Davy College

Analysis of Functions

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14/10/2011

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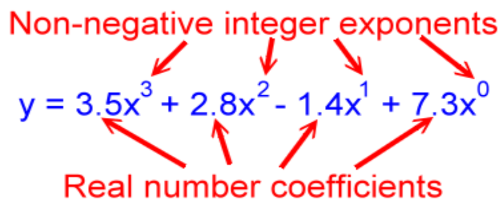
Analyzing Different Functions

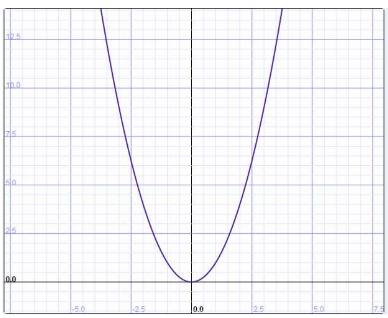
Polynomial Functions:

These are functions with x as an input variable, made up of several terms, each term is made up of two factors, the first being a real number coefficient, and the second being x raised to some non-negative integer exponent.

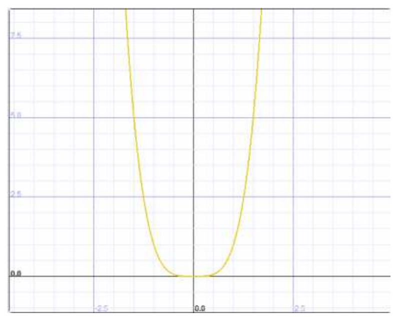


The domain of any polynomial functions are the real numbers set, R. These are some examples with different degrees:

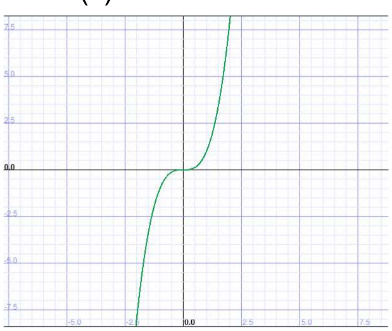
- $f(x) = x^2$



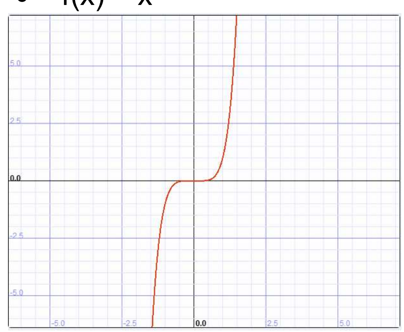
- $f(x) = x^4$



- $f(x) = x^3$



- $f(x) = x^5$



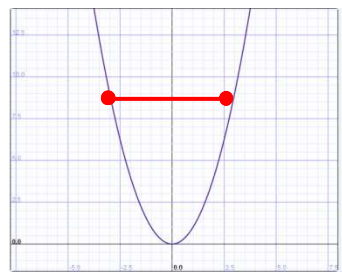
The factors of decreasing and decreasing intervals (in the y axis) in a polynomial function depend on the turning points of the function. The values of y can first be increasing but suddenly caused by the turning point, the values start to decrease. For example the values of y increase on the open intervals (∞ < x < a) but then when b < x < ∞ the intervals start decreasing.

Even polynomial functions have f(x) = f(-x) and odd polynomial functions occur when f(-x)= -f(x)Polynomial functions can be even, odd or neither, as shown with the following ex

$f(x) = x^2$	$f(x) = x^3$	$f(x) = x^2-5x+6$
$f(-x)=(-x)^2$	$f(-x)=(-x)^3$	$f(-x)= x^2+5x+6$
$f(-x)=x^2$	$f(-x)=-x^3$ Odd function	$-f(x) = -x^2+5x-6$
Even function	$\cancel{-f(x) = -(x^3)}$	Neither function
	$-f(x) = -x^3$	

A polynomial function is not periodic in general because a periodic function repeats function values after regular intervals. It is defined as a function for which f(x+a) = f(x), where T is the period of the function. In the case of polynomial functions, clearly the only exception that can't be a periodic function is that there is no definite or constant period like "a". The relation of periodicity, however, holds for any change to x, so it can also be accepted the idea that polynomial functions are periodic functions with no period.

