

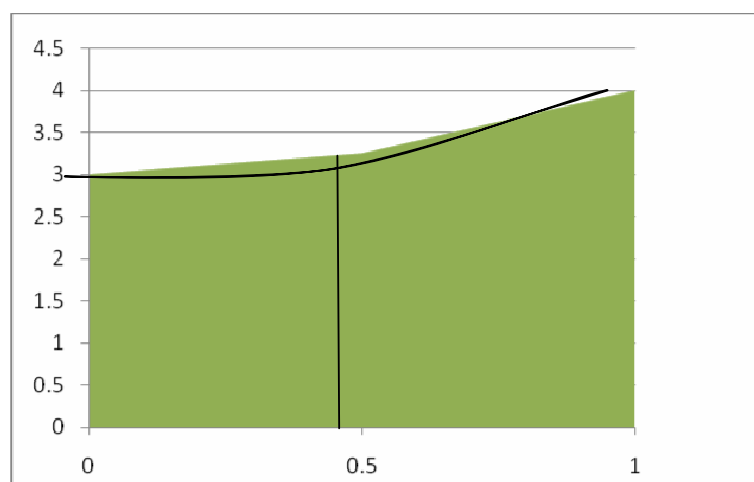
Internal Assessment 1

(05/11/09)

Approximations of areas

The following graph is a curve, the area of this curve is an approximation of two trapeziums from $x=1$ to $x=0$.

Graph 1



The Y-values (N) are found through the function $F(x) = x^2 + 3$ where (x) is (M)

$$F(x) = x^2 + 3$$

$$F(x) = x^2 + 3$$

$$F(x) = x^2 + 3$$

$$F(0) = 0^2 + 3$$

$$F(.5) = .5^2 + 3$$

$$F(1) = 1^2 + 3$$

$$F(0) = 3$$

$$F(.5) = 3.25$$

$$F(1) = 4$$

Table 1

M	N
0	3
0.5	3.25
1	4

I used Microsoft Excel and my Ti-84 calculator to retrieve this data. I graphed my M-values against my N-values in the graph as shown and I drew a line to represent the two trapezoids that are present in my graph. To find the area I will

calculate The area of each trapezoid and then add them to retrieve the area of the graph.

M is obtained depending on the number of trapeziums in a graph. I will show a function to obtain this method during my general statement where you will be able to observe this data.

The area of a trapezium is as shown. $A = \frac{Y1+Y2}{2}(x)$ where Y1 and Y2 are the parallel sides and 'x' 'in this case' is the base of width. For the first set of trapeziums I will not round unless the Ti-84 does it, so that I can keep my answer a little more precise. As we get to the larger trapeziums however I will need to round to three decimal places.

▲Area of a trapezoid (1)
(2)

$$A(1) = \frac{Y1+Y2}{2}(x)$$

$$A(1) = \frac{3+3.25}{2}(0.5)$$

$$A(1) = 1.5625$$

▲Area of Trapezoid

$$\blacktriangle(2) = \frac{Y1+Y2}{2}(x)$$

$$\blacktriangle(2) = \frac{3.25+4}{2}(0.5)$$

$$A(2) = 1.8125$$

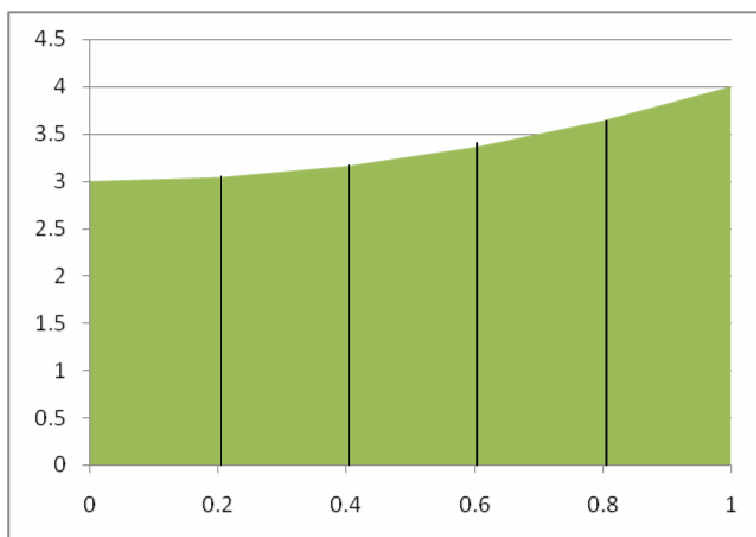
Now I must add the area of Trapezium 1 and Trapezium 2 to get the approximated area of the graph.

