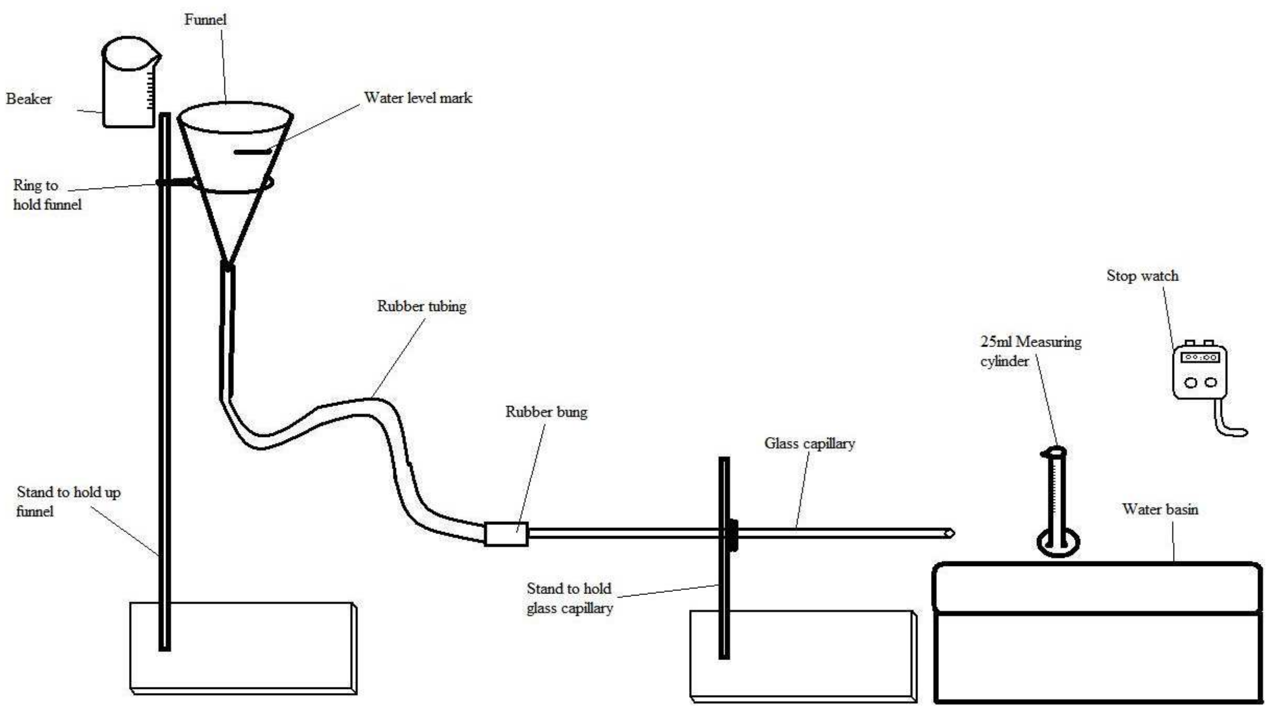
Part 1 – Fluid Dynamics and Pressure Measurement

Aim
The aim of the investigation is to find the viscosity of water using a rigid glass tube (capillary). By further calculations, to find whether the flow of liquid is laminar or turbulent.

Diagram



Apparatus	***Uncertainty***
Water	
Beaker	
Funnel	
Rubber tubing	
Rigid glass tube (capillary)	
Basin	
Travelling Microscope	+/- 0.0005cm
Measuring Cylinder	+/- 0.5cm^3
Stop watch	+/- 0.005s
Ruler	+/- 0.05cm
2 x Clamp stand	

Procedure

The apparatus was set up as shown above. The length of the glass tube was taken and recorded, this was to stay constant. The height of the glass tube from the desk was measured as was the marker of water level on the funnel from the desk. The water was then poured into the funnel up to the marker point using the beaker and allowed to pass through into the basin. This was kept constant by a member of group who poured water into the funnel. The rate at which the water passed through at this particular height was measured. This was carried out by a member collecting water in the measuring cylinder, while another started the stop clock instantly. The clock was stopped as soon as the water reached 25 cm^3 in the measuring cylinder. The volume here was kept constant, so that there were fewer errors in calculations. This was then repeated for another nine times, with varying heights of the water level on the funnel.

Results

Table 1: Results obtained in the experiment.

Height, h (m)	Change in Pressure, ΔP (Pa)	Volume, v(cm^3)	Time, t (s)	Flow Rate,Q (m^3s^{-1})	Viscosity, η	
0.519	5091.39	25.0	25.93	9.6413E-07	8.4585E-04	◄
0.567	5562.27	25.0	23.59	1.0598E-06	8.4069E-04	
0.587	5758.47	25.0	22.91	1.0912E-06	8.4526E-04	
0.605	5935.05	25.0	21.75	1.1494E-06	8.2707E-04	
0.623	6111.63	25.0	20.47	1.2213E-06	8.0155E-04	*
0.671	6582.51	25.0	21.22	1.1781E-06	8.9494E-04	
0.737	7229.97	25.0	19.34	1.2927E-06	8.9588E-04	
0.785	7700.85	25.0	18.30	1.3661E-06	9.0291E-04	
0.840	8240.40	25.0	17.65	1.4164E-06	9.3186E-04	
0.974	9554.94	25.0	14.97	1.6700E-06	9.1644E-04	

◄ = Readings obtained by my group
* = Anomaly in results

Mean, μ 8.7025E-04
Standard Deviation, σ 4.33854E-05

Calculations

Change in Pressure, ΔP

The height (h) of the water from the funnel to the glass capillary was measured. This is firstly used in calculating the pressure change through the system of the apparatus. The density (ρ) of water is required as is the gravitational force.

h = height of water to capillary = 51.9cm = 0.519m
ρ = density of water = 1000 kg m^{-3}
g = gravitational force = 9.81 ms^{-2}

$$\Delta P = h\rho g = 0.519 \times 1000 \times 9.81 = 5091.39\text{Pa}$$

This calculates to be the overall pressure, as the atmospheric pressure is discarded in the calculation. This is due to the fact that there is atmospheric pressure at the funnel, where the water is entered and there is the same pressure at the end of the glass capillary, thus they cancel out.

Flow Rate, Q
This was then measured by allowing a cylinder of volume 25ml, to be filled in a certain time. The time was collected using the stop watch. By converting the volume to cubic metres and dividing this by the time in seconds, the flow rate is calculated.

$$Q = \frac{\text{volume}}{\text{time}}$$
$$= \frac{25/\,1000000}{25.93}$$
$$= 9.6413 \times 10^{-7}\ m^3s^{-1}$$

Viscosity, η
This can be calculated using Poiseuille's Law, which concerns the stationary flow of incompressible uniform viscous liquid through a cylindrical tube with constant circular cross-section. It is defined by:

$$Q = \frac{\pi r^4}{8\eta l} \times \Delta P$$

$\pi = 3.14159265358979$
r = radius of glass capillary = 0.000575m
ΔP = 5091.39Pa
l = length of glass capillary = 0.268m
$Q = 9.6413 \times 10^{-7}\ m^3s^{-1}$

Therefore by transposing the formula, the viscosity can be calculated:

$$\eta = \frac{\pi \times (0.000575)^4 \times 5091.39}{8 \times (9.6413 \times 10^{-7}) \times 0.268}$$
$$= \frac{1.74846771 \times 10^{-9}}{2.06709 \times 10^{-6}}$$
$$= 8.45857567 \times 10^{-4}\ kg / m\ s$$

Mean and Standard Deviation of the Viscosity, η
The sum of all the viscosity values is:
$\Sigma \eta = 8.7024527 \times 10^{-3}$

Therefore the mean value of viscosity can be calculated by dividing the sum by the number of values obtained:

$$\mu = \frac{\Sigma \eta}{n}$$
$$= \frac{8.7024527 \times 10^{-3}}{10}$$
$$= 8.7024527 \times 10^{-4}$$

The standard deviation was calculated using Excel and gives the following value from the obtained values of viscosity:

$\sigma = 4.33854 \times 10^{-5}$

The mean of the viscosity to one standard deviation is calculated by:

$\mu \pm \sigma$
Therefore the range is:
$8.7025 \times 10^{-4} - 4.33854 \times 10^{-5} = 8.2686 \times 10^{-4}$
$8.7025 \times 10^{-4} + 4.33854 \times 10^{-5} = 9.1363 \times 10^{-4}$

$[8.2686 \times 10^{-4}, 9.1363 \times 10^{-4}]$
This proves, from the values obtained in table 1, that they lie in 68% of the mean and show a normal distribution.

The mean of the viscosity to two standard deviations is calculated by:

$\mu \pm 2\sigma$
Therefore, here the range is:
$8.7025 \times 10^{-4} – 2(4.33854 \times 10^{-5}) = 7.8347 \times 10^{-4}$
$8.7025 \times 10^{-4} + 2(4.33854 \times 10^{-5}) = 9.5702 \times 10^{-4}$

$[7.8347 \times 10^{-4}, 9.5702 \times 10^{-4}]$
Therefore, all ten values lie within 95% of mean, showing quite accurate date under this distribution, with out any extreme anomalies.

Percentage uncertainties of recorded results.
The Heisenberg uncertainty principle states that one cannot assign, with full precision, values for certain pairs of observable variables. This would also apply to this investigation where we have uncertainties to the readings taken. All readings would have these uncertainties above and below the reading.

To find the percentage uncertainty we apply the following formula.

$$\% \text{ uncertainty} = \frac{\text{uncertainty}}{\text{reading taken}} \times 100$$

Apparatus	**Uncertainty**	**Reading Taken**
Travelling Microscope	+/- 0.0005cm	0.115cm
Ruler	+/- 0.05cm	51.9cm
Measuring Cylinder	+/- 0.5cm^3	25cm^3
Stop watch	+/- 0.005s	25.93s

Travelling Microscope
This was used to take measurements for the bore of the glass capillary. This was done prior to the investigation.

$$\% \text{ uncertainty} = \pm \frac{0.0005}{0.115} \times 100 = \pm 0.435\ \%$$

Metre rule
The rulers were used to measure both the height of the desk to the water level mark and also the height of the desk to the horizontal glass capillary. As the uncertainty of one ruler is +/- 0.05cm, therefore the overall uncertainty would be = 0.05 + 0.05 = +/- 0.1cm

$$\% \text{ uncertainty} = \pm \frac{0.1}{51.9} \times 100 = \pm 0.19\ \%$$

Measuring cylinder
The volume of water was measured using this cylinder. The volume was kept constant at $25cm^3$.

$$\% \text{ uncertainty} = \pm \frac{0.5}{25} \times 100 = \pm 2\ \%$$

Stop watch
This measured the time taken for the water to fill the measuring cylinder. The time was taken to a hundredth of a second.

$$\% \text{ uncertainty} = \pm \frac{0.005}{25.93} \times 100 = \pm 0.019\ \%$$

Graph 1, drawn using Microsoft Excel, is shown on the next page.

Graph 2, drawn by hand, on the page following graph 1.

Gradient of the slope from Graph 1
Knowing Poiseuille's law as:

$$Q = \frac{\pi r^4}{8\eta l} \times \Delta P$$

A gradient of a linear slope is the division of the change in *y* by the change in *x*.

$$\text{Gradient, m} = \frac{\text{change in 'y' coordinate}}{\text{change in 'x' coordinate}}$$

According to graph 1, the y- axis is the change in pressure and the x- axis is the change in flow rate.

$$\text{Gradient} = \frac{\Delta P}{Q}$$

Therefore looking at Poiseuille's formula, and transposing it so that it equals the gradient, we have:

$$\frac{\Delta P}{Q} = \frac{8\eta l}{\pi r^4} = \text{gradient}$$

Therefore the gradient of the graph is $\frac{8\eta l}{\pi r^4}$ also known as the resistance to flow.

Resistance to flow of a liquid can be characterised by the viscosity of the fluid if the flow of this fluid is smooth.

From Graph 1, the gradient = 5.490608×10^{9}
l = length of glass capillary = 0.268m
π = pi
r = radius of glass capillary = 0.000575m

$$5.490608 \times 10^{9} = \frac{8 \times \eta \times 0.268}{\pi \times (0.000575)^{4}}$$

$$\eta = \frac{5.490608 \times 10^{9} \times \pi \times (0.000575)^{4}}{8 \times 0.268}$$

$$= 8.794616 \times 10^{-4} \text{ kg / ms}$$

The accepted value for the viscosity of water, according to American Society for Testing and Materials (ASTM) is 1.0020 centipoise, cP. To compare the value obtain and the accepted value, they must be in the same units. Below is the obtained value, converted into centipoise.

1 poise = 100 centipoise = 1 g/(cm·s)

$$\eta = \frac{(8.794616 \times 10^{-4}) \times 1000}{100}$$

$$= 8.794616 \times 10^{-3} \text{ poise} \times 100$$

$$= 0.8795 \text{ cP}$$

When using Poiseuille's law to work out the viscosity of water, using the obtained results it was calculated to be 0.8795 cP. This in comparison to the accepted value is about 12% out. This small error can be due to materials used and human error during the course of the experiments.

Reynolds Number

This number is the ratio of inertial forces (resistance) to viscous forces and is used for determining whether a flow will be laminar or turbulent. It is the most important dimensionless number in fluid dynamics. The formula to calculate this number is below.

$$N_R = \frac{2rv\rho}{\eta}$$

If N_R is above 2500, the flow is turbulent.
If N_R is below 2000, the flow is laminar.

r = radius of glass capillary = 0.000575m
v = velocity of fluid
ρ = density of water = 1000 kg m^{-3}

First of all the velocity must be calculated, as this is required in the above equation. To calculate the velocity, the flow rate must be divided by the area of the capillary.

$$V = \frac{Q}{A} = \frac{Q}{\pi r^2}$$

Q = Flow Rate
A = πr^2 = area of capillary

$$V = \frac{9.6413420748E\text{-}07}{\pi (5.75 \times 10^{-4})^2}$$

$$V = \frac{9.6413420748E\text{-}07}{1.038689071E\text{-}06}$$

$$V = 0.9282221546ms^{-1}$$

Therefore Reynolds number is as follows.

$$N_R = \frac{2rv\rho}{\eta}$$

$$N_R = \frac{2 \times 5.75 \times 10^{-4} \times 0.9282221546}{8.689131045E\text{-}04} \times 1000$$

$$N_R = \frac{1.067455478E\text{-}03}{8.689131045E\text{-}04} \times 1000$$

$N_R = 1.228495085 \times 1000$
$N_R = 1228.495$

The Reynolds number calculated above, as 1228.495 is below 2000. This confirms that the velocity of fluid flowing through the glass capillary has not exceeded the critical value. This shows that the flow of the water through the capillary is laminar.

Viscosity
The values of viscosity obtained in this investigation have been proportional to the height of water set up. The viscosity increased as the height was increased. There were anomalies found, for example at the height of 62.3 cm, there was a decrease in time and this in effect caused a decrease in the viscosity. This is due to human error and uncertainties of the readings taken. Reaction time of a person is also an error which has not been taken into account. Such as when measuring the time for the rate of flow of water, one person measures the water and has to say start and stop, whilst another listens and presses the buttons accordingly on the stop watch.

The mean value of the viscosity is a good form of examining the average as the data did lie within certain percentiles of the normal distribution. Although the gradient of the graph gave a different value for the viscosity than that calculated from the raw data, it was a miniscule difference as the numbers are so small, being to the standard form of -4. The difference of these two values was 1%, showing heavy closeness. Graph 1 was drawn to much more accuracy of values, as well as ensuring the regression line passes through the origin. This was done because Poiseuille's law says that at zero height of water fluid, there is zero pressure, and therefore no resistance to flow. For graph 2, this was not possible, and so it would cause heavy errors when calculating resistance to flow, and thus the viscosity would have been inaccurate.

Constituents of blood
Approximately 45% of the blood volume is cells; this includes white cells, platelets and mostly red cells. Plasma makes up the rest of the blood, of which 95% is water. This also contains nutrients such as glucose, fats, proteins, and the amino acids needed for protein synthesis, vitamins, and minerals.

Poiseuille's formula is good at predicting flow with fluids that are homogenous, (where all the particles of the fluid are of molecular size) such as water. Though it can only offer approximations for non-homogenous fluids as it contains molecules of different sizes, such as blood. Viscosity can also be influenced by the concentration of blood cells, if the concentration of blood cells is very high, the viscosity increases up to 5 times that of normal blood.

This law also predicts a linear relationship between blood flow and pressure gradient. It is not found to be correct as the blood vessels are composed of elastin and collagen, which are elastic elements and stretch under pressure and so the vessel offers less resistance to flow than if it were a rigid tube. Under normal conditions the flow of blood in the aorta is lami nar, where $v = 0.3ms^{-1}$. However, during heavy exercise the velocity can increase five times more and the flow becomes turbulent.

Conclusion

Carrying out this investigation, confirms Poiseuille's law for the viscosity of a fluid. It also proves how there are certain factors to be taken into account when using this formula. These factors include the pressure difference, the diameter/ radius of the tube and thus the flow rate of the fluid. There being this number of factors affecting Poiseuille's Law leaves room for error, as there has been in this investigation. Human error is quite significant and accounts for most of the anomalies that occurred. If there were more time, further precautions could have been taken to improve the investigation, and cut out certain sources of error.

Studying Poiseuille's law, it is clear that there is this relation between the height, flow rate, pressure, radius and length. The radius and the length of the capillary were kept constant, though as the pressure increases; it would cause an increase in the viscosity. Also as the flow rate increased, it would cause a decrease in the viscosity. The flow was found to be laminar, though if there were more time to further the investigation there is a possibility the flow may have developed into turbulence.

Bibliography

http://msl.irl.cri.nz/training_&_resources/Technical_guides/TG8.pdf

http://www.uwcsea.edu.sg/chem/IBfolder/IBH/PRACTICAL/ibintropracp1volumetricerrors.html

http://www.saburchill.com/physics/chapters/0068.html

http://encarta.msn.com/media_461516365/Constituents_of_Blood.html

http://hyperphysics.phy-astr.gsu.edu/Hbase/pfric.html