There are two types of maximums and minimums, which are relative and absolute. In polynomial functions there can be both of them, but always just one absolute maximum and one minimum (if the degree of the function is odd), or two maximums or minimums (if the degree is even).the amount of relative max and min depends on the number of tuning points, as there are more turning points, there would be more relatives.



As we can easily observe in the graph of a quadratic function is that with this degree the function is subjective because there is more than one matching, meaning that there are two numbers in set A which has the same number in set B. the same occurs with the functions with degree 4. But in the case of cubic and degree 5 functions the relation can be injective and surjective because they can have a one to one relation but also a many to one so depending on the quotients of the functions.

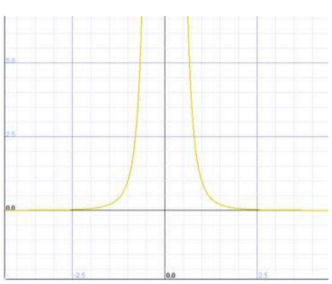
Power Function:

If p is a non-zero integer, then the domain of the power function $f(x) = kx^p$ consists of all real numbers. For rational exponents p, x^p is always defined for positive x, but we cannot extract an even root of a negative number. Any rational number p can be written in the form p = r/s where all common factors of r and s have been cancelled. When this has been done, kx^p has domain:

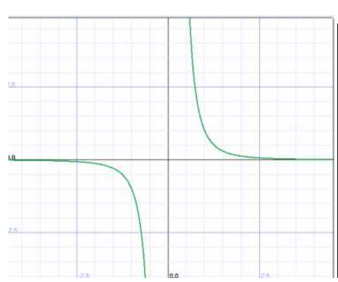
- All real numbers if s is odd
- All non-negative real numbers if s is even.

If p is a real number which is not rational (called an irrational number), then the domain of xp consists of all non-negative real numbers.

In power functions when the degree is positive then there would be always increasing values doesn't matter if it's odd or even, but when the degree is negative then it depends if it's odd or even, to have increasing or decreasing intervals.

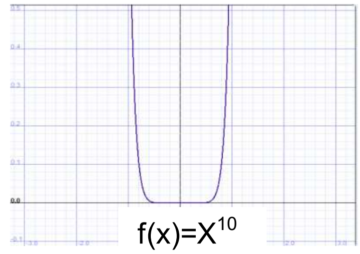


($f(x)=x^{-4}$) for all negative even integers, there would be always an increasing and decreasing



($f(x)=x^{-5}$) for all negative odd2 integers, there would be always two decreasing intervals.

In power functions when the degree are positive integers then the function is subjective because as we can see in the graph there is a many to one relation. But in the case when the degrees of the function are negative integers, the functions turn to be bijective because there is a relation many to one and not all the values of y exists.



$f(x)=X^{10}$

For power functions, the function is even when the exponent is even and odd when the degree is odd: for example $f(x)=x^4$ is even but $f(x)=x^3$ is odd.

Power functions are not periodic because a periodic function repeats function values after regular intervals. It is defined as a function for which f(x+a) = f(x), where T is the period of the function. In the case of power functions, it can't be a periodic function is that there is no definite or constant period like "a". The relation of periodicity, however, holds for any change to x, so it can also be accepted the idea that polynomial functions are periodic functions with no period.

In the case of maximums and minimums for any case of the functions there would be always absolute maximums and minimums. Because there are not turning points the relative max and min don't exist in this type of function.

As we can see in the graphs above, there is a discontinuity when the exponent of the function is negative; the type of discontinuity that is present is the asymptote. For the other cases the asymptote discontinuity is the only type of discontinuity present in the functions. So the asymptotic behavior is to the x and y axes, vertical and horizontal.

