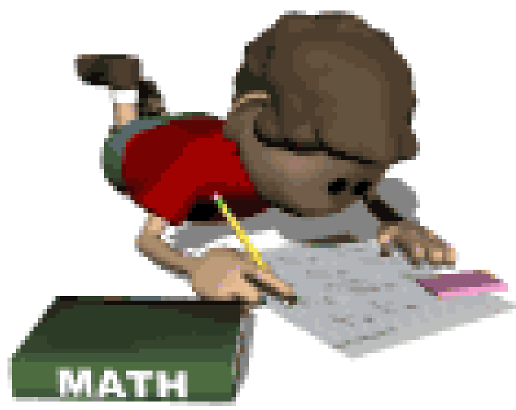
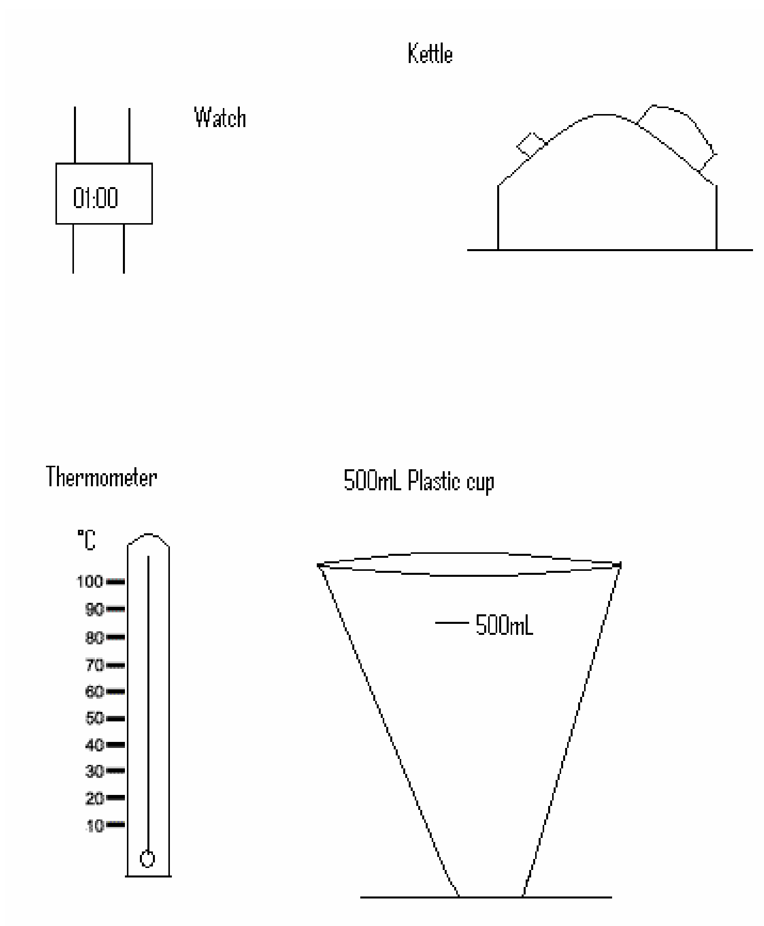
Maths Assignment 1: Exponential Decay in Reality



Instructions to operate

- *Kettle to the boil*
- *Pour 500ml of water in cup (plastic)*
- *Place thermometer in water*
- *Take temperature every minute for the 1st 30 mins and then every 5mins for the next 30 min and then every 10 mins until it is back to room temperature.*
- *Experiment conducted 3 x (results averaged)*
- *Used the exact same equipment for each experiment (water type, container, thermometer, kettle)*



My apparatus is designed to simulate the process of exponential decay. The model was measuring the temperature drop from boiling water. It was considered that the temperature would only ever drop to room temperature, therefore the temperature difference between the water temp and the room temp was measured to get a final reading of zero, which outlines **Newton's cooling law**.

From experimental observations it is known that (up to a ``satisfactory'' approximation) the surface temperature of an object changes at a rate proportional to its relative temperature. That is, the difference between its temperature and the temperature of the surrounding environment. This is what is known as **Newton's law of cooling**.

