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Practice IB Internal Assessment

Introduction: The purpose of interest is for a bank to pay an individual for the use of their money. Interest therefore represents one's return on the investment. To calculate *n* interest compoundings per year, one must utilize the formula:

$$A = P(1 + r/n)nt$$

where A= the investment in dollars
P= the principal
r= the annual interest rate (in decimal form)
t= the number of years the principal is accounted for
n= the frequency of interest compounding.

1. Alan invests $1000 at an interest rate of 12% per year. Copy and complete *Table 1* which shows A, the value of the investment in dollars after t years, assuming that the interest is compounded yearly.

   One must utilize the formula for Compound Interest in order to determine the answer. In the case of Alan, his principle (P) amount of money is $1000, and he collects interest at a rate (r) of 12% per year. To determine the value of the investment in dollars after t years in order to satisfy Table 1, the formula necessary is:

$$A = 1000\,(1 + .12/1)^{(1)(t)}$$

   where P= $1000, r= 0.12, and n=1 because interest is being compounded annually or once a year, and t= the number of years the money is present in the account.

t=0, $A = 1000(1 + 0.12/1)^{(1)(0)} = 1000(1 + 0.12)^{0} = 1000(1.12)^{0} = 1000(1) = 1000$
t=1, $A = 1000(1 + 0.12/1)^{(1)(1)} = 1000(1 + 0.12)^{1} = 1000(1.12)^{1} = 1000(1.12) = 1120$
t=2, $A = 1000(1 + 0.12/1)^{(1)(2)} = 1000(1 + 0.12)^{2} = 1000(1.12)^{2} = 1000(1.2544) = 1254.40$
t=5, $A = 1000(1 + 0.12/1)^{(1)(5)} = 1000(1 + 0.12)^{5} = 1000(1.12)^{5} \approx 1000(1.76234) \approx 1762.34$
t=10, $A = 1000(1 + 0.12/1)^{(1)(10)} = 1000(1 + 0.12)^{10} = 1000(1.12)^{10} \approx 1000(3.10585) \approx 3105.85$
t=20, $A = 1000(1 + 0.12/1)^{(1)(20)} = 1000(1 + 0.12)^{20} = 1000(1.12)^{20} \approx 1000(9.64629) \approx 9646.29$
t=30, $A = 1000(1 + 0.12/1)^{(1)(30)} = 1000(1 + 0.12)^{30} = 1000(1.12)^{30} \approx 1000(29959.92) \approx 29959.92$
*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.

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Table 1 Alan's Investment of $1000 at an Interest Rate of 12% Per Year Compounded Yearly

t	0	1	2	5	10	20	30
A	1000	1120	1254.40	1762.34	3105.85	9646.29	29959.92

Table 1 represents Alan's investment of $1000 at an interest rate of 12% per year over a period of 1, 2, 5, 10, 20, and 30 years. As the data illustrates, it is evident that the longer his investment is in the account, the larger his final sum becomes. This effect can be seen with an extremely large sum of $29,959.92 being accumulated in the thirtieth year of investment, almost thirty times what was originally put into the investment account. Of the $29,959.92 that is in the account at the end of 30 years, $28,959.92 represents the interest earned with Alan's $1000 principal investment.

2. (a) Show that if interest of 12% per year is compounded monthly it is equivalent to an interest rate of approximately 12.68% per year if the interest is compounded yearly.

To calculate a 12% interest rate (r) with a monthly frequency, one must substitute 12 for n instead of 1 because there are 12 months in a year, resulting in the following formula:

$$A= P(1+0.12/12)^{(12)(t)}$$

To calculate a 12.68% interest rate (r) with a yearly frequency, one must the value 1 for n, resulting in the following formula:

$$A= P(1+0.1268/1)^{(1)(t)}$$

12% Interest per Year Compounded Monthly

$r= 0.12$, $n= 12$

$A= P(1+0.12/12)^{(12)(t)}$
$A= P(1+0.01)^{12t}$
$A= P(1.01)^{12t}$
$\boldsymbol{A\approx P(1.12683)^t}$

Example:

$P=$ \$1000, t=1, t=3, t=5

t=1, $A\approx 1000(1.12683)^1$
$\approx 1000(1.12683)$
$\approx$ **1126.83**
t=3, $A\approx 1000(1.12683)^3$
$\approx 1000(1.43079)$
$\approx$ **1430.79**
t=5, $A\approx 1000(1.12683)^5$
$\approx 1000(1.81674)$
$\approx$ **1816.74**

12.68% Interest per Year Compounded Yearly

$r= 0.1268$, $n= 1$

$A= P(1+0.1268/1)^{(1)(t)}$
$A= P(1+0.1268)^t$
$A= P(1.1268)^t$
$\boldsymbol{A= P(1.12680)^t}$

Example:

$P=$ \$1000, t=1, t=3, t=5

t=1, $A\approx 1000(1.12680)^1$
$\approx 1000(1.12680)$
$\approx$ **1126.80**
t=3, $A\approx 1000(1.12680)^3$
$\approx 1000(1.43067)$
$\approx$ **1430.67**
t=5, $A\approx 1000(1.12680)^5$
$\approx 1000(1.81649)$
$\approx$ **1816.49**

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It is clear that 12% interest per year compounded monthly is approximately the same as 12.68% interest per year compounded annually. At first, each formula produces the same interest, however, as time goes on, the exactness between the two rates and frequencies becomes slightly skewed. This, however, is justifiable because it is impossible for these two formulas to yield the exact same results, which is why over time the values differ by less than 1%. Though there is some discrepancy between the two sums, the calculations of a 12.68% interest rate compounded yearly and a 12% interest rate compounded monthly are extremely close in value and will remain this similar even after a large time periods of time.

(b) Copy and complete *Table 2* which shows *B*, the value of the investment in dollars after t years, assuming that the interest is compounded monthly.

To calculate the value of the investment in dollars (*B*) after *t* years, assuming that the interest is compounded monthly (n= 12), one must once again utilize the formula:

$$B= P\,(1 + r/12)^{12t}$$

In Alan's case, P= \$1000 and r= 0.12 so the formula would be altered to look as such:

$$B= 1000(1+0.12/12)^{12t}= 1000(1.01)^{12t}$$

t=0, A= $1000(1.01)^{(12)(0)} = 1000(1.01)^0= 1000(1)$
t=1, A= $1000(1.01)^{(12)(1)} = 1000(1.01)^{12}\approx 1000(1.12683) \approx 1126.83$
t=2, A= $1000(1.01)^{(12)(2)} = 1000(1.01^{24}\approx 1000(1.26973) \approx 1269.73$
t=5, A= $1000(1.01)^{(12)(5)} = 1000(1.01)^{60}\approx 1000(1.81670) \approx 1816.70$
t=10, A= $1000(1.01)^{(12)(10)} = 1000(1.01)^{120}\approx 1000(3.30039) \approx 3300.39$
t=20, A= $1000(1.01)^{(12)(20)} = 1000(1.01)^{240}\approx 1000(10.89255) \approx 10892.55$
t=30, A= $1000(1.01)^{(12)(30)} = 1000(1.01)^{360}\approx 1000(35949.64) \approx 35949.64$
*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.

Table 2 Alan's Investment of \$1000 at an Interest Rate of 12% Per Year Compounded Monthly

