Body Mass Index

SL Math Internal Assessment Type 2

Body mass index (BMI) is a measure of one's body fat. BMI is calculated by taking one's weight (kg) and dividing the square of one's height (m). BMI does not directly measure a person's percentage of body fat but is used to determine of a person is underweight, normal, overweight or obese. BMI can be shown through the following formula:

$$BMI = \frac{weight\ (kg)}{height^2\ (m^2)}$$

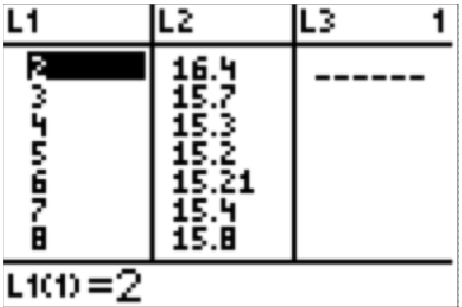
The table below gives the median (the average, found by arranging the values in order and then selecting the one in the middle). BMI for females of different ages in the United States in the year 2000.

Table 1:

Age (years)	BMI
2	16.40
3	15.70
4	15.30
5	15.20
6	15.21
7	15.40
8	15.80
9	16.30
10	16.80
11	17.50
12	18.18
13	18.70
14	19.36
15	19.88
16	20.40
17	20.85
18	21.22
19	21.60
20	21.65

(Source: http://www.cdc.gov)

Using a TI-84 calculator (or any other graphing tool) one can set up a list where list #1 is age (represented on the x-axis) and list #2 is the BMI (represented on the y-axis). Below is a screen shot of the ages and their respective BMIs.



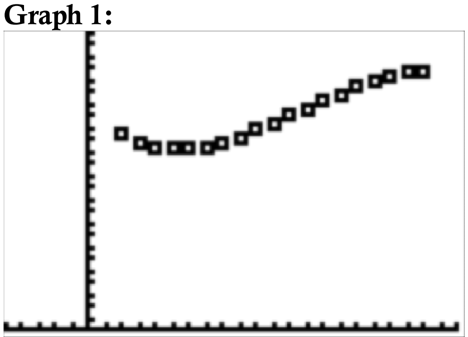
L1	L2	L3 1
2	16.4	------
3	15.7	
4	15.3	
5	15.2	
6	15.21	
7	15.4	
8	15.8	

L1(1)=2

Once the list is in place, activate the "stat plot" function and graph the points on the graph.

Then the screen shows the points plotted on the graph. Below is a screen shot of the plotted points:

Graph 1:



The parameters of the plotted points include the y-minimum (y=15.2 when x=5), the y-maximum (y=21.56 when x= 20) as well as the x-minimum (x=2) and the x-maximum (x=20).

This graph represents the BMIs of females in the US between the ages of 2 and 20.

From **Graph 1** one can conclude that the sine curve (which has been shifted to the right) best models the behavior represented on the graph.

The general sine function is defines as:

$$y = A\sin\ B(x - C) + D$$

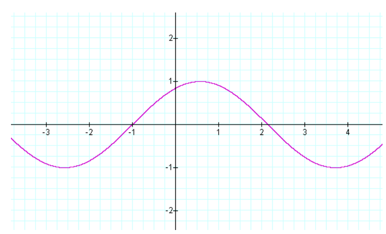


Figure 1: graph of the sine function: y=sin(x)

Where:
A determines the amplitude[1]
B determines the period[2]
C determines the horizontal translation[3]
D determines the vertical translation[4]

In order to find an equation (model) that fits the graph one has to find A, B, C and D.

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[1] Amplitude: the distance between the maximum and minimum y-values divided by 2
[2] Period: the interval from one repetition to the next
[3] Horizontal translation: a movement of the graph along the x-axis
[4] Vertical translation: movement of the graph along the y-axis

To find the **amplitude** (A) one has to subtract the minimum y-value from the maximum y-value. Then one must divide the result by 2 in order to find the half the distance between the two y-values. The equation is modeled below:

$$Amplitude = \frac{(y\max - y\min)}{2} \rightarrow \quad Amplitude = \frac{(21.65 - 15.20)}{2}$$

$$Amplitude = 3.225$$

The amplitude or A for this particular function is therefore 3.225.

Next, in order to find the **period** (B) of this function we must again subtract the y-minimum from the y-maximum and divide by 2. This will give us the amplitud e (as well as half the period), which we then have to multiply by 2 in order to find the full period. The period must be in radian, however, so we must again multiply the number but this time by $\pi/180$. This final number is then the period of the function. The calculations are represented below:

$$Period = \frac{(y\max - y\min)}{2} \times 2 \times \frac{\pi}{180} \rightarrow$$

$$\frac{(21.65 - 15.26)}{2} \times 2 \times \frac{\pi}{180} = 0.225$$

Therefore, the period or B for this function is 0.225.