The ending behavior when the exponents are positive is to be positive infinite when x gets to negative and positive infinite. When the degree is negative the ending behavior changes to have an asymptotic behavior to the x axis when the x values go to negative infinity and the same when is goes to positive infinity.

Rational functions:

The domain in the nominator can be the set of all real numbers, but in the dominator it is different because in fractions the denominator can't be equal to zero so here are some examples of the domains with different types of rational functions:

$$f(x)=\frac{2}{x+6}$$

Domain= R, x ≠

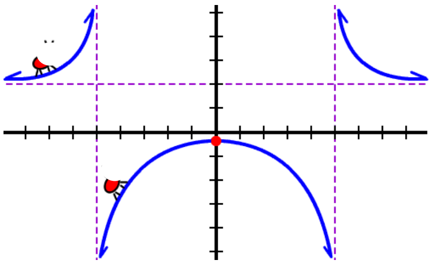
$$f(x)=\frac{3x+4}{5x-9}$$

Domain=R, x ≠

$$f(x)=\frac{x+4}{x^2-3x-10}$$

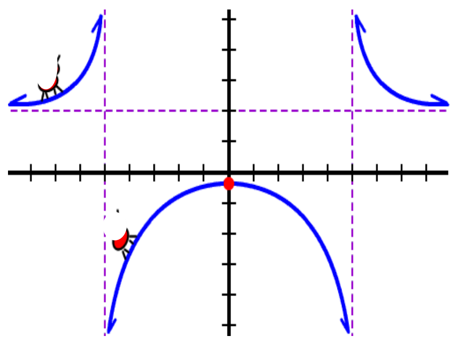
Domain=R, (x ≠ 5)(x ≠

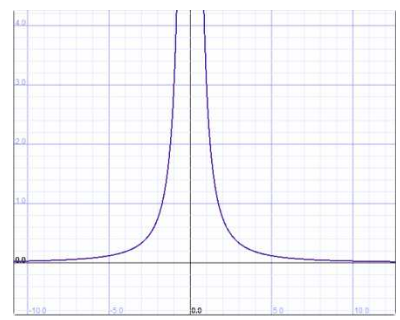
The increasing and decreasing intervals depends in the degree of the nominator and the denominator here are some examples:



$\frac{2x^2+5}{2x^2-25}$ In this function there are different increasing and decreasing intervals, so as we can see first it increases, then increases again but after it happends the oppsite. The increasing values are all. In rational functions the increasing and decreasing values are always present, there can be both, increasing and decreasing values, as there can also be just decreasing or just increasing, like in the cases bellow. The domains are from cero to positive or negative infinite.

All rational functions are injective beause there a one to one relation and the range is part of B, being A its domain. Here are some examples:





$f(x)= 5/x$ $f(x) = \frac{2x^2+5}{2x^2-25}$ $f(x)= 3/x^2$

This type of function is not periodic for the same reason explained before with the other functions, in which the variable A is not constant.

As we can see with the three graphs above, rational functions have always symmetry. The line of symmetry can be horizontal, vertical or diagonal, these is because in these type of function parabolas are form, so parabolas have always symmetry at any place they were.

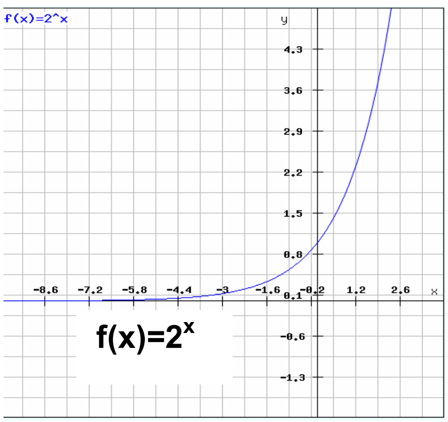
In rational functions the increasing and decreasing values are always present, there can be both, increasing and decreasing values, as there can also be just decreasing or just increasing, like in the cases above. The domains are from cero to positive or negative infinite.

