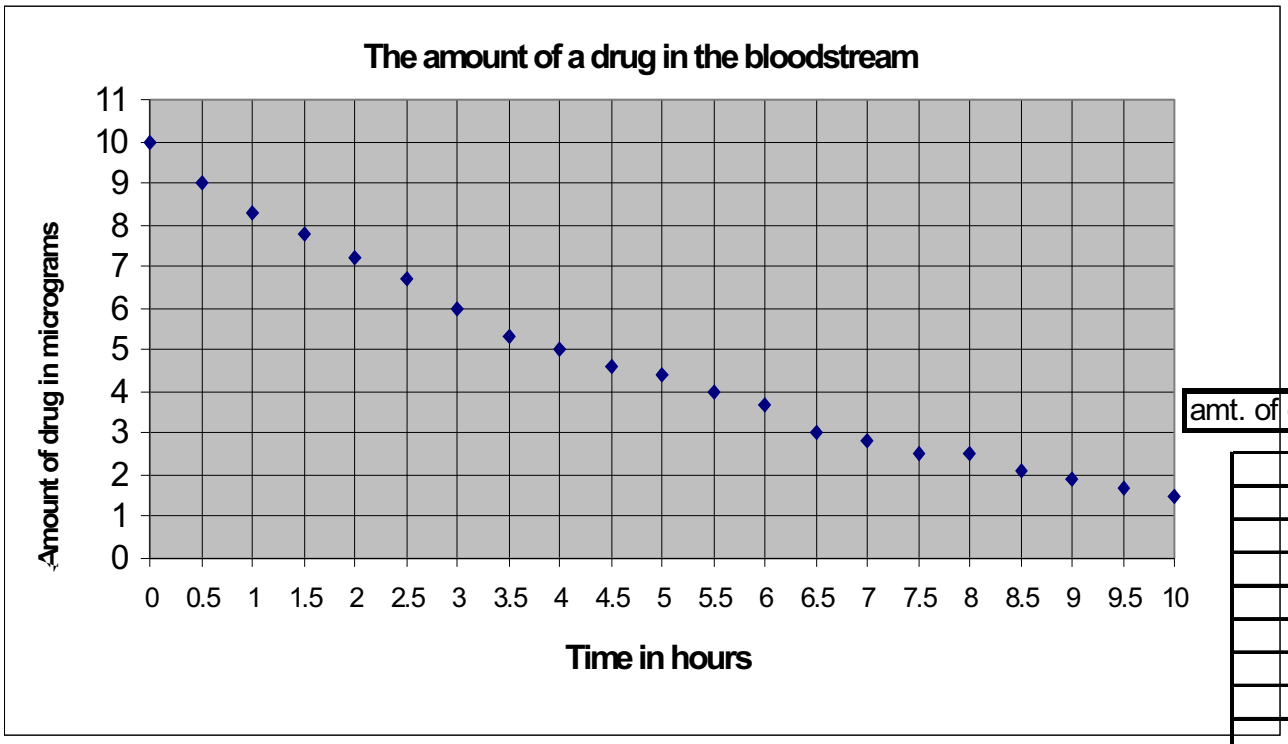
IB SL I
4/19/07

Modeling the amount of a drug in the bloodstream

Description of task

The graph below records the amt of a drug for treating malaria in the bloodstream over 10 hrs following an initial dose of 10 micrograms. Student must find a suitable function to model the data given and compare and analyze the student's model to the given data. Finally, interpret the data by stating limitation and modification for the model and how the model can be applied to real life situations.

Original Data Given:



amt. of drug	time (hrs)
0	10
0.5	9
1	8.3
1.5	7.8
2	7.2
2.5	6.7
3	6
3.5	5.3
4	5
4.5	4.6
5	4.4
5.5	4
6	3.7
6.5	3
7	2.8
7.5	2.5
8	2.5
8.5	2.1
9	1.9
9.5	1.7
10	1.5

Part A

Find a suitable function for your graph and comment on it.