Next, to find the **horizontal translation** (C) of the function one must find the peak and trough of the function. The maximum y-value of the function occurs at x=20, the peak is therefore 20. The minimum y-value of the function occurs at x=5, so the trough is 5. In order to find the horizontal translation we must add the peak and trough values and divide by 2. The number that is produced will indicate how far the function has moved from its original position, which is at (0,0).

$$Horizontal \quad translation = \frac{(peak + trough)}{2} \rightarrow \quad = \frac{(20 + 5)}{2} = 12.5$$

The horizontal translation is therefore 12.5.

Lastly, in order to find the **vertical translation** (D) we can observe that the general sine function passes through (0,0) and so we simply take the y-value of the median (or midpoint) between the peak and the trough and that number will indicate how far vertically the function as moved from its original position at (0,0) or the origin.

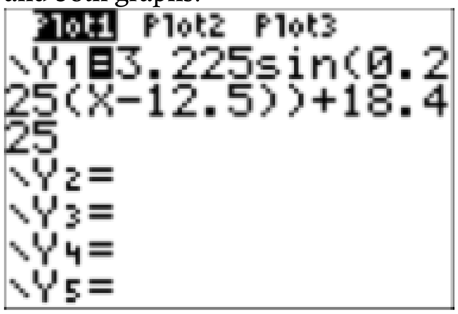
$$Vertical \quad translation = \frac{(y\max + y\min)}{2} \rightarrow \quad \frac{(21.65 + 15.2)}{2} = 18.425$$

And so the vertical translation of the function is 18.425.

Therefore the equation (or model) that fits **Graph 1** is:

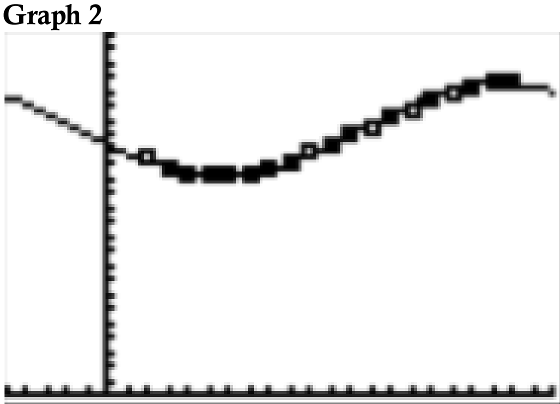
$$y = 3.225\sin 0.225(x - 12.5) + 18.425$$

To check whether the equation above fits the points plotted previously I graphed them both on the calculator. First, one must correctly (using brackets) plot the equation above and then the points plotted. Below are screenshots of the equation and both graphs.



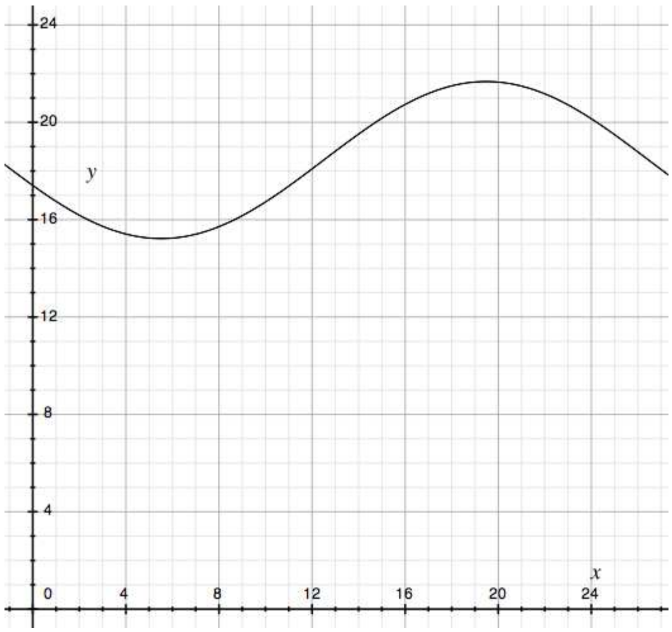
The equation above put into the calculator.

Graph 2



The data form **Table 1** plotted as well as the equation above. From **Graph 2** it is justified to say that the sine function best fits the data presented in **Table 1**.

