Population Trends In China

IB SL Mathematics Type II

An investigation of different functions that best model the population of China.

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CONTENTS

INTRODUCTION

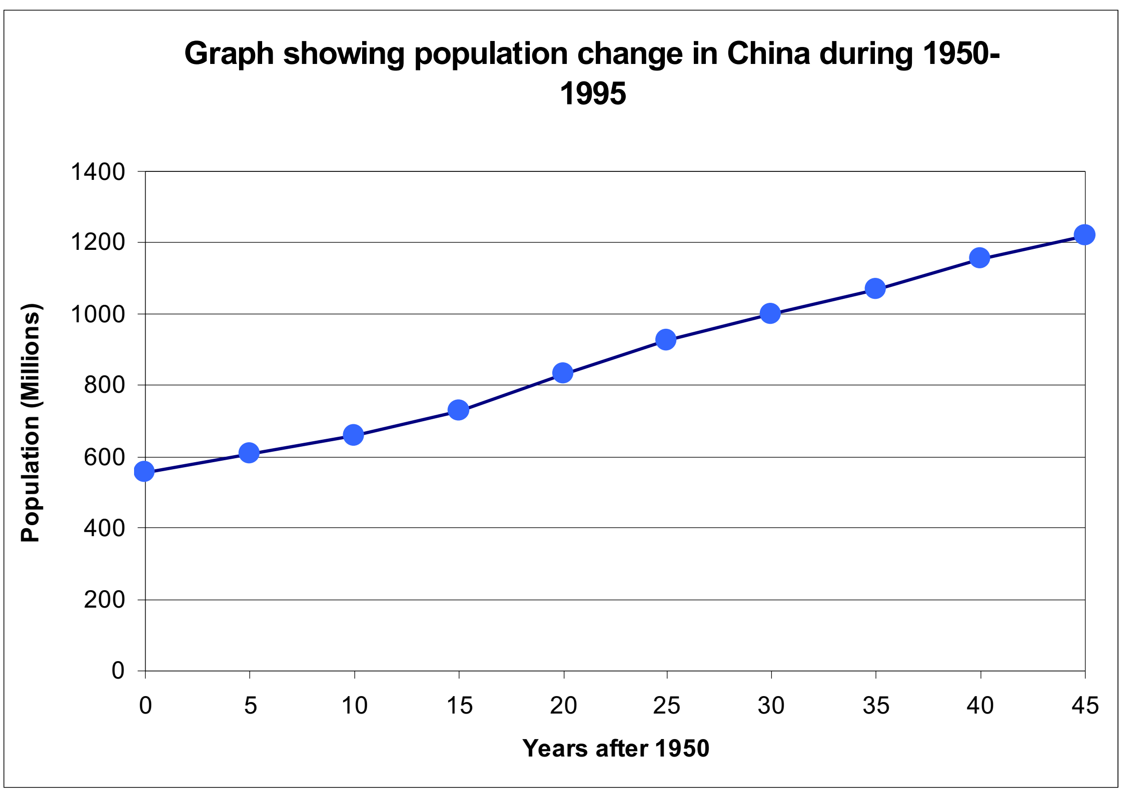
In this portfolio, I will be investigating a variety of functions in order to find out which ones best model the population of China from 1950 to 1955. In order to do this, I will be using a number of different technological methods which will help my investigation, with all my findings contained in this portfolio.

MODELLING THE POPULATION OF CHINA

The following table shows the population of China from 1950 to 1995.