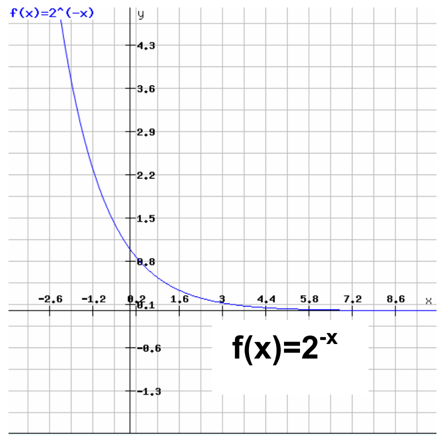
The maximums and minimums in these types of functions can be from 0 to negative or positive infinite real numbers, which also includes the relative max and min.

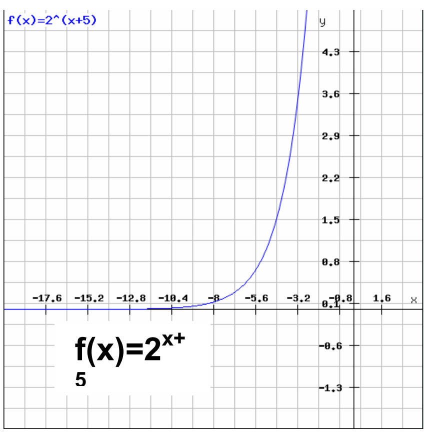
These functions are not continuously; all have an asymptotic discontinuity, horizontal and vertical. The asymptotes are present in all functions, having an asymptotic ending behavior also, to the x and y axis.

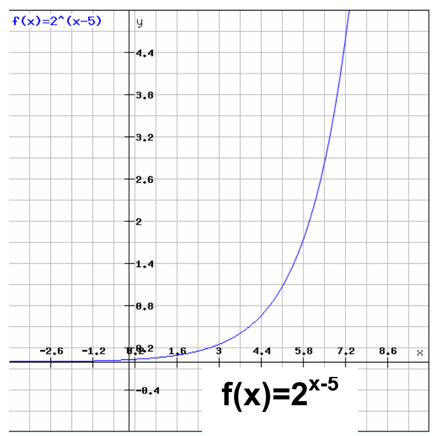
Exponential functions:

The domain of exponential functions in form of $f(x) = a^x$, is the set of all real numbers. The base, a, should be larger than 0 because the reason that a > 0 is that if it is negative, the function is undefined for -1 < x < 1. Restricting a to positive values allows the function to have a domain of all real numbers.









As we can see in the graphs above, there are increasing values in the four different situations, from zero to positive infinite.

These functions are injective because there is a one to one relation and the range are not all the values of y, the range are only the values larger than zero to positive infinite.

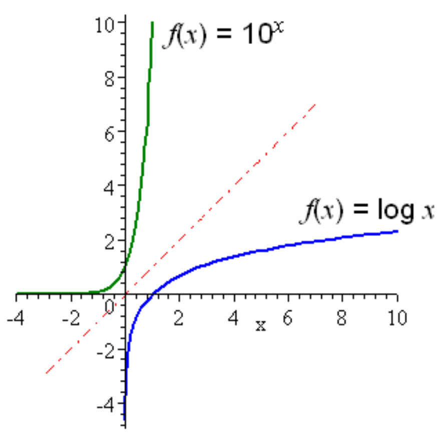
There is not periodicity in any of these functions because in a function f(x+a)=f(x), a is not constant so there is not periodicity.

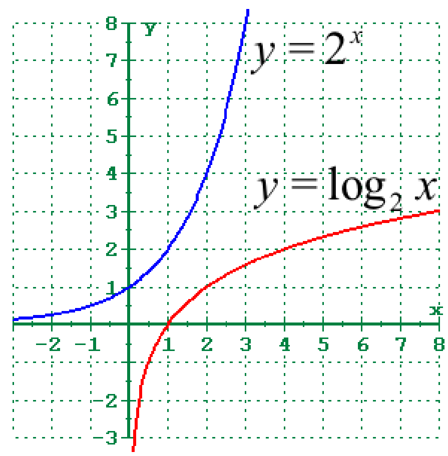
As we can see in the graphs above, there is no symmetry in these functions. It looks from a simple look that there is a symmetric line but if the window is increase, we can observe the asymptotic behavior to the x axis and at the other end x and y continue increasing constantly.