The graph can also be plotted on the computer using the application “Graph”:
Graph 3



The sine function is not the only type of function that can model the data presented in **Table 1**. Again using the calculator the cosine function can also fit the data. This is because the sine function is merely the cosine function shifted to the right by $\pi/2$ radians as demonstrated below:

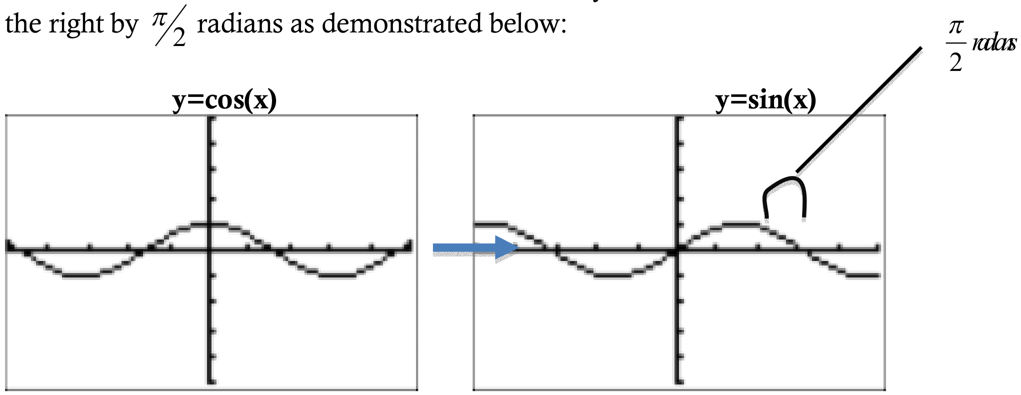


Figure 2: the cosine and sine functions graphed on the TI calculator

The general cosine function is similar to the sine function:

$$y = A\cos B(x - C) + D$$

In this case, the values of A, B and D will remain the same as the amplitude, period and horizontal translation of the function will not change. The only difference between the cosine and sine function is the vertical translation described above and so it is the vertical translation that will be a different value.

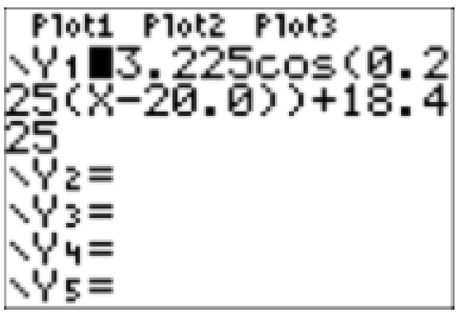
Therefore A = 3.225, B = 0.225 and D = 18.425.

In order to find the vertical translation we can observe that the general cosine function starts at x = 0 (which is the function's peak) shown in **Figure 2**. Looking back at **Graph 2** we can notice that the greatest y-value is at x = 20. This tells us that the peak moved from x = 0 to x = 20. Therefore the horizontal translation is 20.

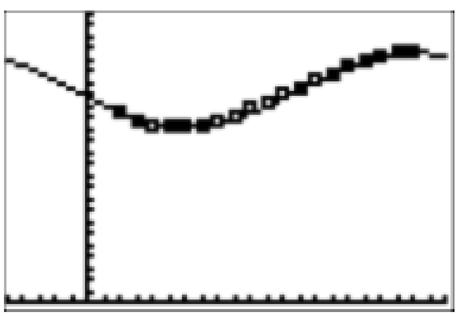
This makes the cosine equation for the data presented in **Table 1**:

$$y = 3.225 \cos 0.225 (x - 20.0) + 18.425$$

To check whether the equation above fits the points plotted previously I graphed them both on the calculator. First as with the sine function, one must correctly (using brackets) plot the equation above and then the points plotted. Below are screenshots of the equation and both graphs.



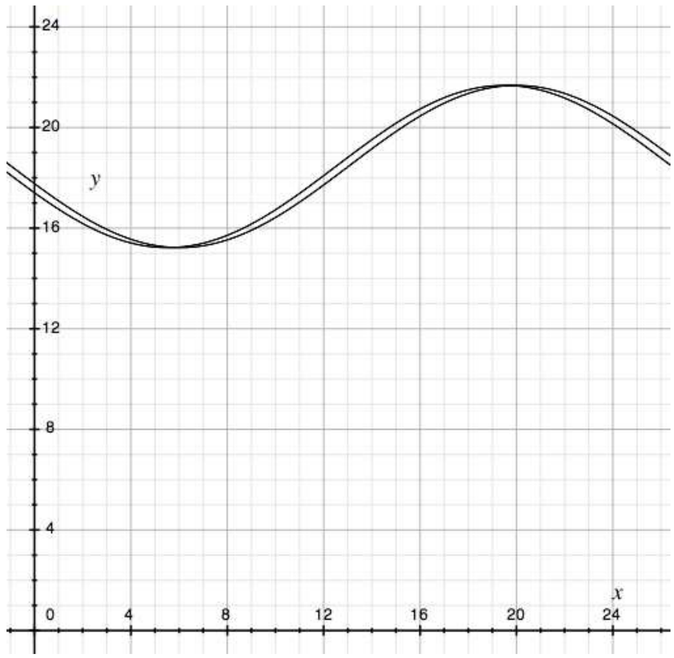
The cosine equation put into the calculator.