$$\blacktriangle = \blacktriangle(1) + \blacktriangle(2)$$

$$\blacktriangle = 3.375$$

The approximated area of the graph using two trapeziums is 3.375 units ²

Graph 2



Here I repeat the process of Graph 1 to obtain the following data, $F(x) = x^2 + 3$. Here the number of trapeziums are 5 so the M values change, I will show an equation to monitor this data soon.

Table 2

M	N
0	3
0.2	3.04
0.4	3.16
0.6	3.36
0.8	3.64
1	4

Again I repeat the process of Graph 1 but instead of adding 2 trapezoids, I add 5. Remember the width or base is the same for all the trapezoids.

Area of a trapezoid (1)
(3)

$$A(1) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(1) = \frac{3 + 3.04}{2}(0.2)$$

$$A(1) = .604$$

Area of Trapezoid (2)

$$A(2) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(2) = \frac{3.04 + 3.16}{2}(0.2)$$

$$A(2) = .620$$

Area of Trapezoid

$$A(3) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(3) = \frac{3.16 + 3.36}{2}(0.2)$$

$$A(3) = .652$$

▲Area of trapezoid (4)

$$A(4) = \frac{Y1+Y2}{2}(x)$$

$$A(4) = \frac{3.36+3.64}{2}(0.2)$$

$$A(4) = .700$$

▲Area of trapezoid (5)

$$A(5) = \frac{Y1+Y2}{2}(x)$$

$$A(5) = \frac{3.64+4}{2}(0.2)$$

$$A(5) = .764$$

▲Again we add all Trapezoids to retrieve the approximated area of the graph.

$$\blacktriangle = \blacktriangle(1)+\blacktriangle(2)+\blacktriangle(3)+\blacktriangle(4)+\blacktriangle(5)$$

$$\blacktriangle = .604+.62+.652+.7+.764$$

$$\blacktriangle = 3.34$$

The approximated area of the graph using 5 trapeziums is 3.34 units²

Now using technology I will create two more graphs with an increasing number of trapezoids to find a pattern between the approximations of the areas so far.

The first will be an approximation using 8 trapezoids.

Graph 3

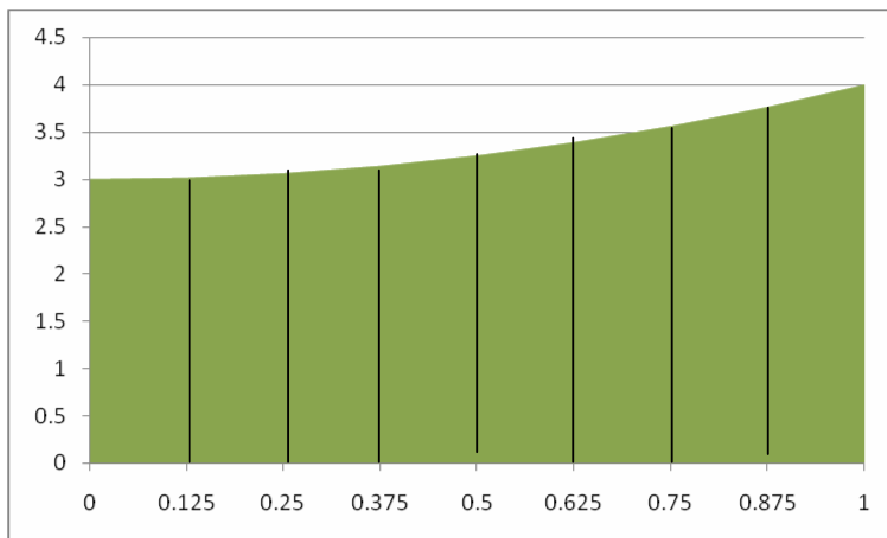


Table 3

M	N
0	3
0.125	3.015625
0.25	3.0625
0.375	3.140625
0.5	3.25
0.625	3.390625
0.75	3.5625
0.875	3.765625
1	4

Again as shown in Graph 1 and Graph 2, N is obtained through a process of $F(x) = x^2 + 3$, but now there are eight trapezoids. For these N-values, it is unfortunately not exact as the previous values, they are rounded and therefore the answer is an even more approximated area.