The maximums of these functions are larger than zero to positive infinite, but the minimums are greater than zero, as there is an asymptotic behavior to the y axis. As it has been mentioned before, the discontinuity in these functions is an asymptotic behavior y=0. The ending behavior of these functions is that the values of y is positive infinite for the vertical asymptotes and in some situations when x goes to positive infinite, and for the horizontal asymptotes, the values of y get closer to zero but never reaches it.

Logarithmic functions:

The domain of a logarithmic function, $\log_a x=b$, is the set of positive real numbers larger than zero, (R)−{0}. This is because, as we can understand the logarithm as $a^b=x$, so x can't be zero because if b equals zero, because of a rule x should be one and not zero, and with any positive or negative number, x would never be zero.





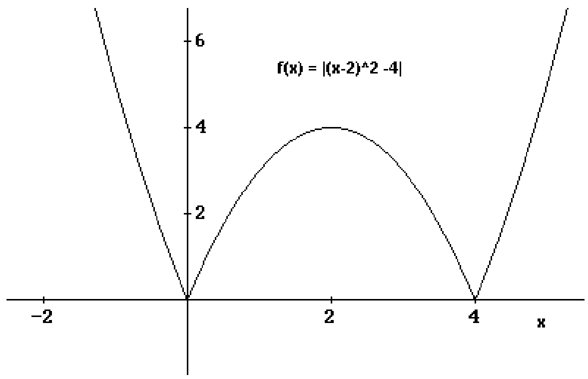
So as we can see with the examples above logarithmic functions and exponential functions are inverses, so the increasing intervals are the same, because if the function is increasing its inverse will be still increasing, it just changes direction.

As well as the exponential, these functions are injective because there is a one to one relation and the ranges are not all the values of y.

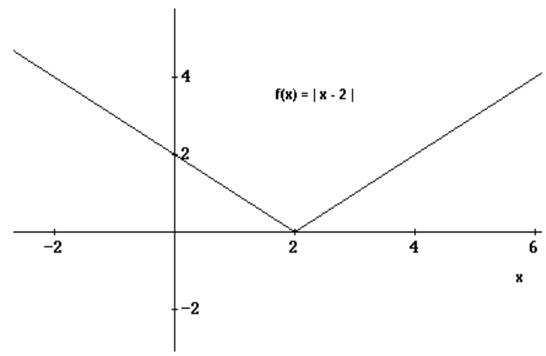
There is not periodicity among these functions because of the same reason of all the above functions. There is not symmetry within these functions because it is just a curve which doesn't get to be a parabola. The maximum is infinity, while in the other hand the minimum is greater than zero. The only discontinuities are the asymptotes to the y axis. And finally the ending behavior is positive infinite when the values of x go to positive infinite and an asymptotic behavior to the y axis when the values of x get closer to zero.

Absolute value functions:

The domain of this type of function is the set of all real numbers, because for every value I can get is absolute value.



$f(x)= |X^2-4x|$



$f(x)= |X-2|$

As we can see in the graphs above, there are increasing and decreasing intervals, the intervals can be major caused because there can't be negative values in the range of the functions, so every part of the function that it should normally be negative, because of the absolute value, changes direction , causing a change in the increasing and decreasing intervals.

These types of functions are bijective because they have a many to one relation but also the range is not all but just the numbers greater or equal than zero.

These functions are not periodic because in a situation of f(x+a)=f(x), a is not constant so this makes it not periodic.

In the cases above, quadratic and linear functions are clearly symmetric specially because, when their values of y are supposedly negative, they change to the opposite of all the values, making them symmetric to the other part, this is in the case of linear, but for quadratic occurs nearly the same, not affecting the symmetry of a parabola.

The absolute maximums of these functions would always be infinite, while in the other hand, for minimums the absolute minimum is 0. There would be still relatives but I only in the case of maximums.