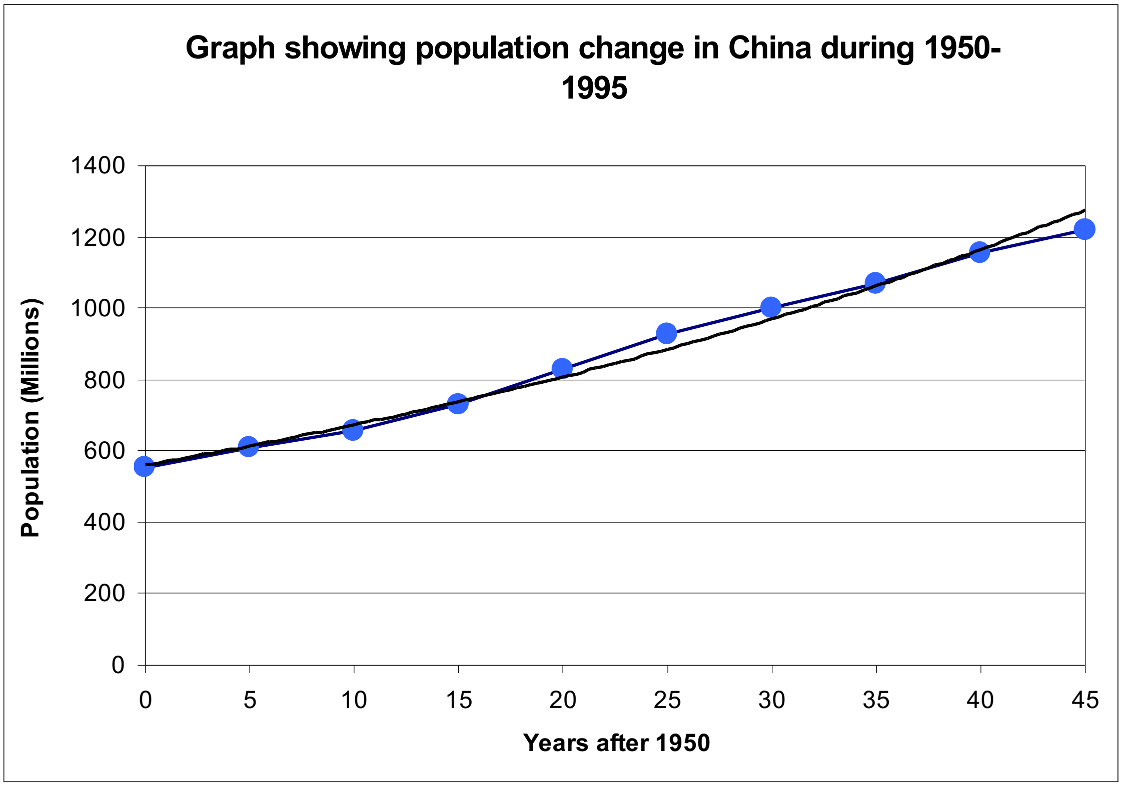
Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Population in Millions	554.8	609.0	657.0	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

As one can see from the data, there are two variables – the year and the population in millions. The year will be represented by x and the population will be represented by y There are restrictions that also need to be set; the year as well as the population can never be anything below 0. My parameter for time will be that for each year, "t" will equal the number of years af ter 1950. Therefore, for 1950, "t" will equal 0, for 1955 "t" will equal 5 and so on. Below is a graph that plots the above data points.