In the process of undertaking this experiment many **assumptions** were made.

- Taking into account the water was poured from the kettle, a reading of the temperature of the water was not taken, until it was in the cup, therefore I assumed that the initial temperature was of 100˚C
- The experiment was conducted 3 times to obtain the best results in doing so; I assumed that the water ingredients remained the same for each experiment.
- It was also considered that the container used did not make a significant difference in the results.
- It was assumed that the thermometer was accurate
- The room temperature remained the same
- The volume of water used remained the same (500mL)

It was considered during the development of the model that the following, **refinements** must be done to ensure the best and most accurate results

- The thermometer must be placed in the same position in the cup
- The ingredients of the liquid used remained the same because all of the water used from the experiment was from the same bottle
- The volume of water used was accurate because it was measured by measuring cup
- The room temperature was managed through the use of a controlled environment (air conditioning)

It was also determined that the shape of the container, determines how fast the temperature decay's.

Obtaining a model: Exponential model

Exponential Reg.

G.C

Stat > Enter > Type "Time" into L_1, "Temperature Average" into L_2 > Stat > Calc > 0: ExpReg

This will give you the mathematical model ($y = ab^x$)

y= temperature difference average (˚C)

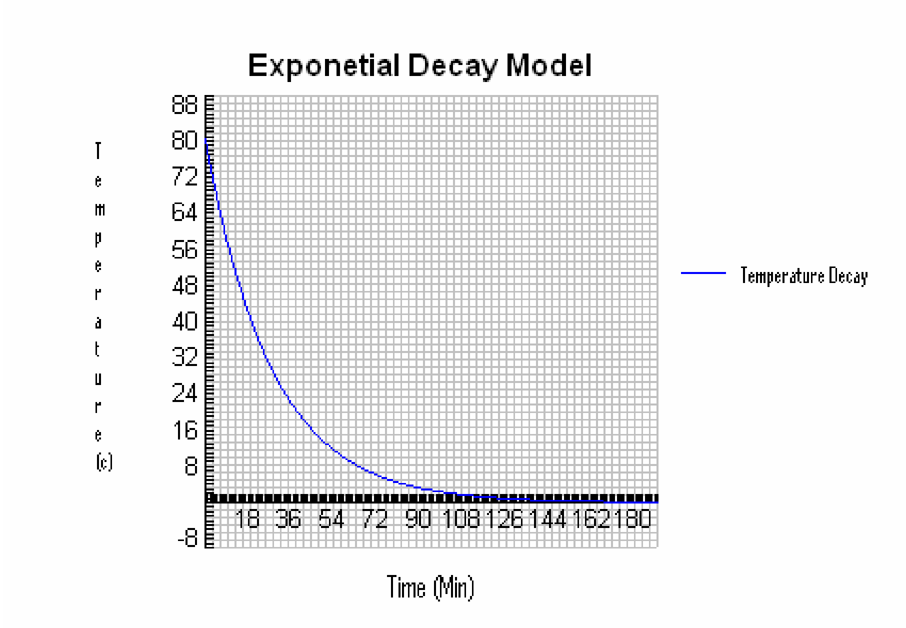
x= time (minutes)

$y=80.412 \times .964^x$

a= 80.412

b= 0.965

$r^2=.80$



Comparing the exponential model to other models

Quad Reg.

$y=ax^2 + bx + c$

$y=.0043x^2 + -1.04x + 58.90$

$r^2=.912$ graph

Between t = 90 to 152 graph indicates a negative temp, after t=121 graph rises which is unrealistic, therefore this is not an accurate model, even though the r^2 value is better then the exponential.

Cubic Reg.

$y=ax^3 + bx^2 + cx + d$

$y= -5x^3 + .017x^2 + -1.80x + 66.89$

$r^2=.98$

At t = 91 from 136 the graph rises and after t=136 the model drops unrealistically. Therefore this is not an accurate model; even though the r^2 value is better then the exponential.

Strengths

- Models cooling temperature difference realistically as this model tends to zero as time increases
- Never negative Temperature difference
- Graph continues to fall (never rising)

Weaknesses

- R^2(coefficient of determination) value wasn't as close to 1 as the other models investigated
- Never reaches zero
- Initial temperature(model) = 80.4117˚C
  In reality = 100˚C

In terms of k

The adjusted exponential model in the form of $y = ab^x$ is:
$y=80.412 \times .965^x$
$y=a \times b^x$
$a= 80.412$
$b= 0.965$

This can now be altered into the natural log exponential which has the form $y = ae^{-kx}$

$\therefore b=e^{-k}$
$0.965= e^{-k}$
$\ln 0.965= -k$
$-2.34 = -k$
$k= 2.34$

$\therefore y= a \times e^{-kx}$
$y =80.412 * e^{-2.34x}$

Half Life

By observing the table various half-life values were found and the time it takes for each temperature value to halve.

Time (min)	Temp. Difference (˚C)
18	71˚ - 35.5˚
19	68˚ - 34˚
20	62˚ - 31˚
25	30˚ -15˚
20	4˚- 2˚
24	40˚ - 20˚
30	10˚ - 5˚
25	26˚ - 13˚
22	52˚ - 26˚
21	56˚ - 28˚

Average half-life from **table** = 22.4 minutes

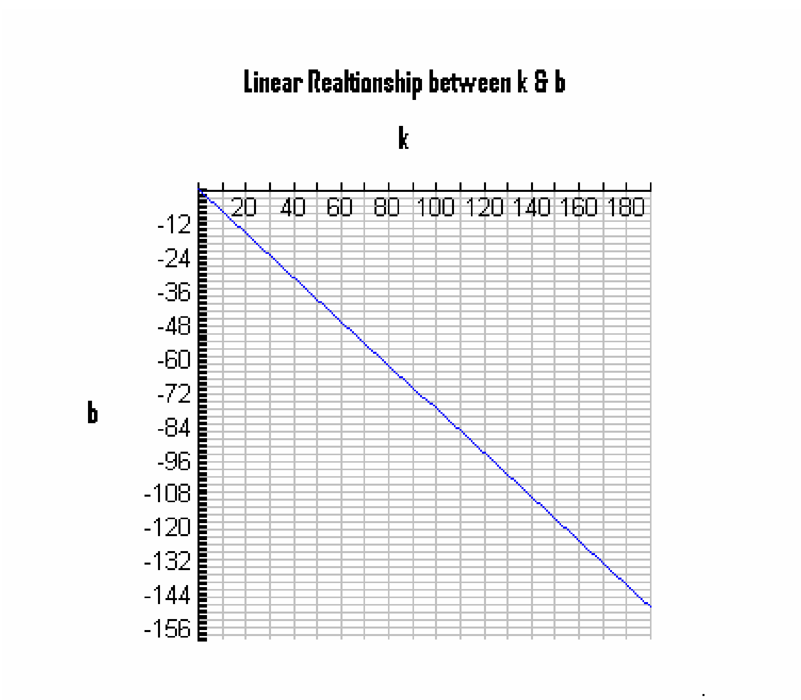
Model Half life
$y=80.412 \times .964^{x}$
Half life occurs when the initial temp is halved in value and the time is calculated from the model the initial temp is 80.412(let x =0) this value is divided b y 2 and substituted for y.
80 / 2 = 40.206
$y=80.412 \times .964^{x}$
$40.206=80.412 \times .964^{x}$
$.5=.965^{x}$
$\text{Log } .5=\log .965^{x}$
$= x \log .965$
$\text{Log } .5/ \log .965 = x$
x = 19.46
Time equals 19.46
Comparing the half life of the model with that from the table, it does not appear a close approximation
Difference =22.4 – 19.46
= 2.94
% error = 2.94/22.4 * 100
= 13.13% error

To find a linear relationship between k & b use
b= e^-k

From G.C
Stat > Calc >Press 4(LinReg),
Y=> Vars > 5: statistics> EQ
2^{nd} Table or Graph

y= ax + b
b= -.785k + .987
r^2= .995

k	b
.1	.905
.2	.819
.3	.741
.4	.670
.5	.607



Trial 1

Time (min)	Temperature (C)	Temp. Difference
0	100	75
1	96	71
2	93	68
3	90	65
4	87	62
5	84	59
6	81.5	56.5
7	79	54
8	77	52
9	75.1	50.1
10	73.2	48.2
11	71	46
12	69.7	44.7
13	68	43
14	66.7	41.7
15	65.2	40.2
16	64	39
17	63	38
18	61.5	36.5
19	60.6	35.6
20	59.7	34.7
21	58.2	33.2
22	57.5	32.5
23	56.2	31.2
24	55.3	30.3
25	54.8	29.8
26	54	29
27	53.3	28.3
28	52.6	27.6
29	51.7	26.7
30	51	26
35	48.5	23.5
40	44.3	19.3
45	42.8	17.8
50	40.5	15.5
55	39	14
60	36.6	11.6
70	35	10
80	33	8
90	31	6
100	29.7	4.7
110	28.5	3.5
120	27.8	2.8
130	27.1	2.1
140	26.7	1.7
170	25	0

Trial 2

Time (min)	Temperature (C)	Temp. Difference
0	100	75
1	96	71
2	93	68
3	89.7	64.7
4	87	62
5	84.8	59.8
6	82	57
7	80	55
8	78.2	53.2
9	76.4	51.4
10	74.7	49.7
11	72.3	47.3
12	70.5	45.5
13	68.9	43.9
14	67.2	42.2
15	66	41
16	64.6	39.6
17	63	38
18	62	37
19	60.5	35.5
20	59.8	34.8
21	58.5	33.5
22	57.2	32.2
23	56.5	31.5
24	55.6	30.6
25	54.2	29.2
26	53	28
27	52.2	27.2
28	51.5	26.5
29	50.7	25.7
30	50.2	25.2
35	46.3	21.3
40	43.2	18.2
45	40.8	15.8
50	38.1	13.1
55	36.2	11.2
60	35	10
70	33.8	8.8
80	32	7
90	30.3	5.3
100	29	4
110	28.2	3.2
120	27.3	2.3
130	26.5	1.5
140	26	1
170	25	0

Trial 3

Time (min)	Temperature (C)	Temp. Difference
0	100	75
1	96.2	71.2
2	93	68
3	90	65
4	87	62
5	84.2	59.2
6	81.5	56.5
7	79.5	54.5
8	77	52
9	75	50
10	73.5	48.5
11	71.9	46.9
12	70	45
13	68.7	43.7
14	67.2	42.2
15	66	41
16	64.8	39.8
17	63.5	38.5
18	62.4	37.4
19	61.4	36.4
20	60.3	35.3
21	59.4	34.4
22	58.5	33.5
23	57.7	32.7
24	56.8	31.8
25	55.8	30.8
26	55	30
27	54.5	29.5
28	53.8	28.8
29	53	28
30	52.4	27.4
35	49.3	24.3
40	46.7	21.7
45	44.3	19.3
50	42	17
55	40.5	15.5
60	39	14
70	37.7	12.7
80	36	11
90	35.2	10.2
100	33.3	8.3
110	31.6	6.6
120	30	5
130	29	4
140	28.2	3.2
170	25.8	0.8
180	25	0

Trial
Average

Time (min)	Average Temp. Difference
0	75
1	71.06666667
2	68
3	64.9
4	62
5	59.33333333
6	56.66666667
7	54.5
8	52.4
9	50.5
10	48.8
11	46.73333333
12	45.06666667
13	43.53333333
14	42.03333333
15	40.73333333
16	39.46666667
17	38.16666667
18	36.96666667
19	35.83333333
20	34.93333333
21	33.7
22	32.73333333
23	31.8
24	30.9
25	29.93333333
26	29
27	28.33333333
28	27.63333333
29	26.8
30	26.2
35	23.03333333
40	19.73333333
45	17.63333333
50	15.2
55	13.56666667
60	11.86666667
70	10.5
80	8.666666667
90	7.166666667
100	5.666666667
110	4.433333333
120	3.366666667
130	2.533333333
140	1.966666667
170	0.266666667
180	0