Trapezium (1)

$$A(1) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(1) \approx \frac{3 + 3.015625}{2}(.125)$$

$$\blacktriangle(1) \approx .376$$

Trapezium (4)

$$A(4) = \frac{Y_1 + Y_2}{2}(x)$$

Trapezium (2)

$$A(2) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(2) \approx \frac{3.015625 + 3.0625}{2}(.125)$$

$$A(2) \approx .380$$

Trapezium (5)

$$A(5) = \frac{Y_1 + Y_2}{2}(x)$$

Trapezium (3)

$$A(3) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(3) \approx \frac{3.0625 + 3.140625}{2}(.125)$$

$$\blacktriangle(3) \approx .388$$

Trapezium (6)

$$A(6) \approx \frac{Y_1 + Y_2}{2}(x)$$

$$A(4) \approx \frac{3.140625 + 3.25}{2} (.125)$$

$$A(1) \approx .399$$

Trapezium (7)

$$A(7) \approx \frac{Y_1 + Y_2}{2} (X)$$

$$A(7) \approx \frac{3.5625 + 3.765625}{2} (.125)$$

$$A(7) \approx .458$$

$$A(5) \approx \frac{3.25 + 3.390625}{2} (.125)$$

$$A(5) \approx .415$$

Trapezium (8)

$$A(7) \approx \frac{Y_1 + Y_2}{2} (X)$$

$$A(8) \approx \frac{3.765625 + 4}{2} (.125)$$

$$A(8) \approx .485$$

$$A \approx A(1) + A(2) + A(3) + A(4) + A(5) + A(6) + A(7) + A(8)$$

$$A \approx .376 + .380 + .388 + .399 + .415 + .435 + .458 + .485$$

$$A \approx 3.336$$

The approximated area of the graph under the curve is approximately 3.336 units²

So far the approximations we have are 3.375 for 2 trapezoids, 3.34 for 5 trapezoids and 3.336 for 8 trapezoids.

Graph 4

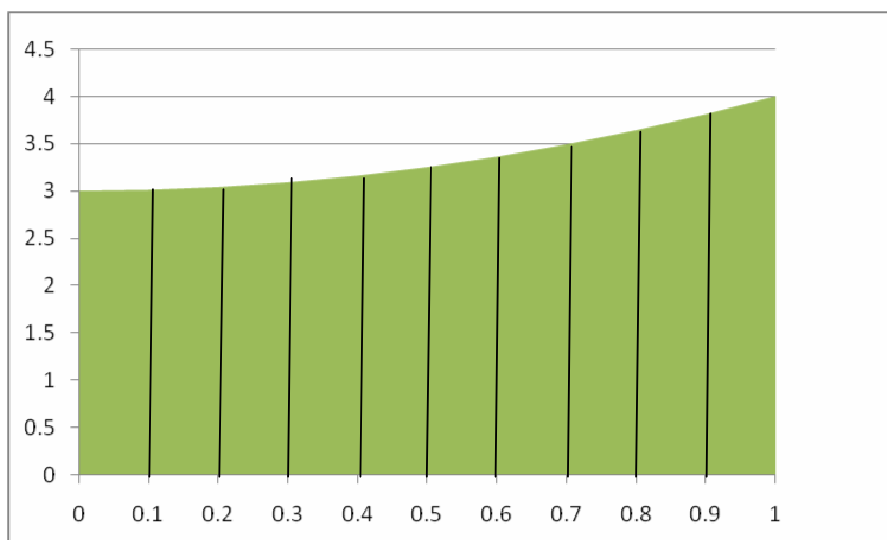


Table 4

M	N
0	3
0.1	3.01
0.2	3.04
0.3	3.09
0.4	3.16
0.5	3.25
0.6	3.36
0.7	3.49
0.8	3.64
0.9	3.81
1	4

Trapezoid (1)

$$A(1) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(1) = \frac{3 + 3.01}{2}(.1)$$