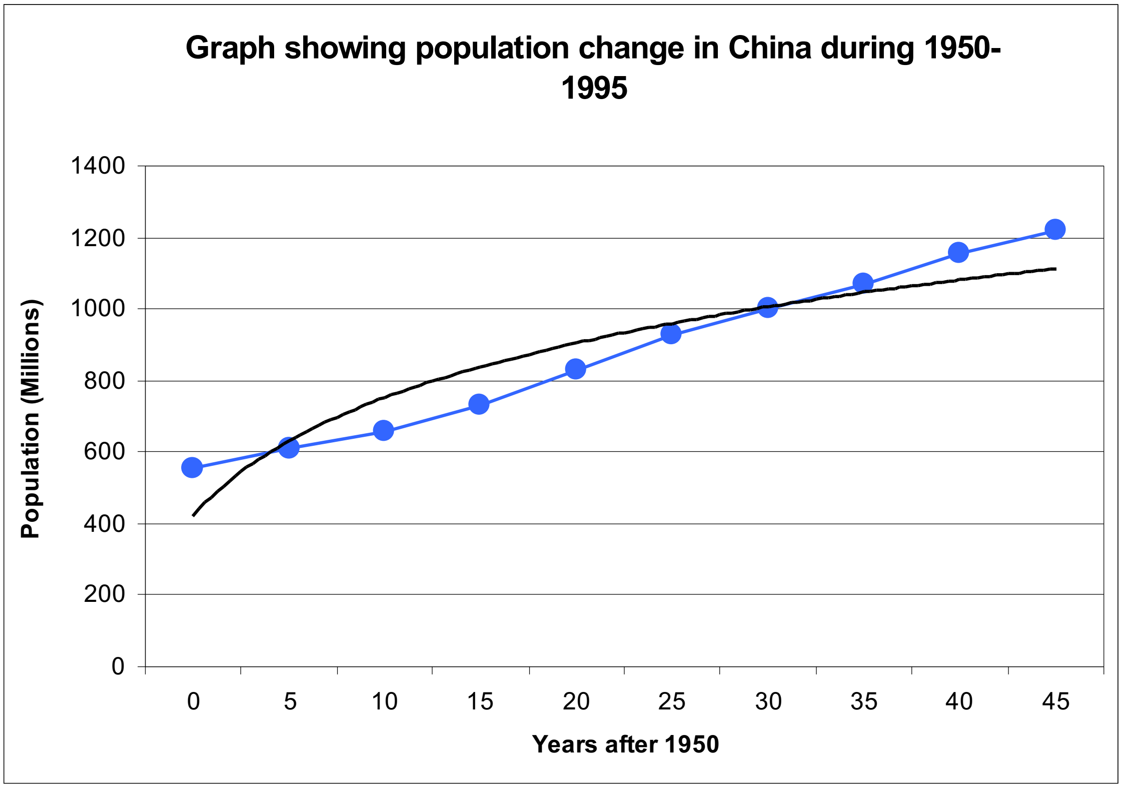


Looking at the graph, created using Microsoft Excel, one can see a gradual increase in population between the years 1950 to 1995 at intervals of five years. Below I have used several functions to decide which model will best fit the data.