MODEL A

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3	6
3.5	5.3
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4.5	4.6
5	4.4
5.5	4
6	3.7
6.5	3
7	2.8
7.5	2.5
8	2.5
8.5	2.1
9	1.9
9.5	1.7
10	1.5

MODEL A

A1. This model demonstrates the exponential decay relationship between the time in hours and the amount of drug left in the bloodstream. According to the model data, the amount of hours increases as the amount of drug in the blood stream decrease. The function of the trend line is y= 10.391e^-1863x. The coefficient of determination is .9942. "The rate of decrease of the drug is approximately proportional to the amount remaining."

A2. Method

First, I start by making a scatter plot using the Excel Program. I enter the x and y values (data) given on the task sheet into two columns. Then I add the exponential trend line for the line of best fit by right clicking the data points on the scatter plot and "add trend line". I chose the exponential trend line because after looking at the other available trend lines in the Excel program, the line that will provide me with the r^2 value closest to 1 is the exponential decay trend line with the r^2 value of .9942. A trend line is most reliable and reasonable when the r^2 value is at one or very close to one. Also, the exponential line looks like it is the most reasonable and best fitting. The graph I made is very similar to the model given. The only difference is the original data didn't have the line of best fit on it. It still contains the same data points and scale.

The r^2 or the coefficient of determination displays how closely my model fits the data. The coefficient of determination is "the ratio of the explained variation to the total variation. The coefficient of determination represents the percent of the data that is the closest to the line of best fit." "The coefficient of correlation of a set of pairs of quantities is equal to the sum of the products of the deviation of each quantity in the pairs from its respective mean, divided by the product of the number in the set and the standard deviations. It indicates the strength and direction of a linear relationship between two random variables. The coefficient of determination is a measure of how well the regression line represents the data. If the regression line passes exactly through every point on the scatter plot, it would be able to explain all of the variation. The further the line is away from the points, the less it is able to explain."(Math bits)

We can use the following formula to calculate r^2:

$$\frac{\text{Explained Variance}}{\text{True Variance}} \quad \text{OR} \quad r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2}\sqrt{n(\sum y^2) - (\sum y)^2}}$$

The closer the value it is to 1.00, the more suitable the line is for the graph. I just used the Excel program to calculate the r^2 value for me.

A3. Interpretation

The reasonableness of the data is acceptable for the most part with minor discrepancy. The data is pretty reasonable until it goes beyond 10 or more hours when the amount of drug in blood reaches below 0 with negative numbers which are impossible.

Also, in the data given there is a little minor mistake. Between the 7.5^{th} hr to the 8^{th} hour after the initial dose of drug, the amount in the blood stream remains the same. I thought it might be a human error because I didn't think the amount of drug would stay the same for that specific hour. It should have a small decrease.