$$A(1) \approx .301$$

Trapezoid (2)

$$A(2) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(2) = \frac{3.01 + 3.04}{2}(.1)$$

$$A(2) \approx .303$$

Trapezoid (3)

$$A(3) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(3) = \frac{3.04 + 3.09}{2}(.1)$$

$$A(3) \approx .306$$

Trapezoid (4)

$$A(4) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(4) = \frac{3.09 + 3.16}{2}(.1)$$

$$A(4) \approx .313$$

Trapezoid (5)

$$A(5) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(5) = \frac{3.16 + 3.25}{2}(.1)$$

$$A(5) \approx .320$$

Trapezoid (6)

$$A(6) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(6) = \frac{3.25 + 3.36}{2}(.1)$$

$$A(6) \approx .331$$

Trapezoid (7)

$$A(7) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(7) = \frac{3.36 + 3.49}{2}(.1)$$

$$A(7) \approx .342$$

Trapezoid (8)

$$A(8) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(8) = \frac{3.49 + 3.64}{2}(.1)$$

$$A(8) \approx .357$$

Trapezoid (9)

$$A(9) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(9) = \frac{3.64 + 3.81}{2}(.1)$$

$$A(9) \approx .372$$

Trapezoid (10)

$$A(10) = \frac{Y_1 + Y_2}{2}(x)$$

$$A(10) = \frac{3.81 + 4}{2}(.1)$$

$$A(10) \approx .390$$

$$A \approx .301 + .303 + .306 + .313 + .321 + .330 + .342 + .357 + .372 + .390$$

$$A \approx 3.335$$

The approximation under the curve using 10 trapezoids is approximately 3.335 units²

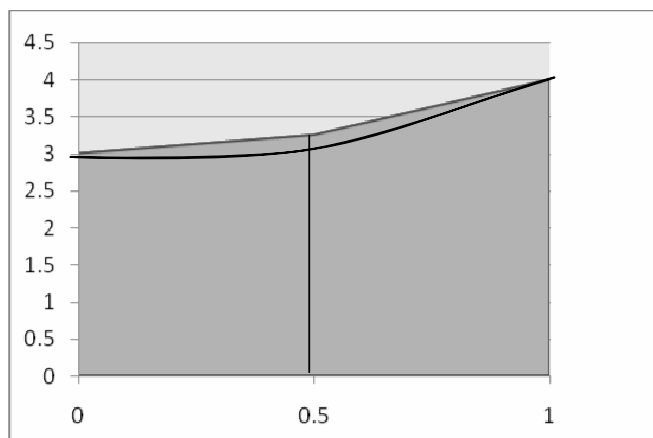
So far the approximations we have are 3.375 for 2 trapezoids, 3.34 for 5 trapezoids, 3.336 for 8 trapezoids and 3.335 for 10 trapeziums.

Table 6

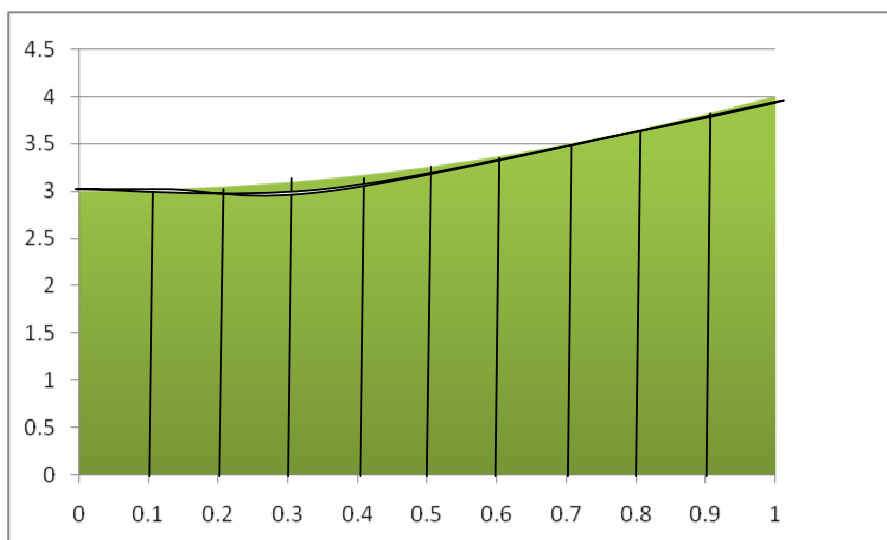
T	▲
2	3.375
5	3.34
8	3.336
10	3.335

We notice that although they don't differ by much, they do decrease as the number of trapeziums increase. ▲ And this makes sense. Let's go back to the first graph. Now as you can see we are actually trying to find the area of the graph **under** the curve (the black line). But using trapeziums, we end up calculating a little over it as well, though the approximation is close, it cannot be exact.

