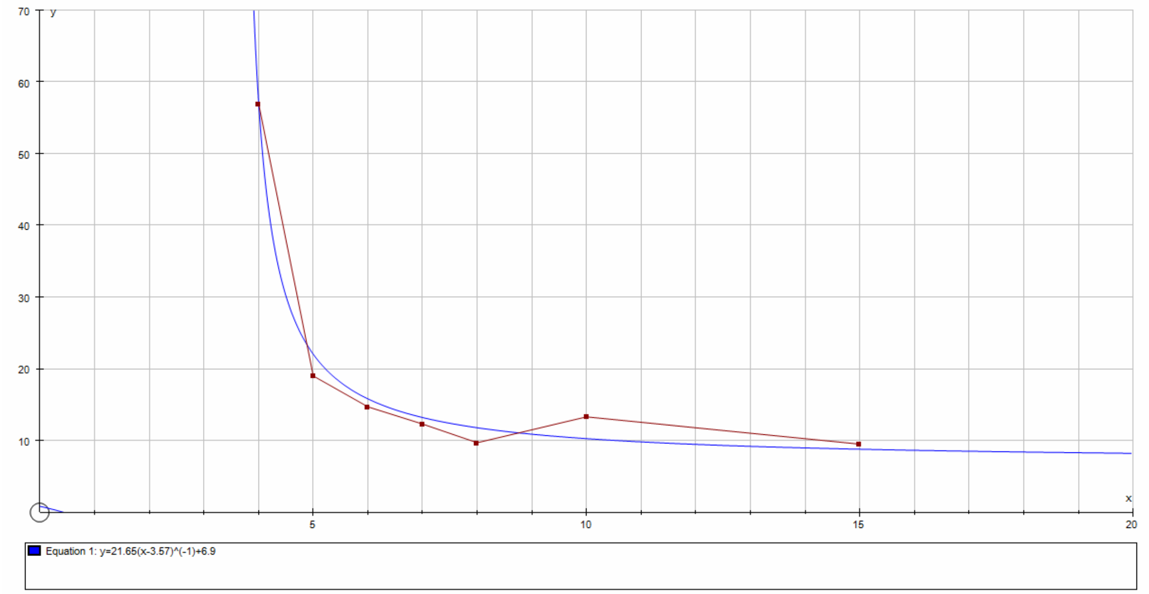
Height of drop in meters *(□)*	1.5	2.0	3.0	4.0	5.0	6.0	7.0	8.0	10.0	15.0
Number of drops *(□)*	-	-	-	57.0	19.0	14.7	12.3	9.7	13.3	9.5

Graph showing the medium nuts (red line) versus □=□□.□□(,□−□.□□)-−□.+□□.□□□□ (blue line)



We will manipulate the graph (via the trial and error method) by manipulating the function in the □=,□(□+□)-−□.+□ format.
We need to increase the values of □ and □ and decrease the value of □ in □=,□(□+□)-−□.+□

□=□□.□□(,□−□.□□)-−□□.+□□.□□



We are only able to get the general shape because of the structure of the original data points, which does not make very accurate. The limitations of the model is that it is done by trial and error. So it will never be exact no matter what we do to improve it. Also, because of the irregularity of the line graphs of the small and medium nut graphs, the models will never be able to fully replicate them. The medium and small nut graphs are not real curves, and because the models that have been created will always be curves, it will be impossible to create an ideal graph.

The function □=□□.□□(,□−□.□□)-−□□.+□□.□□□□ does not particularily depict the medium and small nut graphs very well. However, by using the method of trial and error, we have been able to manipulate the graphs into the general shape of the original graphs. As said earlier, an ideal graph will never be created, but by this method the slope, area under the curve, and other properties will be similar to that of the originals'. A limitation is that the graph will continue almost straight up. In reality and in the medium and small nut graphs, as the height of the drop (□−□□□□) decreases so will the number of drops (□−□□□□). In this case, the models will never work.