t	0	1	2	5	10	20	30
B	1000	1126.83	1269.73	1816.70	3300.39	10892.55	35949.64

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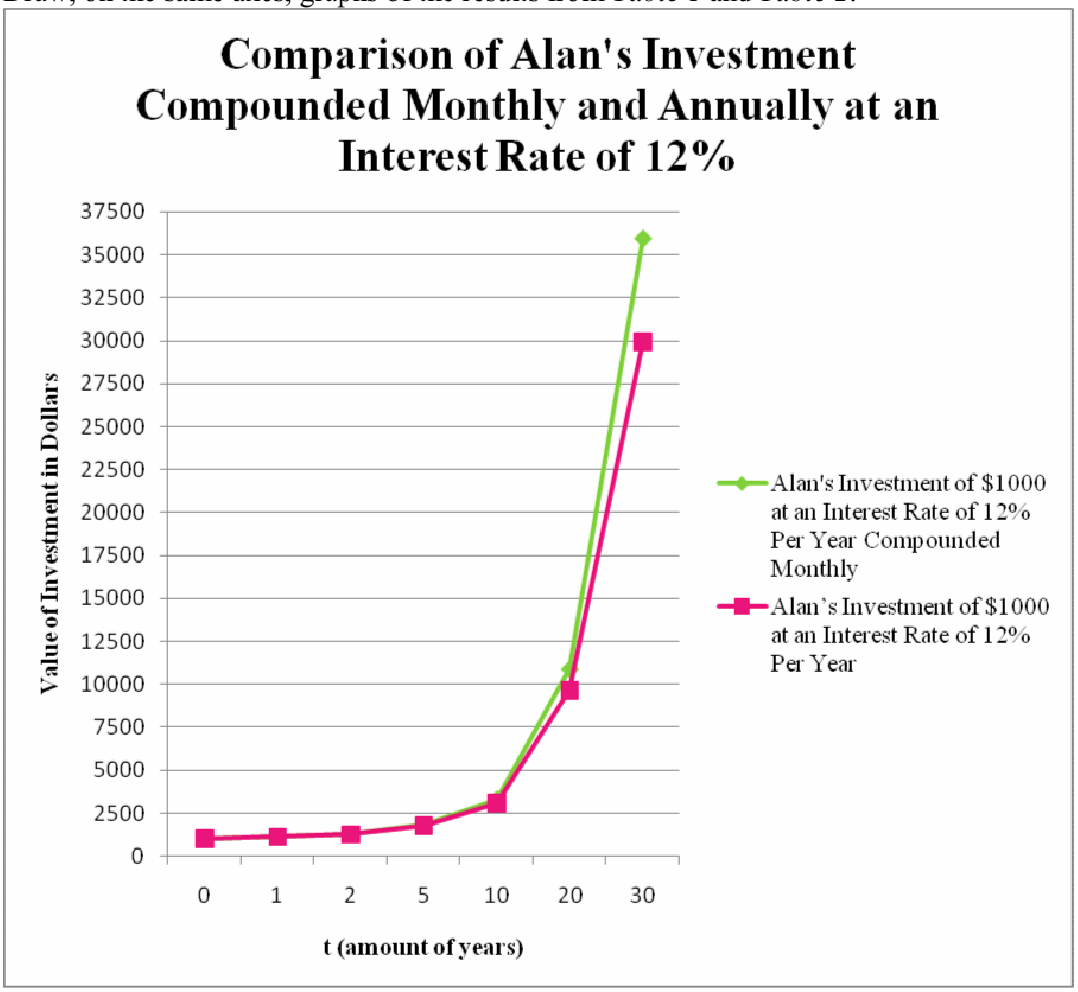
3. Draw, on the same axes, graphs of the results from *Table 1* and *Table 2*.



Based on the visual representation of the difference between monthly and annual frequencies, it is clear that Alan's investment gains more interest and value when compounded monthly. As a result of the monthly frequency, Alan's interest is collected more frequently, adding up to a larger final value for his investment.

4. Investigate the effect on the value of the investment if interest is compounded more frequently.

For Alan's investment, P= \$1000. To calculate the value of Alan's investment at a fixed 12% interest rate (r) compounded in different frequencies, one must change the value of n.

Annually
Refer to *Table 1*.

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Bi-annually

With bi-annual compounding, the formula for Alan's investment would look as follows:

$$A = 1000(1+0.12/2)^{2t} = 1000(1.06)^{2t}$$

$t = 0$, $A = 1000(1.06)^{(2)(0)} = 1000(1.06)^0 = 1000(1) = 1000$

$t = 1$, $A = 1000(1.06)^{(2)(1)} = 1000(1.06)^2 = 1000(1.1236) = 1123.60$

$t = 2$, $A = 1000(1.06)^{(2)(2)} = 1000(1.06)^4 \approx 1000(1.26248) \approx 1262.48$

$t = 5$, $A = 1000(1.06)^{(2)(5)} = 1000(1.06)^{10} \approx 1000(1.79085) \approx 1790.85$