Graph 5



Now if we take four trapeziums instead of 2, less of the outer area is calculated because the lines drawn from one $f(x)$ value to another is not as high, and is therefore more minimal.



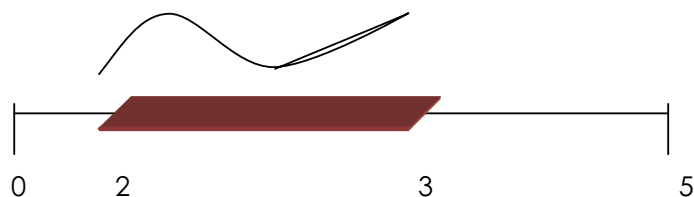
The trapezoids here are much more fit for the curve and will therefore give you a much better estimation. Using this theory we can conclude that as the base width gets smaller, or the number of trapeziums gets higher, we will receive a better estimation, and to get an exact answer we must have a width of basically 0.

To find a general expression for the following graphs we did we simply follow the processes.

- 1) We found the M-value, the width of the base. Also known as (x) in our trapezoid equation.

To find this we need our x-limits. In these problems, it goes from $0 \leq x \leq 1$. We minus them to get the difference or just the part of the graph we will be using. We will let 'a' represent the starting point and 'b' represent the end point,
 $a \leq x \leq b$

ex.



If we subtract 2 from 3 we will get the only the distance of what we want.
 $b-a$.

This part will unfortunately give us the base of the whole thing, or one trapezoid. If we want the exact base of a certain number of trapezoids, we must divide $(b-a)$ by 'n' number of trapezoids.

$\frac{b-a}{n}$ This will be our width for each equation.

2) The area of a trapezoid is $A = \frac{Y1+Y2}{2}(x)$. We know what (x) represents now.

Y1 and Y2 are the sides of the trapezoid. I will show you how we can calculate the sides with variables, using five lines.

$$\begin{array}{ccccccccc} a & a+1\left(\frac{b-a}{n}\right) & a+2\left(\frac{b-a}{n}\right) & a+3\left(\frac{b-a}{n}\right) & a+4\left(\frac{b-a}{n}\right) & a+5\left(\frac{b-a}{n}\right) & \text{or } b \\ \hline 0 & & & & & & 1 \end{array}$$

Now in this case $\frac{b-a}{n}$ is the following:

$$\frac{1-0}{5}$$

$$\frac{1}{5}$$

The width is always constant, but the location of the line changes, therefore we must change the coefficient. We must put in function notation because we need the height and not the x-value.

$$\begin{array}{ccccccccc} F\left(a+0\left(\frac{1}{5}\right)\right) & F\left(a+1\left(\frac{1}{5}\right)\right) & F\left(a+2\left(\frac{1}{5}\right)\right) & F\left(a+3\left(\frac{1}{5}\right)\right) & F\left(a+4\left(\frac{1}{5}\right)\right) & F\left(a+5\left(\frac{1}{5}\right)\right) & \text{or } b \\ \hline \end{array}$$

Now we see that the coefficient increases at a constant rate, adding one each time, and a and b, the starting and ending points, are the same. Also The width value remains the same and is repeated constantly in every function value. Using these functions we find the area of each trapezoid.