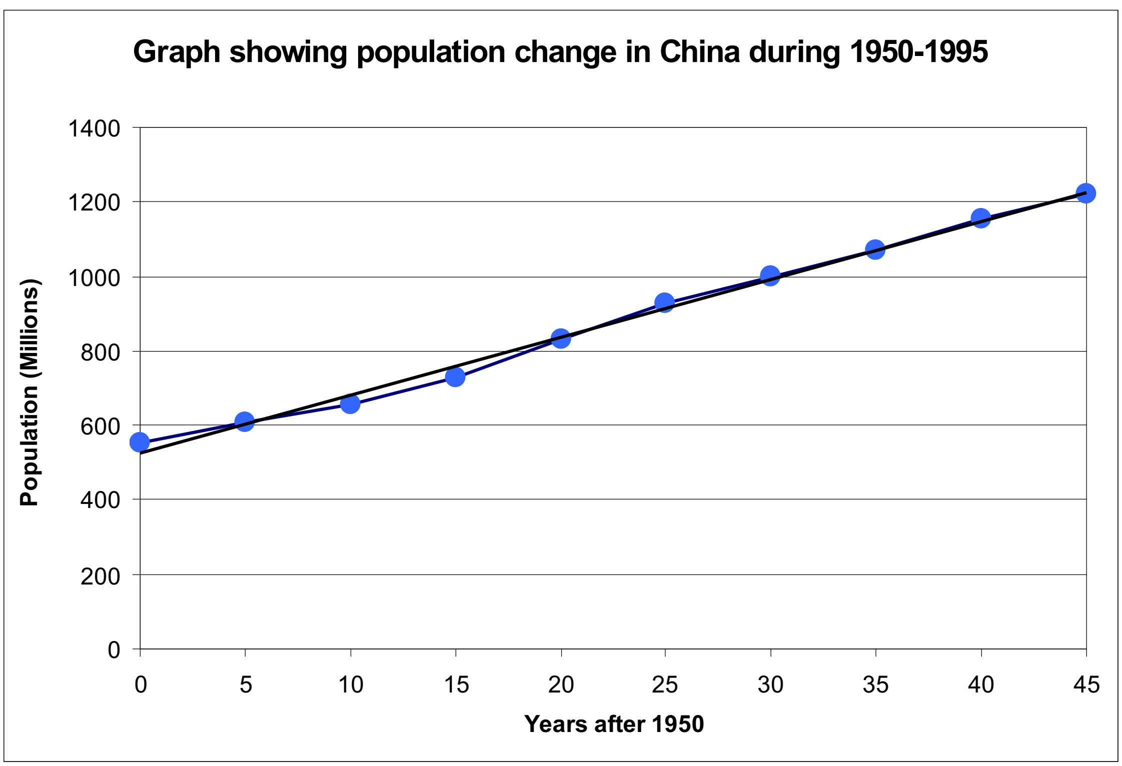
Exponential



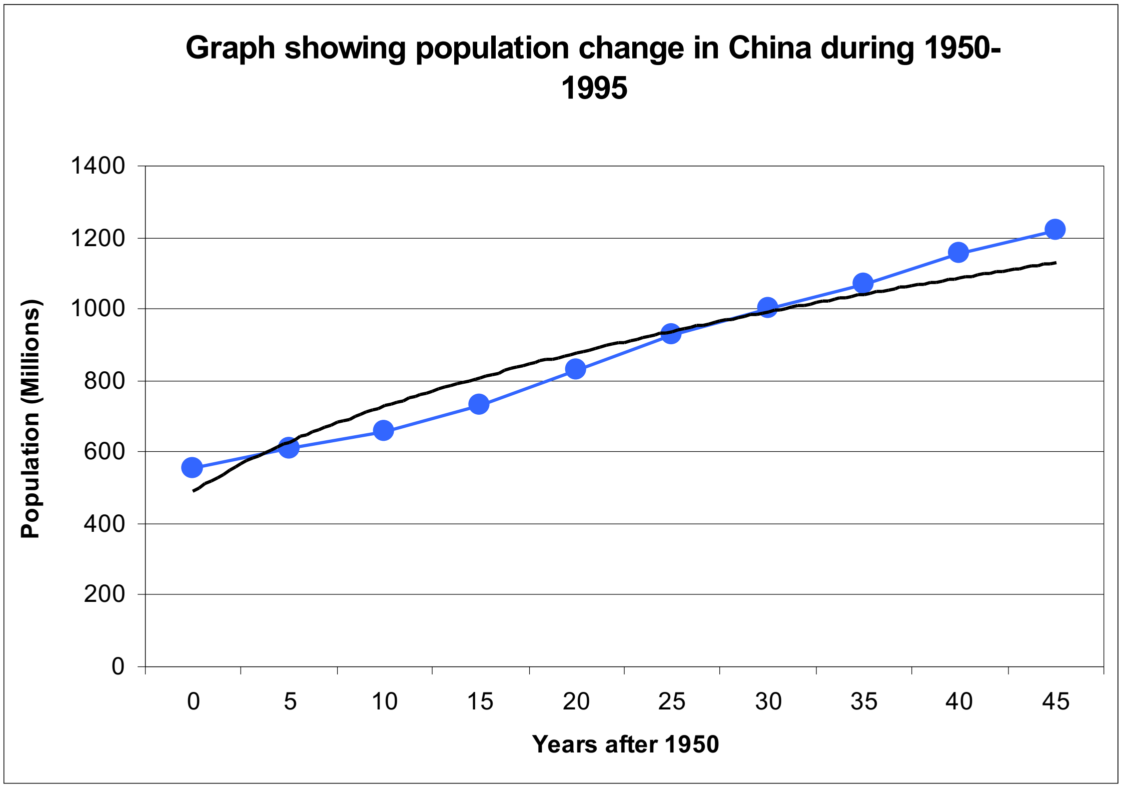
Logarithmic



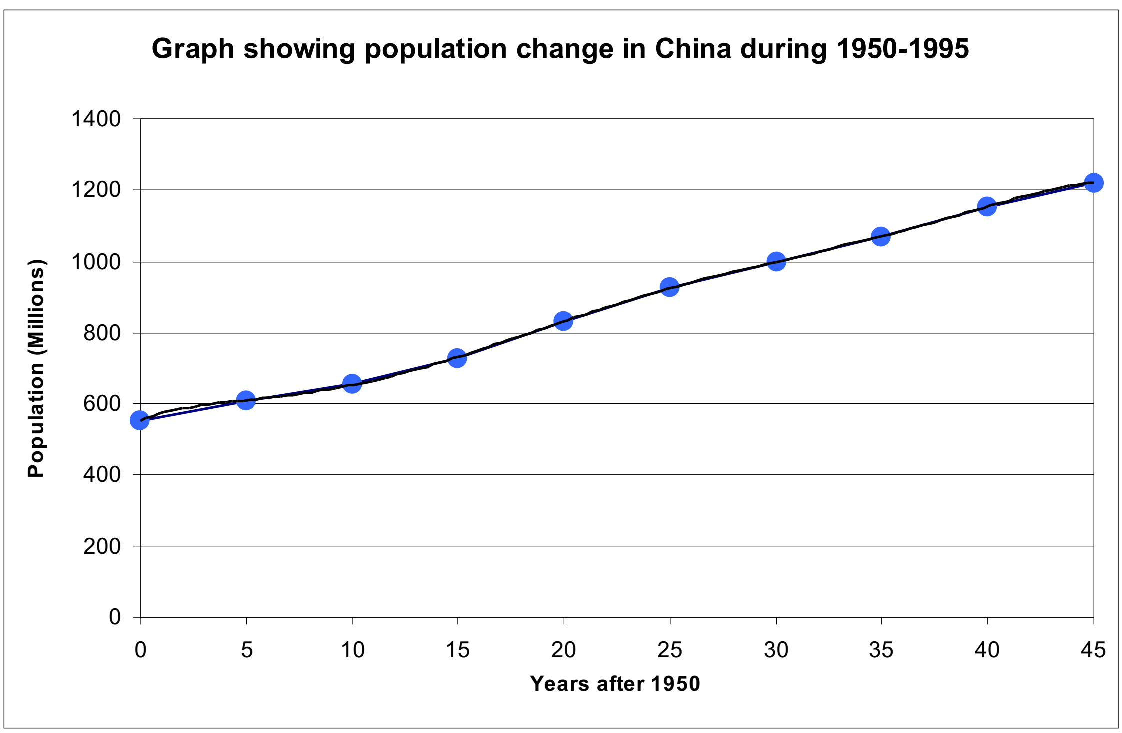
Linear



Power of x



Polynomial (6th degree)



Indeed the rate of this increase constantly chan ges with it getting faster and slower. Therefore, it would not make sense to use an exponential, or indeed a linear model to best fit the data. An exponential model would not make sense as it would not represent the slowing down of the population growth an d the linear model would not allow the rate of population increase to change as it does on the graph.

However polynomials can be integrated to have a higher precision and so I have decided that this will best model the population growth of China. This also eliminates the use of logarithmic and power of x functions as they wouldn't be as precise. The best polynomial would be one to the 6th degree as it is the highest, and most precise degree that the excel software has to offer. This means that it will follo w this general equation:

$y = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g$

From this I have obtained the following function:

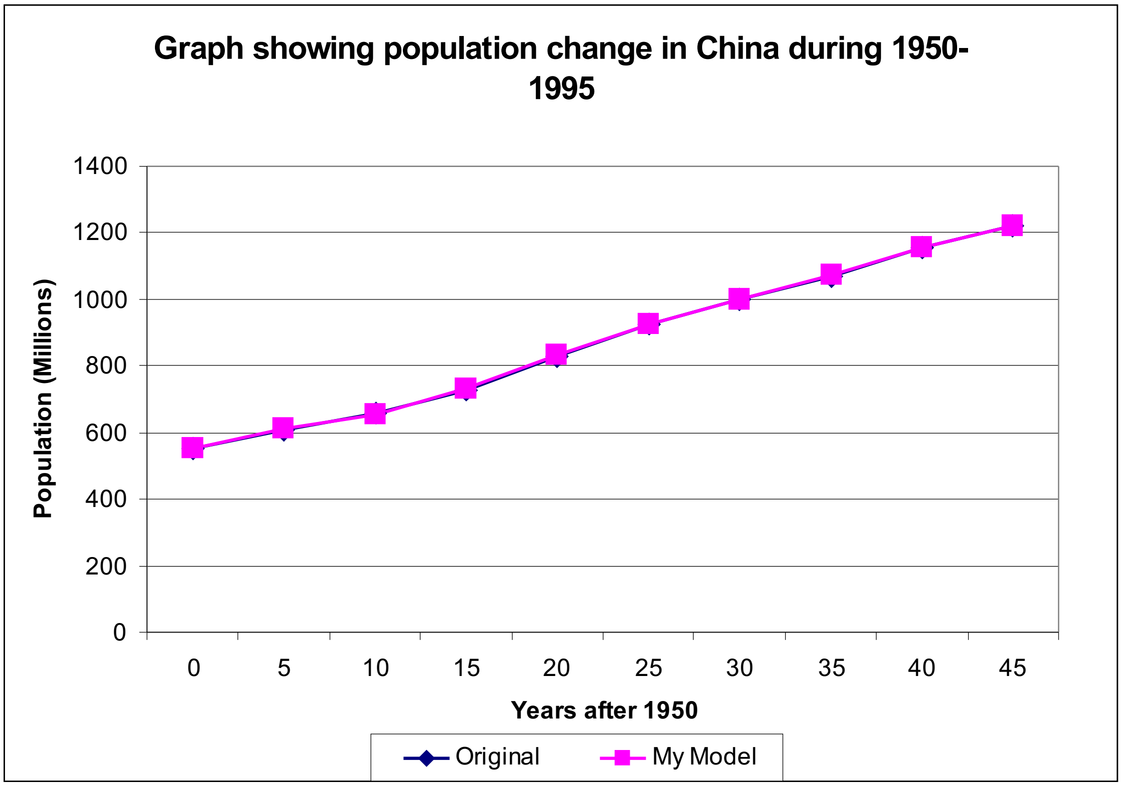
$y = -0.0000022702x^6 + 0.0003191282x^5 - 0.0167831966x^4 + 0.3982148012x^3 - 3.8791438329x^2 + 22.6459713893x + 554.4475579557$

As you can see, I have

If I substitute the elapsed time into the above equation, I get the following values:

Years After 1950	0	5	10	15	20	25	30	35	40	45
Population in Millions (My Model)	554.4	610.9	653.0	732.1	832.0	924.5	999.8	1071.3	1154.4	1220.9
Population in Millions (Original)	554.8	609.0	657.0	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

We can now see my model in comparison to the other model.



As one can see, my model has proven to be very accurate to the original data given the very minimal difference seen in the data and the graph. Indeed it is not necessary to revise this model.

RESEARCHER'S MODEL FOR THE POPULATION OF CHINA

A researcher has suggested that the population, "P" at time "t" can be modelled by:

$$P(t) = \frac{K}{1 + Le^{-Mt}}$$

We need to use a Graphical Display Calculator (GDC) to work out K, L and M. Given the five year intervals of the data, one can make "t" the number of years after 1950 and we already know that "P" means population. Therefore we have to input this into our GDC:

x(t)	0	5	10	15	20	25	30	35	40	45
y(P)	554.8	609.0	657.0	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5

The researcher's model is a logistic function and so we need to use the "Regression" then the "Logistic" function to find out the estimates for values of K, L and M. These are as follows:

K= 3230.36868

L=4.77937

M=0.02431256

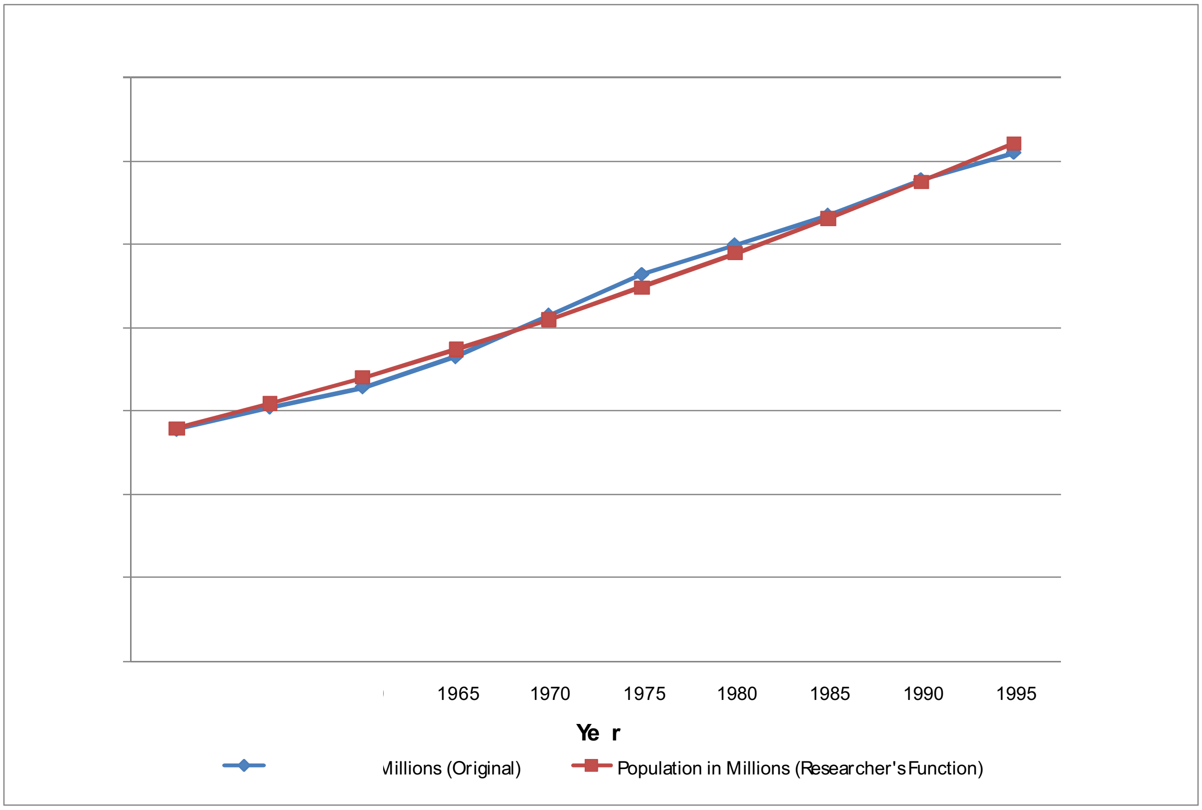
Therefore:

$$P(t) = \frac{3230.36868}{1 + (4.77937)e^{-0.02431256t}}$$

Having calculated the population using the researcher's model, we can see that the table looks like this.

Year	1950	1955	1960	1965	1970	1975	1980	1985	1990	1995
Population in Millions (Original)	554.8	609.0	657.0	729.2	830.7	927.8	998.9	1070.0	1155.3	1220.5
Population in Millions (Researcher's Function)	558.9	617.4	680.4	748	820.1	896.7	977.5	1062.3	1150.7	1242.3

I have now created a graph to compare the two above results on the same axis. This can be seen below.



As one can see from the graph, the researcher's model fits the original data relatively accurately, however, being an exponential model, it does not reflect the points in which the original data slows and down and instead increases at a fairly constant rate.

Indeed each of these models suggests that the population will continue to increase at a similar as it has done the past 45 years. I found however, that with my model, the population actually decreased showing that in the long te rm, my model inaccurate. With the researcher's model, I found that the data did not reflect any possible changes in the rate of population change but is in the long run more accurate then my model.

Additional Data

Let's now look at additional data on popul ation trends in China from the 2008 World Economic Outlook in conjunction with my model and the researcher's model .