Quarterly

With quarterly compounding, the formula for Alan's investment would look as follows:

$$A = 1000(1+0.12/4)^{4t} = 1000(1.03)^{4t}$$

$t = 0$, $A = 1000(1.03)^{(4)(0)} = 1000(1.03)^0 = 1000(1) = 1000$

$t = 1$, $A = 1000(1.03)^{(4)(1)} = 1000(1.03)^4 \approx 1000(1.12551) \approx 1125.51$

$t = 2$, $A = 1000(1.03)^{(4)(2)} = 1000(1.03)^8 \approx 1000(1.26677) \approx 1266.77$

$t = 5$, $A = 1000(1.03)^{(4)(5)} = 1000(1.03)^{20} \approx 1000(1.80611) \approx 1806.11$

Monthly

Refer to *Table 2*.

Comparison of Alan's Investment at 12% Interest Rate Compounded at Different Frequencies*

t	Annually	Bi-annually	Quarterly	Monthly
0	1000	1000	1000	1000
1	1120	1123.60	1125.51	1126.83
2	1254.40	1262.48	1266.77	1269.73
5	1762.34	1790.85	1806.11	1816.70

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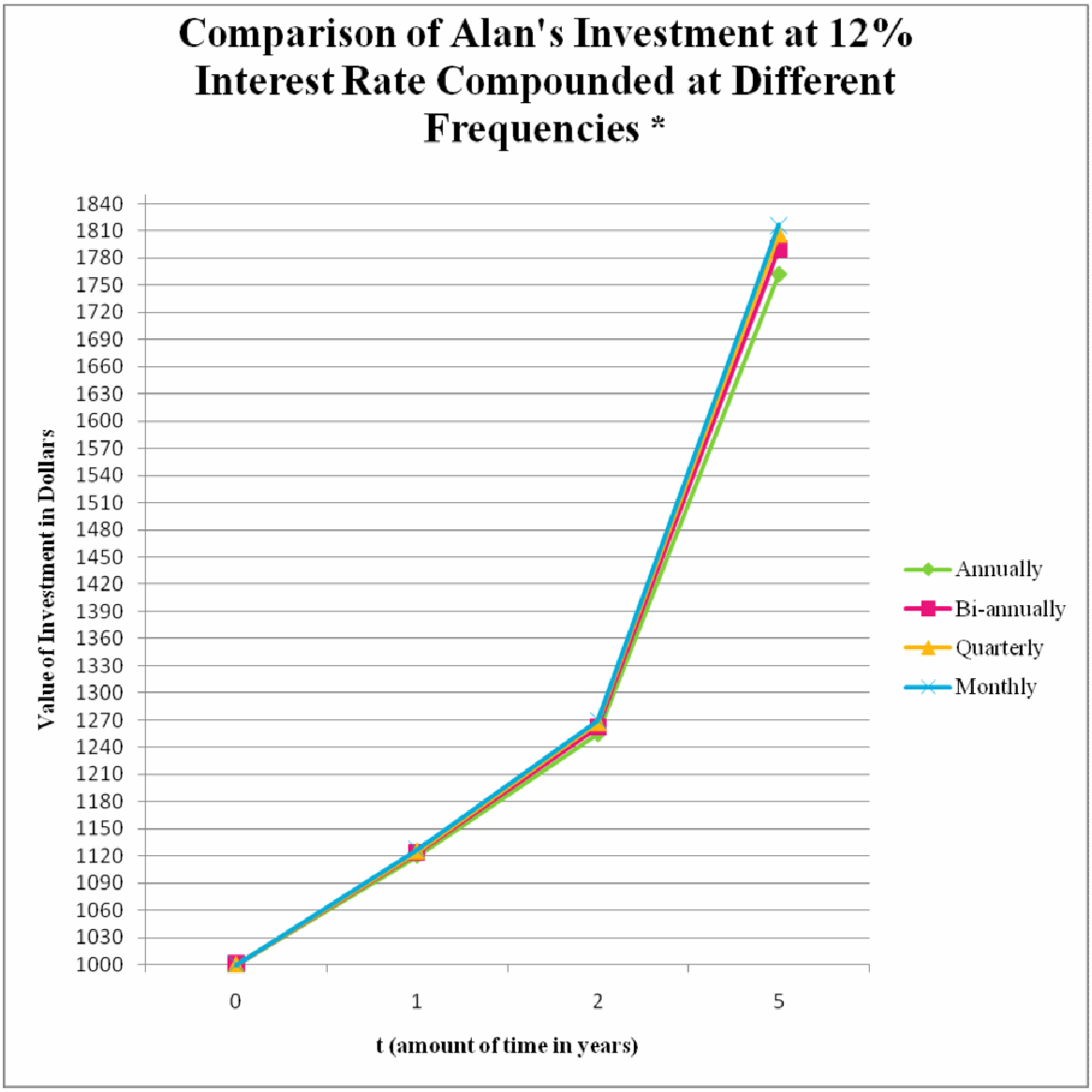