So for example $f\left(\frac{a+b}{2}\right)\left(\frac{b-a}{n}\right)$ will give is the value of one trapezoid. But since this equation will give us only one trapezoid approximation, we must use the coefficients. In this equation you have basically put all the trapezoids into one expression.

$$f\left(\frac{a+a+1\left(\frac{1}{5}\right)+a+1\left(\frac{1}{5}\right)+a+2\left(\frac{1}{5}\right)+a+2\left(\frac{1}{5}\right)+a+3\left(\frac{1}{5}\right)+a+3\left(\frac{1}{5}\right)+a+4\left(\frac{1}{5}\right)+a+4\left(\frac{1}{5}\right)+b}{2}\right)\left(\frac{b-a}{n}\right)$$

In the following equation given, we have fractions on fractions. But not all of them have fractions so we must separate them to make it easier.

You will notice that the coefficients are repeating and this makes sense because when we calculated the trapezoids, you will notice previously that we used every notation except the first and last twice to calculate the trapezoid.

For example if you reference Graph 1, you will find that $f(0.5)$ or 3.25 was used twice, to calculate the first and well and the second trapezoid.

Since a and b , only the beginning and end ones, are the ones that are only being divided by we will sum it up into the following.

$$F\left(\frac{a+b}{2}\right)$$

The rest of them have a few things in common. The first is that they are all added to the common variable a , and multiplied by the coefficient.

In this equation $\left(\frac{1}{n}\right)$ represents $\left(\frac{b-a}{n}\right)$, since only the fraction is again divided by 2 we can write it as $\left(\frac{b-a}{2n}\right)$,

We can make it common by changing the coefficient to the variable ' k '. This also gives the advantage of not give the number of trapezoids, making it more diverse in its uses.

$$F\left(a+k\left(\frac{b-a}{2n}\right)\right).$$

Since ' k ' is always changing we must add a summation notation which also allows us to determine the approximation according to our ' n '. Since $\left(\frac{b-a}{n}\right)$ at the end of the equation is multiplied by the entire function and is not repeatedly multiplied we can factor it out of the summation notation as already shown.

$$\left(\frac{b-a}{n}\right) \left[\left(\frac{a+b}{2}\right) + \sum_{k=1}^{n-1} f\left(\frac{a}{2} + 2k \frac{b-a}{2n}\right) \right]$$

Remember, the end has to $(n-1)$ because $a + n \frac{b-a}{2n}$ would equal b , and b is already added at the beginning of the statement. Since everything in the brackets are divided by 2 we can factor that out as well.

$$\left(\frac{b-a}{2n}\right) \left[f(a+b) + \sum_{k=1}^{n-1} f\left(2\left(a + k \frac{b-a}{n}\right)\right) \right]$$

This is the general statement that we can use for all functions.

I will test the validity of this equation by performing the 5 trapezoid function.

$$\left(\frac{1-0}{2(5)}\right) \left[f(0+1) + \sum_{k=1}^{5-1} f\left(2\left(a + k\frac{1}{5}\right)\right) \right]$$

$$\left(\frac{1-0}{2(5)}\right) \left[(3+4) + f\left(2\left\{a + \frac{1}{5} + a + \frac{2}{5} + a + \frac{3}{5} + a + \frac{4}{5}\right\}\right) \right]$$

*recall table 2 which has the N-values already calculated according to the given function.

$$\frac{1}{10} [7 + f(2\{3.04 + 3.16 + 3.36 + 3.64\})]$$

$$\frac{1}{10} [7 + f(2\{3.04 + 3.16 + 3.36 + 3.64\})]$$

$$\frac{1}{10} [33.4]$$

3.34

Recall that the area found with 5 trapezoids was also 3.34, therefore the general statement works.

Equation vs. Ti-83

The area of Y_1 for 8 trapezoids

*For the following areas and most of the points, they will be rounded, they are not exact but I will leave them as shown in the calculator to keep it as exact as possible.

$$Y_1 = \left(\frac{x}{3}\right)^{\frac{2}{3}}, 1 \leq x \leq 3$$

1)

Find the x-values

against the y-values. This is found again by using the function already given.

Recall the first part of the Assessment to find the example shown.

$$\left(\frac{b-a}{n}\right) \rightarrow m$$

$(\frac{1}{4}) \rightarrow$ Therefore each base is .25

Table Y1

M	N
1	0.629961
1.25	0.731004
1.5	0.825482
1.75	0.914826
2	1
2.25	1.081687
2.5	1.160397
2.75	1.236522
3	1.310371

2)
statement.

Sub into general

$$\Delta = \left(\frac{b-a}{2n}\right) \left[f(a+b) + \sum_{k=1}^{n-1} f\left(2\left(a+k\frac{b-a}{n}\right)\right) \right]$$

$\Delta \approx$

$$\left(\frac{3-1}{2(8)}\right)$$

$$\left[f\left(1+3\right) + f\left[\left(2\left(1+1\left(\frac{2}{8}\right)\right) + \left(2\left(1+2\left(\frac{2}{8}\right)\right) + \left(2\left(1+3\left(\frac{2}{8}\right)\right) + \left(2\left(1+4\left(\frac{2}{8}\right)\right) + \right.\right.\right.\right. \\ \left.\left.\left. \left(2\left(1+5\left(\frac{2}{8}\right)\right) + \left(2\left(1+6\left(\frac{2}{8}\right)\right) + \left(2\left(1+7\left(\frac{2}{8}\right)\right)\right) \right] \right] \right]$$

Sub in the sigma notation values

$\Delta \approx$

$$\left(\frac{2}{16}\right)$$

$$\left[(0.629961 + 1.310371) + f[(2(1.25) + (2(1.5)) + (2(1.75) + (2(2) + (2(2.25) + \right. \\ \left. (2(2.5) + (2(2.75)). \right]$$

- Find the functions

$\Delta \approx$

$$\left(\frac{1}{8}\right)$$

$$\left[(1.940332) + (2(0.731004 + 0.825482 + 0.914826 + 1 + 1.081687 + 1.160397 + \right. \\ \left. 1.236522)) \right]$$

- Solve inside bracket

$$\Delta \approx$$

$$\left(\frac{1}{8}\right)$$

$$\llbracket (1.940332) + (2(0.731004 + 0.825482 + 0.914826 + 1 + 1.081687 + 1.160397 + 1.236522)) \rrbracket$$

$$\Delta \approx \frac{1}{8}(15.840168)$$

$$\Delta \approx 1.980021$$

Now as said before, this is an approximation. To find the actual Area we must use integration and calculus with our TI-84. I will show this task in more detail in the appendix at the end.

If we sub the equation into the calculator and solve it we get a precise answer.

$$\text{Sub it in form } \int_y^U \left(\frac{x}{3}\right)^{\frac{3}{2}} d(x)$$

$$\Delta \approx 1.986909$$

Now as you can see the approximations are extremely close to each other, but obviously not exact. Again, you can only get an exact answer if the width of your trapezoids are 0, but this equation will get you very close, and you can make it as close as you want by changing the 'n' on your equation.

I will perform two more examples of the type, and compare them with the actual value shown by my Ti-84.

The area of Y_2 using 8 trapezoids

$$Y_2 = \frac{9x}{\sqrt{9+x^3}}, 1 \leq x \leq 3$$

Table Y2

M	N
1	2.84605

1.25	3.399253
1.5	3.837613
1.75	4.156356
2	4.365641
2.25	4.484456
2.5	4.534134
2.75	4.534087
3	4.5

$$\left(\frac{b-a}{2n}\right) \left[f(a+b) + \sum_{k=1}^{n-1} f\left(2\left(a+k\frac{b-a}{n}\right)\right) \right]$$

▲ ≈

$$\left(\frac{3-1}{2(8)}\right)$$

$$\left[f\left(1+3\right) + f\left[2\left(1+1\left(\frac{2}{8}\right)\right) + 2\left(1+2\left(\frac{2}{8}\right)\right) + 2\left(1+3\left(\frac{2}{8}\right)\right) + 2\left(1+4\left(\frac{2}{8}\right)\right) + 2\left(1+5\left(\frac{2}{8}\right)\right) + 2\left(1+6\left(\frac{2}{8}\right)\right) + 2\left(1+7\left(\frac{2}{8}\right)\right) \right] \right]$$

▲ ≈

$$\left(\frac{2}{16}\right)$$

$$\left[(2.84605 + 4.5) + f[2(1.25) + 2(1.5) + 2(1.75) + 2(2) + 2(2.25) + 2(2.5) + 2(2.75)] \right]$$

▲ ≈

$$\left(\frac{1}{8}\right)$$

$$\left[(7.34605) + (2(3.399253 + 3.837613 + 4.156356 + 4.365641 + 4.484456 + 4.534134 + 4.5)) \right]$$

$$\text{▲} \approx \left(\frac{1}{8}\right) (65.96913)$$

$$\text{▲} \approx 8.24914125$$

Using the calculator I will now find an approximation.

$$\int_y^u \frac{9x}{\sqrt{9+x^3}} d(x)$$

$$\blacktriangle \approx 8.2597312$$

▲ Again as shown in the previous example the value is very close but not exact.