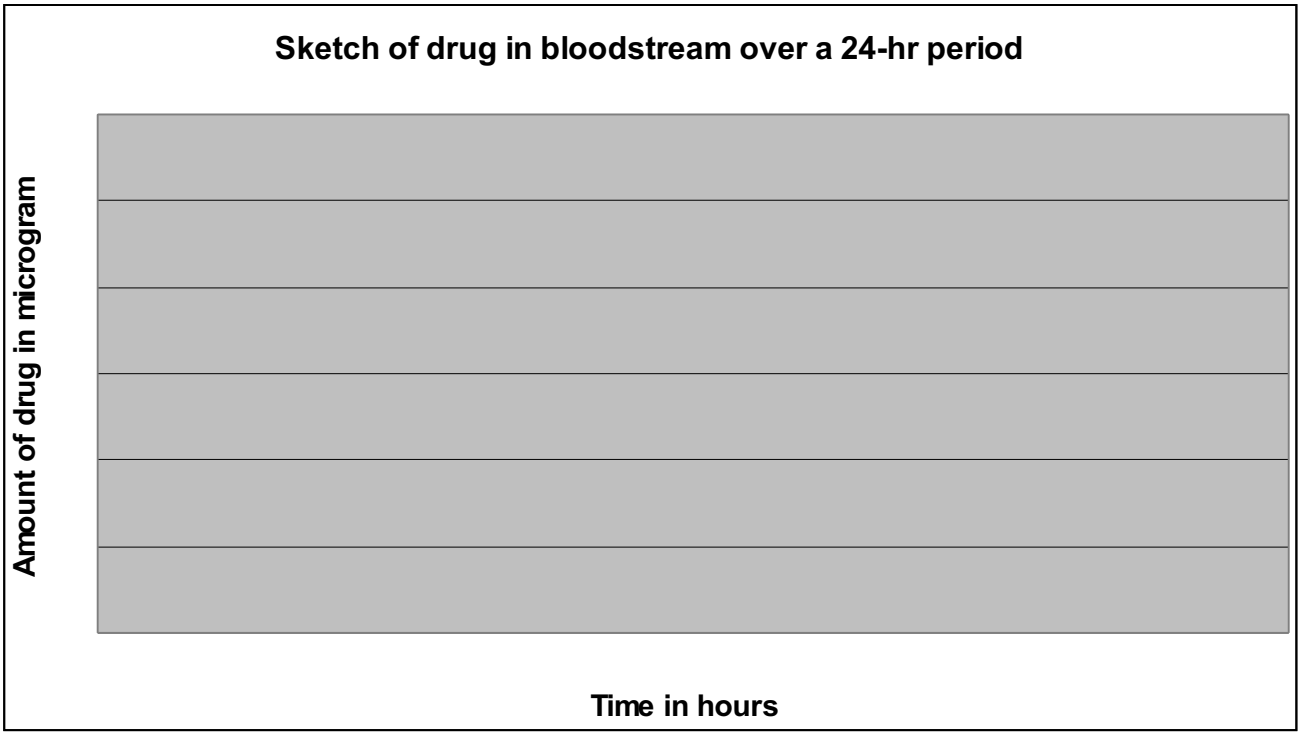
The suitability of this graph is good. The r^2 is .9942. The r^2 being .9942 allows me assume that the trend line fits very well near actual data points. From just looking at the graph it self, it looks like most of the data points are on the line. This is the most suitable one I saw out of my other line of best fit options. Example, the linear line of best fit didn't even fit on about half of the data points. Even thought the polynomial trend line graph is not bad, the r2 is not as close as the exponential line of best fit.

Part B

A patient is instructed to take 10 microgram of this drug every six hours.

1. Sketch… over a 24 hour period

Model B



a. I assume that the amount of drug in the slope of the curve after the initial dose would be steeper. I assume that because the starting amount of the drug at 6 hours is different than the amount of drug starting at zero hours. For a person's body, I think the decay would not be the same rate if the initial amount of drug already in the body is increased. The "rate of decrease of the drug is approximately proportional to the amount remaining." The more drugs there is to start with, the fraction of the amount before is not going to be the same. If the rate doesn't change, then the relationship would not be proportional. Therefore, I assume that the slope of the trend line would be steeper.

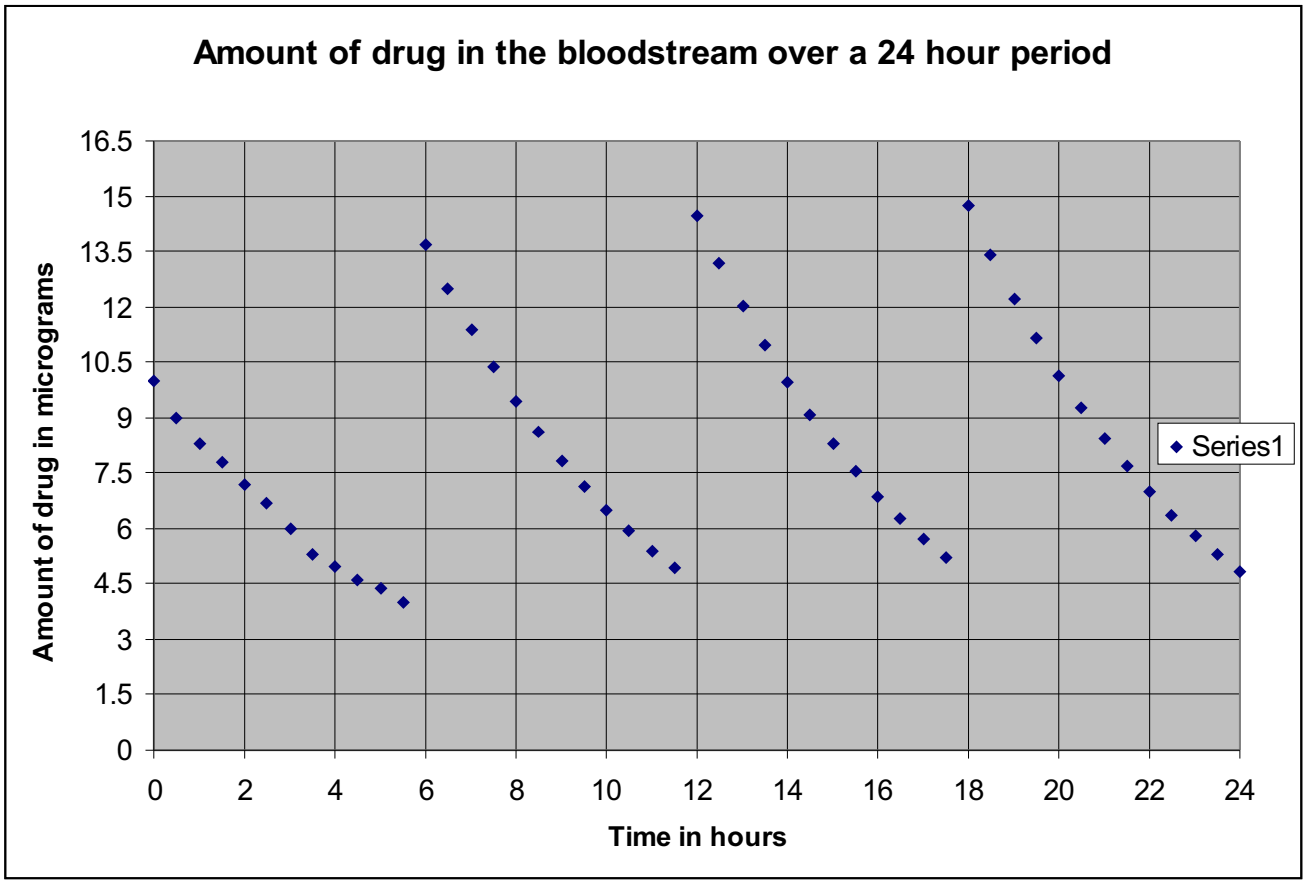
b. I also assume that there is left over drug in the bloodstream from the previous dose while taking the next dose. The drug is supposed to be taken every 6 hours and according to the given data, there is still drug in the bloodstream after 6 hours. Therefore, the amount of drug left in the bloodstream before the next dose would get a little higher because more drugs is added before all the drug was gone in the bloodstream. I assume that the left over drug would start building up in amounts in the body. Example, after 6 hours, there is 4 microgram of drug left in the bloodstream. After 12 hours there will be 4.3 micrograms left in the bloodstream, and it goes on building up higher. Another reason I assume that is because some medication labels would something like "take this drug for 5 days and if the illness continues, stop taking this drug and see your doctor." I predict that the manufacturer of the drug do not want the person taking the drug to have excess amount of the drug inside his/ her bloodstream because the patient is continuously taking it.

2. Data points for model B (model B on next page)

0	10
0.5	9
1	8.3
1.5	7.8
2	7.2
2.5	6.7
3	6
3.5	5.3
4	5
4.5	4.6
5	4.4
5.5	4
6	13.7
6.5	12.48141
7	11.391
7.5	10.35988
8	9.4384
8.5	8.599
9	7.8341
9.5	7.1373
10	6.5025
10.5	5.92417
11	5.3923
11.5	4.9172
12	14.4799

12.5	13.192
13	12.01867
13.5	10.9497
14	9.9758
14.5	9.0885
15	8.2802
15.5	7.54369
16	6.8731
16.5	6.26145
17	5.7046
17.5	5.19716
18	14.7349
18.5	13.42432
19	12.23
19.5	11.1425
20	10.15
20.5	9.2486
21	8.42597
21.5	7.6765
22	6.9938
22.5	6.3717
23	5.805
23.5	5.2887
24	4.818286

Model B



This graph models the amount of drug in bloodstream over a 24hr period of time. On the graph, I can see that the dose is added every six hours and the slope gets slightly steeper every time because of the increasing gap between the first two dots after each new dose. The amount of drug in the bloodstream increases and decreases repeatedly.

Method

I started by entering the data given up to the sixth hour. At the sixth hour, I added 10 to the left over amount of drug in bloodstream. I use the original exponential function to figure out what the amounts should be. I know the y value and the x value. The y value is the left over amount at 6 hours plus 10 for the new dose to figure out the new amount of drug in the bloodstream. All I have to do is change the "10.391"To z and solve for it in a graphing calculator. Afterwards, graph the equation on a graphing calculator. *(NOTE: I'm sorry, but I do not have the software to copy the ti-83 screen to the computer so I can't show pictures of the screen from the ti-83. I try to explain as best as I can with words on the next page))*

Step 1: Orginial function: $y=10.391e^{-.1863x}$
Step 2: replace x and y with know values and change the 10.391 to z.
X: 6 (because it's the time in hours that the new dose is going to be taken)
Y: 3.7 +10.0 = 13.7 because it's left over amount of drug from previous dose plus 10 (new dose) for a total of 13.7 micrograms.
New equation: $13.7= z e^{(-.1863*6)}$
Step 3: solve for z
a.) -.1863*6=-1.1178; b.)e^-1.1178=.3260; c.)13.7/ .3260=41.896=z
Step 4: graph the old equation replacing the 10.391 with z (41.896) in a graphing calculator.
$Y= 41.896e^{-.1863x}$
Step 5: find the new values
Use the table to find the new values up till the next dose at the 12 hour and repeat the same procedure here.

Interpretation

The reasonableness of Model B is logical, for all my assumptions are true according to Model B. The data seems scientifically reasonable. It makes sense and there are no unreasonable amounts in the values. The model did show a steeper slope and an excess buildup of leftover drug. I didn't add a line of best fit because there wouldn't be one that would fit all of the data points in a suitable way. If I do each separately, they should be similar to each other.

Approach

3. Maximum and Minimum

Maximum is at the 18^{th} hour at the beginning of the 4^{th} dose of drug with the amount of 14.7349 microgram in the bloodstream. With the excess buildup of leftover drug, the maximum is at the beginning of the last dose taken, in this case of 24 hours, the 18hr would be the last dose.

Minimum is at the 5 and a half hour with the amount of 4 micrograms in bloodstream. With no buildups from previous doses, the amount at the end of the first dose would be the lowest.

4. What would happen to the values during these periods?

(a) No further doses are taken

If no further doses are taken, then according to the model chart, the amount of drug in the bloodstream would continue to decrease until the amount is too small to measure. The model is an exponential decay which means the line will never touches zero, but in this case, I can assume that the drug would eventually be completely gone from the body even though the model shows that the amount would never reach zero. I just assume the line would reach to the amount when it can't be measured and call it the zero mark.

(b) Doses continue to be taken every 6 hours.

If dose continues, then the graph would continue. The left over amount of drug in the body would continue get higher and higher. Even though it increases and decreases, there is still going to be buildup of drugs. I am assuming that if this person or patient takes too much of this drug, he/ she might get into an overdose situation with all the excess amount of drug. The values in this graph will be completely opposite of the graph in

situation a.

Real life situations along with limitations and modification

For this model to accurately represent the model, the process the drug is inserted has to be the same. Example if this malaria drug was injected into the bloodstream by a needle, then any other drug that is going to follow this model must also have the drug be injected into the body the same way for the model to fit the data. Also, for the model to fit the situation the drug must have the same rate of decay and I guess the particles or something that makes up the drug must be similar. If the rate of decay is different because the drug is made from different particles, then the graph would be different.

Everyone's biological processes in the body are different. Every drug will not have the same effect in everyone. Some, the drug might help them recover fully from this malaria after taking the drug for a week while some people have to continuously taking the drug for months to get the same results.

What if the person was taking another drug along with this current drug? I think that if another drug is taken while this drug is also taken, one drug might cause the other drug to decay faster in the body. The model can not accurately represent what will happen because often, people take drugs without first asking a doctor. A common person would not know what would happen if two different drugs were taken together.

There are too many different factors that needed to be controlled for the model to work, so I do not think this is a very accurate model. Maybe the model is accurate to a certain extent. It should only be an approximate model of amount of drug in the bloodstream. Currently, I can not think of anything to modify this model to make it more accurate.