*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.

Based on the investigation of the effects of different frequencies when using compound interest on Alan's investment, it is clear that the more frequently interest is compounded, the higher the value of the investment. This is evident by comparison of annual and monthly compounding, where at first there is only about a six dollar difference, however, as time goes on, the difference between the two frequencies grows larger. Additionally, this finding is especially evident in the graph pictured above, which visually illustrates the large difference in the interest earned when compounded annually, bi-annually, quarterly, and monthly. The

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data from each frequency of compounding covers a wide range, and thus the lower frequencies remain closely similar while the higher frequencies vary in the value of Alan's investment.

5. Barbara invests $200 at the start of each year for a period of 5 years, at an interest rate of 12% per year. Thereafter, she stops adding $200 at the start of each year but leaves her investment in the bank to earn interest at 12% per year.

   Table 3 shows *C*, the value of the investment in dollars after *t* years, assuming that the interest is compounded yearly.

   Table 3 Barbara's Investment of $200 with an Additional $200 Added to the Sum the First Five Years at an Interest Rate of 12% Compounded Yearly

t	0	1	2	5	10	20	30
C	200.00	424.00	674.88	1423.04	2507.88	7789.09	24191.72

   Draw a new table showing the value *D*, Barbara's investment in dollars after *t* years, assuming that the interest in compounded monthly. Label it *Table 4*.

   To calculate Barbara's investment in dollars with a monthly compounding, one must substitute 12 for the value of n because there are 12 months in a year. In addition to changing the value of n, one must account for the $200 added to the sum for the first five years, causing the formula for the first five years of her investment appear as follows:

$$D= [P(1+0.12/12)^{(12)(t)}]+200$$

   where the additional $200 is added to the final sum of the end of year interest collected. Thus, the formula for Barbara's investment is:

$$D= [200(1+0.12/12)^{(12)(t)}] + 200= [200(1.01)^{12t}] + 200$$

   however, for the first five years, one must substitute 1 for *t* for the first five years because of the additional $200 Barbara adds to her investment. Also, one should not start adding $200 until after *t*= 0, because it only consists of the primary $200 invested.

   When:

   $t= 0$, $D= 200(1.01)^{(12)(0)} = 200(1.01)^0= 200(1) = 200$

   $t= 1$, $D= \{[200(1.01)^{(12)}]+200 = \{[200(1.01)^{12}]+200\}\approx 225.365006+200\approx 425.37$

   ↑ *P* ↑ *r* ↑ *n* ↑ Additional 200 ↑ Final at End of Year

   $t= 2$, $D= \{[425.37(1.01)^{(12)}]+200\}\approx 479.32 + 200\approx 679.32$

   ↑ Amount from Prior Year

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t= 3, D= {[679.32(1.01)^{12}]+200}≈ 765.47 + 200≈ 965.47
t= 4, D= {[965.47(1.01)^{12}]+200}≈ 1087.92 + 200 ≈ 1287.92
** One must calculate the third and fourth year of Barbara's investment in order to determine the principle (P) at the end of the fifth year.
t= 5, D= [1287.92(1.01)^{12}] ≈ 1451.26
*On the fifth year, Barbara stops adding $200 to her investment; therefore one must stop adding an additional 200 to the equation. The sum accumulated at the end of the fifth year becomes the principal (P) for further years to come, however, when trying to determine the interest earned from this amount, one must now subtract 5 from t in order to remove the first 5 years of Barbara's investment since there was no set principal during this time period. The formula for Barbara's monthly compound interest rate of 12% with her previous investment is as follows:

$$D= P(1.01)^{[(t-5)(12)]}$$

where P= Barbara's investment at the end of the fifth year
t= the number of years minus the first five years and multiplied by twelve to correspond to the monthly frequency.

thereby producing the formula:

$$D= 1451.25(1.01)^{(12t-60)}$$

Thus, one can now complete the necessary findings for *Table 4*
t= 10, D= 1451.26(1.01)$^{[(12)(10)-60]}$ = 1451.26(1.01)^{60} ≈ 1451.26(1.816696699) ≈ 2636.50
t= 20, D= 1451.26(1.01)$^{[(12)(20)-60]}$ = 1451.26(1.01)^{180}≈ 1451.26(5.995801975) ≈ 8701.47
t= 30, D= 1451.26(1.01)$^{[(12)(30)-60]}$ = 1451.26(1.01)^{300}≈ 1451.26(19.78846626) ≈ 28718.21

Table 4 Barbara's Investment of $200 with an Additional $200 Added to the Sum the First Five Years at an Interest Rate of 12% Compounded Monthly

t	0	1	2	5	10	20	30
D	200.00	425.37	679.31	1451.25	2636.50	8701.47	28718.21

*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.

6. If Barbara starts her investment at the beginning of 1990 so that her final premium of $200 is paid at the beginning of 1994 and Alan's single investment is made at the beginning of 1994, draw on the same axes the graphs of the values of Alan's and Barbara's investments during the interval of years from 2000 to 2010. You may assume that interest is compounded yearly.

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Explain why Barbara's investment is worth more than Alan's.

To determine the value of Barbara's investment of $200 compounded yearly at an interest rate of 12% with an additional $200 is added each year for the first five years, one must utilize the same formula that was used in question 5, however one must now substitute 1 for *n* to make the frequency a yearly rate, thereby creating the following formula:

$$A = \{[P(1.12)^{t}]+200\}$$

however, one must remember to not add 200 until $t = 1$, and substitute 1 for *t* and *n* for the first five years because of the additional $200 Barbara adds to her investment.

One must first calculate the money accumulated in the first five years of Barbara's investment to determine the final principal (*P*).

$t = 0$, $A = 200(1.12)^{(1)(0)}] = 200\ (1.12)^{0} = 200\ (1) = 200$

$t = 1$, $A = \{[200(1.12)^{(1)}]+ 200\} = 224+200 = 424$

$t = 2$, $A = \{[424(1.12)^{(1)}]+ 200\} \approx 474.88+200 \approx 674.88$

$t = 3$, $A = \{[674.88(1.12)^{(1)}]+ 200\} \approx 755.8656+200 \approx 955.87$

$t = 4$, $A = \{[955.87(1.12)^{(1)}]+ 200\} \approx 1070.5695+200 \approx 1270.57$

$t = 5$, $A = [1270.57(1.12)^{(1)}] \approx 1270.57(1.12) \approx 1423.04$

↑

final principal (*P*)

Now one must utilize the regular yearly compounding interest formula, but must not include the first five years of the investment, creating the following formula:

$$A = 1423.04(1.12)^{[(t-5)(1)}$$

To calculate the value of Barbara's investment during the interval of years from 2000 to 2010, one must calculate how many years this interest compounding is taking place.

$$2000 - 1990 = 10$$

As a result, one must start with $t = 10$, using the final principal (*P*), and add 1 with every additional year.

Therefore:

Year:

2000 → t= 10, $A = 1423.04\ (1.12)^{[(10-5)(1)]} = 1423.04\ (1.12)^{5} \approx 1423.04\ (1.762341683) \approx 2507.88$

2001 → t= 11, $A = 1423.04\ (1.12)^{[(11-5)(1)]} = 1423.04\ (1.12)^{6} \approx 1423.04\ (1.973822685) \approx 2808.83$

2002 → t= 12, $A = 1423.04\ (1.12)^{[(12-5)(1)]} = 1423.04\ (1.12)^{7} \approx 1423.04\ (2.210681407) \approx 3145.89$

2003 → t= 13, $A = 1423.04\ (1.12)^{[(13-5)(1)]} = 1423.04\ (1.12)^{8} \approx 1423.04\ (2.475963176) \approx 3523.39$