The area of Y_3 using 8 trapezoids

$$Y_3 = 4x^3 - 23x^2 + 40x - 18, 1 \leq x \leq 3$$

Table Y3

M	N
1	3
1.25	3.875
1.5	3.75
1.75	3
2	2
2.25	1.125
2.5	0.75
2.75	1.25
3	3

$$\left(\frac{b-a}{2n}\right) \left[f(a+b) + \sum_{k=1}^{n-1} f\left(2\left(a+k\frac{b-a}{n}\right)\right) \right]$$

$$\blacktriangle \approx$$

$$\left(\frac{3-1}{2(8)}\right)$$

$$\left[f\left(1+3\right) + f\left[2\left(1+1\left(\frac{2}{8}\right)\right) + 2\left(1+2\left(\frac{2}{8}\right)\right) + 2\left(1+3\left(\frac{2}{8}\right)\right) + 2\left(1+4\left(\frac{2}{8}\right)\right) + 2\left(1+5\left(\frac{2}{8}\right)\right) + 2\left(1+6\left(\frac{2}{8}\right)\right) + 2\left(1+7\left(\frac{2}{8}\right)\right) \right]$$

$$\blacktriangle \approx$$

$$\left(\frac{2}{16}\right) \left[(6) + f(2(1.25) + (2(1.5)) + (2(1.75) + (2(2) + (2(2.25) + (2(2.5) + (2(2.75)). \right]$$

$$\blacktriangle \approx \left(\frac{1}{8}\right) [(6) + (2(3.875 + 3.75 + 3 + 2 + 1.125 + .75 + 1.25))]$$

$$\blacktriangle \approx \left(\frac{1}{8}\right) (37.5)$$

$$\blacktriangle \approx 4.6875$$

$$\int_y^u 4x^3 - 23x^2 + 40x - 18 \, d(x)$$

$$\blacktriangle \approx 4.\overline{6}$$

The following examples demonstrate to us, that this equation can find the approximation of many different curves. For exponential as shown in Y_1 , In radicals as shown in Y_2 and also in cubic and in extension quadratic as shown in Y_3 . This equation will get you a very close approximate to your answer, and again we can make it as approximate as we want by choosing the right amount of 'n' trapezoids.

Scopes and Limitations

The limitations, b and a, of the graph are very important. They are your limitations and show you how much you need to calculate, even with your calculator.

The only true problem you will find with this equation is the time spent on it, and you accuracy is most likely not 100% correct.

$M, N \in \mathbb{N}$.

$K \in I$

K must be an integer because it is calculating how many widths you are adding, if it is not an integer, then the width of your trapezoid will change.

It is wrong to get a negative integral since we are calculating the area.

Appendix

Microsoft Word XP and Microsoft Excel was used to type this assignment. Microsoft office's "Equation" option was used to type the equations. The " \blacktriangle Area" graphs were used in Excel to draw them. The tables and graphs were all formed using Microsoft Excel. I also used it's "function" option to give the answers for the tables.

\blacktriangle All calculations were done using Ti-84.

The following will describe how the Ti-84 was used to perform the functions and find the area from section 'Equations vs. Ti-84'. I will show it according to the first problem, Y_1 .

I went first to 'Y=' and typed in my equation, $(\frac{x}{2})^{\frac{2}{3}}$. After it was drawn I went to 'WINDOW' and set it accordingly, I changed my 'Xmin' to 1 and 'xmax' to 3.

After that I pressed '2ND' and 'Trace' to get to 'CALC'. I went to option '7' which is the integration function. After I clicked that it took me back to the graph where I typed in my end points.

Lower limit '1' 'ENTER'

Upper limit '3' 'ENTER'

And on the bottom it then showed me approximation which in this case was ≈ 8.2597312 .

I used Paint to create the following screen shots of the process described above.