Year	1983	1992	1997	2000	2003	2005	2008
Population in Millions (Original)	1030.1	1171.7	1236.3	1267.4	1292.3	1307.6	1327.7
Population in millions (My Model)	1030.1	1227.5	1171.7	1125.9	1125.9	1097.6	1125.9

Population in millions (Researcher's Model)	1045.6	1173.6	1225.2	1256.8	1305.2	1326.4	1348.9

As one can see, the best model for the new data is the researcher's model. However, I need to modify this model so that will best fit the data from 1950 to 2008.

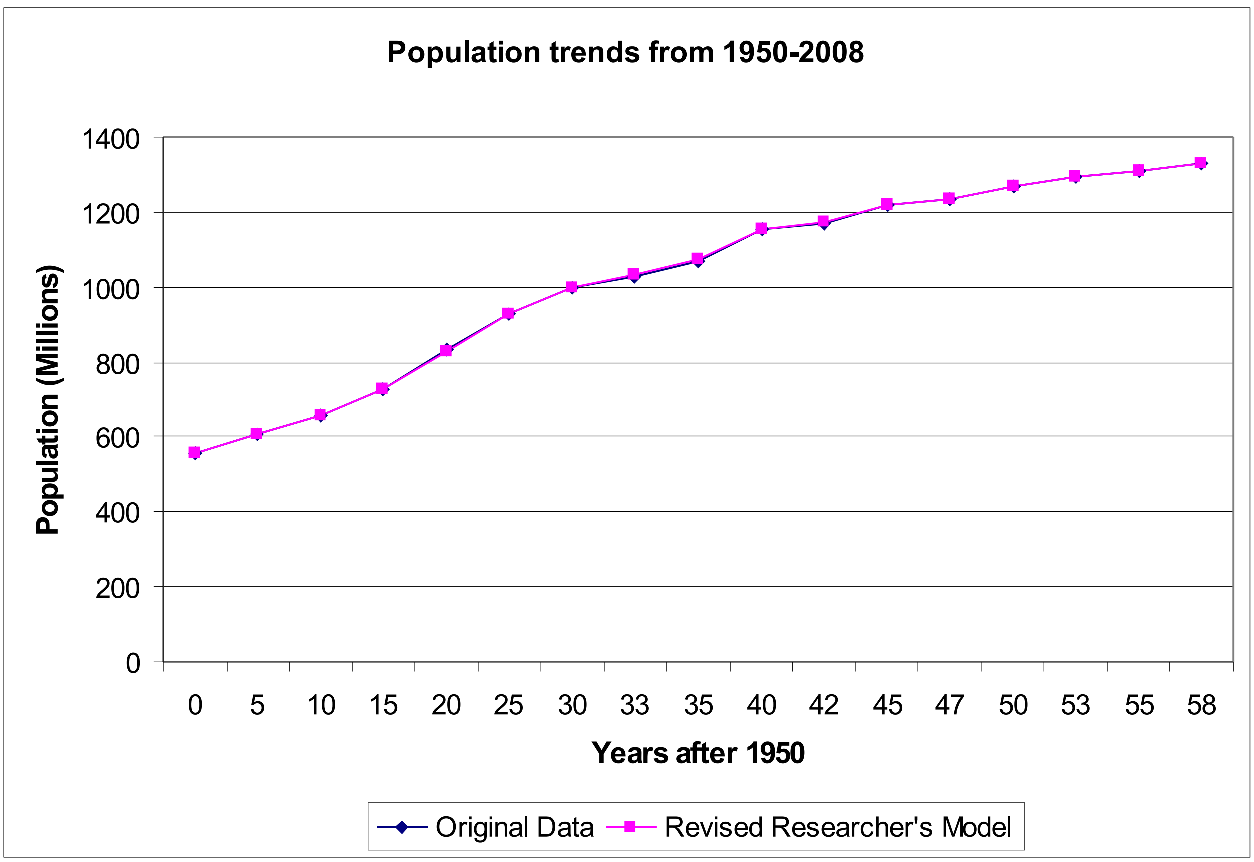
I have found these knew values of K, L and M:

K= 1809.69011

L= 2.31173963

M= 0.03216208

With my revised model I have created this graph:



As one can see, the modified model fits all the data relatively accurately, proving that the researcher's model, when revised works best.

[illegible] used Microsoft Excel and GDC for my work.