2004 → t= 14, $A = 1423.04\ (1,12)^{[(14-5)(1)]} = 1423.04\ (1.12)^{9} \approx 1423.04\ (2.773078757) \approx 3946.20$

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2005 → t= 15, A= 1423.04 $(1.12)^{[(15-5)(1)]}$ = 1423.04 $(1.12)^{10}$ ≈ 1423.04 (3.105848208) ≈ 4419.75

2006 → *t*= 16, *A*= 1423.04 $(1.12)^{[(16-5)(1)]}$ = 1423.04 $(1.12)^{11}$ ≈ 1423.04 (3.478549993) ≈ 4950.12

2007 → *t*= 17, *A*= 1423.04 $(1.12)^{[(17-5)(1)]}$ = 1423.04 $(1.12)^{12}$ ≈ 1423.04 (3.895975993) ≈ 5544.13

2008 → *t*= 18, *A*= 1423.04 $(1.12)^{[(18-5)(1)]}$ = 1423.04 $(1.12)^{13}$ ≈ 1423.04 (4.363493112) ≈ 6209.43

2009 → *t*= 19, *A*= 1423.04 $(1.12)^{[(19-5)(1)]}$ = 1423.04 $(1.12)^{14}$ ≈ 1423.04 (4.887112285) ≈ 6954.56

2010 → *t*= 20, *A*= 1423.04 $(1.12)^{[(20-5)(1)]}$ = 1423.04 $(1.12)^{15}$ ≈ 1423.04 (5.473565759) ≈ 7789.10

Table 5 Barbara's Investment of $200 with an Additional $200 Added to the Sum the First Five Years at an Interest Rate of 12% Compounded Yearly in the Interval of Years 2000 to 2010*

Year	Amount
2000	2507.88
2001	2808.83
2002	3145.89
2003	3523.39
2004	3946.20
2005	4419.75
2006	4950.12
2007	5544.13
2008	6209.43
2009	6954.56
2010	7789.10

*Investment started in 1990

*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.

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To calculate the value of Alan's investment, one must utilize the regular formula of yearly compounded interest and apply it to Alan's principal:

$$A = P\,(1.12)^{nt} \rightarrow A = 1000\,(1.12)^{t}$$

Also, one must calculate the starting year, making sure that the right amount of time is compounded.

$$2000 - 1994 = 6$$

Therefore, when it is the year 2000, t= 6. One must add one to each corresponding year in order to correctly determine the amount of interest and money gained.

Year:

When:

2000 → t= 6, A=1000(1.12)^{6}≈ 1000 (1.973822685)≈ 1973.82
2001 → t= 7, A=1000(1.12)^{7}≈ 1000 (2.210681407)≈ 2210.68
2002 → t= 8, A=1000(1.12)^{8} ≈ 1000 (2.475963176)≈ 2475.96
2003 → t= 9, A=1000(1.12)^{9}≈1000 (2.773078757)≈ 2773.08
2004 → t= 10, A= 1000(1.12)^{10}≈ 1000 (3.105848208)≈ 3105.85
2005 → t= 11, A=1000(1.12)^{11}≈ 1000 (3.478549993)≈ 3478.55
2006 → t= 12, A=1000(1.12)^{12}≈ 1000 (3.895975993)≈3 895.98
2007 → t= 13, A=1000(1.12)^{13} ≈ 1000 (4.363493112)≈ 4363.49
2008 → t= 14, A=1000(1.12)^{14} ≈ 1000 (4.887112285)≈ 4887.11
2009 → t= 15, A=1000(1.12)^{15}≈ 1000 (5.473565759)≈ 5473.57
2010 → t= 16, A=1000(1.12)^{16} ≈ 1000 (6.13039365)≈ 6130.39

Table 6 Alan's Investment of $1000 at an Interest Rate of 12% Compounded Yearly in the Interval of Years 2000 to 2010*

Year	**Amount**
2000	1973.82
2001	2210.68
2002	2475.96
2003	2773.08
2004	3105.85
2005	3478.55
2006	3895.98
2007	4363.49
2008	4887.11

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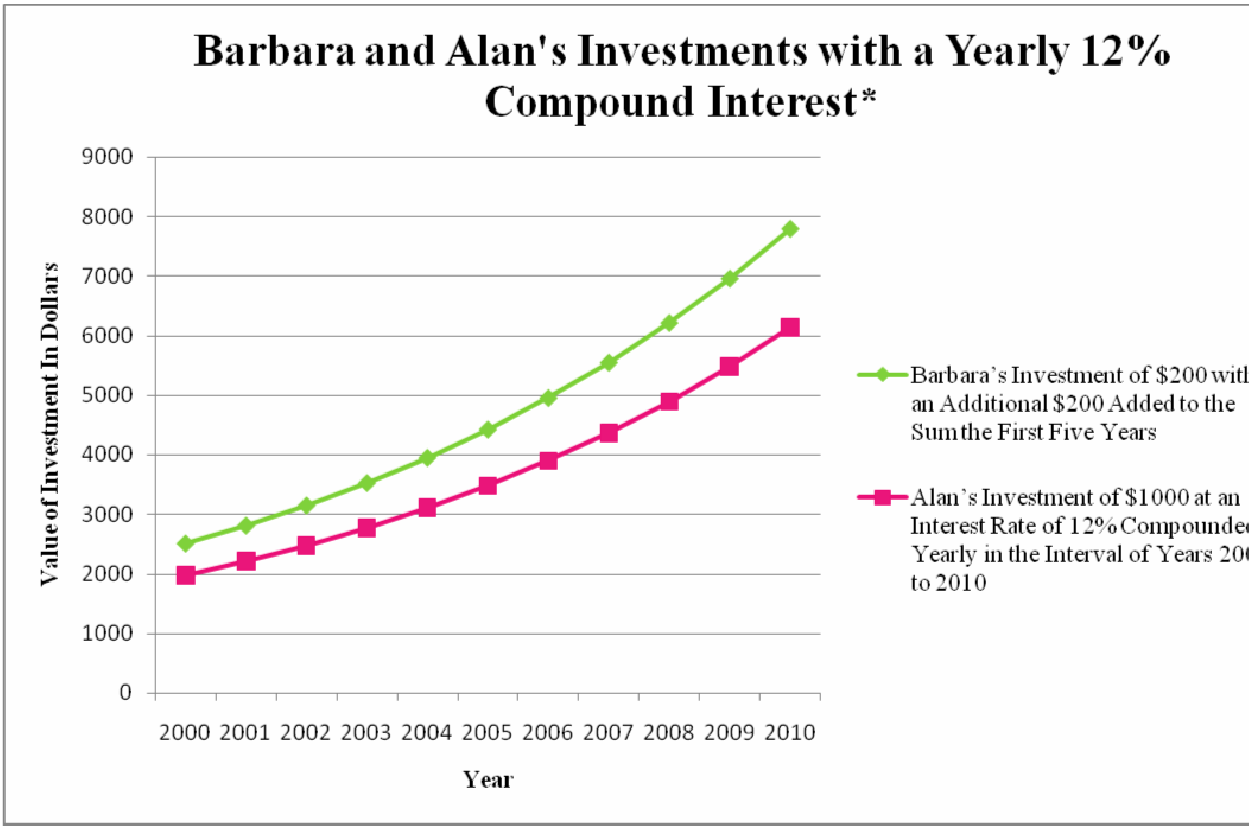
2009	5473.57
2010	6130.39

*Investment started in 1994
*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.



*Final answers rounded to the nearest hundredth in order to comply with the general money standard of cents.

The reason why Barbara's investment is worth more than Alan's, even though she began with only $200 in her account while Alan started with $1000, is because Barbara collected interest for four years longer than Alan, and furthermore added money to her investment for the first five years. Due to Barbara's addition of $200 to her final investment at the end of each year for the first five years, she collected a new principal. At the end of the first five years of her investment, instead of having $200 as her principal, which would have been the case had she done a single investment, Barbara thus had the larger principal of $1423.04. With this as the new principal for Barbara during her investment for the following years, Barbara was able to

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collect more interest than Alan, making her investment worth more than Alan's by $1658.71, or approximately 21.3%. As a result, the larger one's principal is, the more interest will be collected, creating a larger final investment.