The data from **Table 1** plotted as well as the cosine equation graphed. As one can see, the function does not go through less of the points plotted than the sine function.

The following graph is both the sine and cosine functions plotted together. Notice the inconsistencies; the graphs are not identical.

Graph 4



If we were to estimate the BMI of a 30-year old woman in the US using our model her BMI would be 18.64 kg/m^2. In order to get this number we substitute the x in the function $y = 3.225 \sin 0.225(x - 12.5) + 18.425$ with 30 (the age of the woman) as follows:

$$y = 3.225 \sin 0.225(30 - 12.5) + 18.425$$
$$y = 18.64$$

This result would not be reasonable or very convincing for many BMI researches. This is because in most cases an average 30 year old woman is not in better shape than a 13 year old girl. According to the different categories of BMI, a women that has an 18.64 BMI is almost underweight and it is safe to say that the average 30 year old woman is not underweight. This means that this model is only accurate to a certain age.

From the internet I found statistics regarding the BMI of women in the United Kingdom from the ages of 0-15 in 2002.

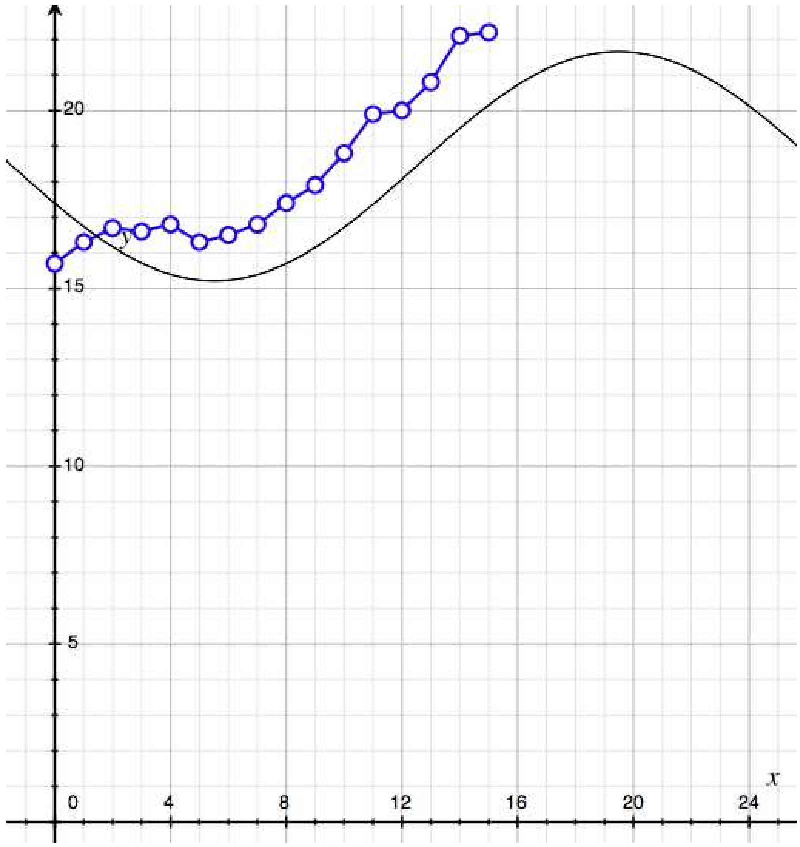
Table 2

Age (years)	BMI
0	15.7
1	16.3
2	16.7
3	16.6
4	16.8
5	16.3

6	16.5
7	16.8
8	17.4
9	17.9
10	18.8
11	19.9
12	20.0
13	20.8
14	22.1
15	22.2

(Source: http://www.erpho.org.uk/viewResource.aspx?id=9001)

Graph 5



These plots do not correspond with the curvature of the sine function used for the data in **Table 1**. Also, the data in **Table 2** does not fit the original equation (model) as the data only ranges from 0-15 and not from 2-20 (this required an adjustment of the x-axis). Another limitation of the model is that the data in **Table 1** are BMI medians and the data in **Table 2** are BMI means. Therefore, the original equation, $y = 3.225 \sin 0.225(x - 12.5) + 18.425$, does not apply to all sets of BMI data but has to be adjusted in order to fit specific data.