From the data gathered from the graphs above, we can say that absolute value functions are continuous, having any type of discontinuity. So there is not any asymptotic behavior.

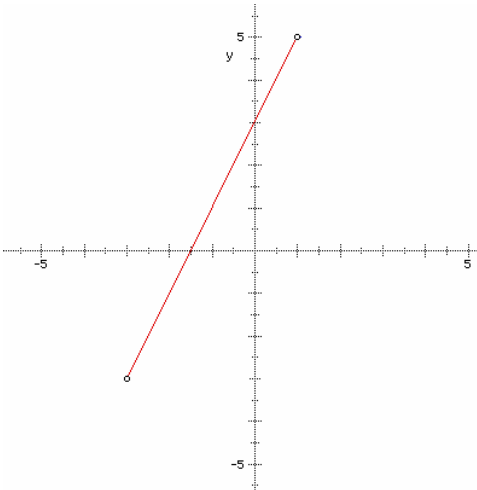
The ending behavior of these functions is that when x's values go to negative or positive infinity, the values of y would always go to positive infinity.

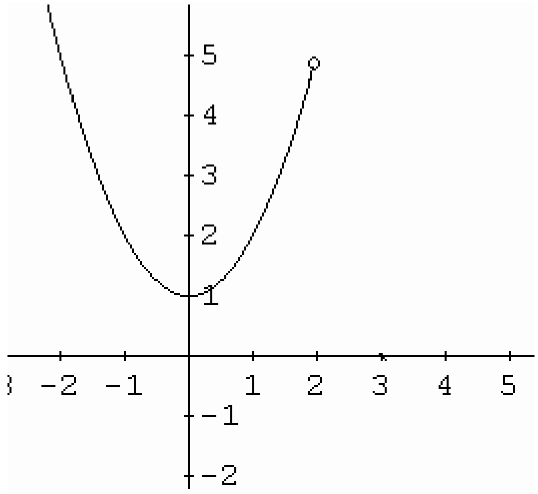
Piecewise functions:

f1(x) = x + 5	if -3 <= x <= 2
f2(x) = -x + 3	if 3 < x <= 5
f3(x) = -3	if x > 5

Having these functions as examples, the domain of the functions is determined by the "if", which for example in functions one the domain are all the numbers between

-3 and 2, [-3, 2], in case of the third function, the domain is (5, ∞) .





f(x)= 2x + 3 on the interval (-3, 1)

f(x)= x 2 + 1 on the interval (-∞,2)

As we can see in the graphs above the increasing and decreasing intervals depends on the degree of the function, for example in the case of the linear ones, the sign of the slope would tell if the line is increasing or decreasing.

These functions can be injective, in the case of linear functions, but also bijective for degree 2 or more functions.

Even functions have f(x) = f(-x) and odd functions occur when f(-x)= -f(x)Polynomial functions can be even, odd or neither, piecewise functions can be any of these three as shown with the following examples:

$f(x) = x^2$	$f(x) = x^3$	$f(x) = x^2-5x+6$
$f(-x)=(-x)^2$	$f(-x)=(-x)^3$	$f(-x)= x^2+5x+6$
$f(-x)=x^2$	$f(-x)=-x^3$ Odd function	$-f(x) = -x^2+5x-6$
Even function	$-f(x) = -(x^3)$	Neither functions
	$-f(x) = -x^3$	

These type of functions are not periodic because in the formula f(x+a)=f(x), a is not constant, so it can't be periodic.

There can be symmetry in some functions like the quadratic, linear, cubic, etc. but it's important to mention that not always there would be symmetry among all these degrees, it depends on the quotients of the function. Similar as the polynomial functions the maximums can be absolute an relative, depending on the degree of the function. Is important to have in consideration that all these characteristics depend on the domain given before, because with the domain just a part of the function is considerate and not all, which can contradict what, was said before.

Gerald Veliz

There can be discontinuity but the functions can be also continuous all depends on the degrees or form of the function but as well the domain given. So he ending behavior follows the same conditions.

Hybrid functions: