Infinity

Infinity Most everyone is familiar with the infinity symbol, the one that looks like the number eight tipped over on its side. Infinity sometimes crops up in everyday speech as a superlative form of the word many. But how many is infinitely many?

How big is infinity? Does infinity really exist? You can't count to infinity.

Yet we are comfortable with the idea that there are infinitely many numbers to count with; no matter how big a number you might come up with, someone else can come up with a bigger one; that number plus one, plus two, times two, and many others. There simply is no biggest number. You can prove this with a simple proof by contradiction. Proof: Assume there is a largest number, n. Consider n+1. n+1*n. Therefore the statement is false and its contradiction, "there is no largest integer," is true. This theorem is valid based on the "Validity of Proof by Contradiction." In 1895, a German mathematician by the name of

Georg Cantor introduced a way to describe infinity using number sets. The number of elements in a set is called its cardinality. For example, the cardinality of the set {3, 8, 12, 4} is 4. This set is finite because it is possible to count all of the elements in it. Normally, cardinality has been detected by counting the number of elements in the set, but Cantor took this a step farther. Because it is impossible to count the number of elements in an infinite set, Cantor said that an infinite set has No elements; By this definition of No, No+1=No. He said that a set like this is countable infinite, which means that you can put it into a 1-1 correspondence. A 1-1 correspondence can be seen in sets that have the same cardinality. For example, {1, 3, 5, 7, 9}has a 1-1 correspondence with {2,

4, 6, 8, 10}. Sets such as these are countable finite, which means that it is possible to count the elements in the set. Cantor took the idea of 1-1 correspondence a step farther, though. He said that there is a 1-1 correspondence between the set of positive integers and the set of positive even integers. E.g. {1, 2, 3, 4, 5, 6, ...n ...} has a 1-1 correspondence with {2, 4,

6, 8, 10, 12, ...2n ...}. This concept seems a little off at first, but if you think about it, it makes sense. You can add 1 to any integer to obtain the next one, and you can also add 2 to any even integer to obtain the next even integer, thus they will go on infinitely with a 1-1 correspondence. Certain infinite sets are not 1-1, though. Canter determined that the set of real numbers is uncountable, and they therefore can not be put into a 1-1 correspondence with the set of positive integers. To prove this, you use indirect reasoning. Proof:

Suppose there were a set of real numbers that looks like as follows 1st

4.674433548... 2nd 5.000000000... 3rd 723.655884543... 4th 3.547815886... 5th

17.08376433... 6th 0.00000023... and so on, were each decimal is thought of as an infinite decimal. Show that there is a real number r that is not on the list.

Let r be any number whose 1st decimal place is different from the first decimal place in the first number, whose 2nd decimal place is different from the 2nd decimal place in the 2nd number, and so on. One such number is r=0.5214211...

Since r is a real number that differs from every number on the list, the list does not contain all real numbers. Since this argument can be used with any list of real numbers, no list can

include all of the reals. Therefore, the set of all real numbers is infinite, but this is a different infinity from No. The letter c is used to represent the cardinality of the reals. C is larger than No. Infinity is a very controversial topic in mathematics. Several arguments were made by a man named Zeno, a Greek mathematician who lived about 2300 years ago. Much of

Cantor's work tries to disprove his theories. Zeno said, " There is no motion because that which moved must arrive at the middle of its course before it arrives at the end. And, of course, it must traverse the half of the half before it reaches the middle, and so on for infinity." Another argument that he stated was that, " If Achilles (a Greek Godlike person) can run 1000 yards a minute, he will never overtake a turtle that runs 100 yards a minute." Once

Achilles has advanced 1000 yards, the turtle is 100 yards ahead of him. By the time Achilles covers these 100 yards, the turtle is still ahead of him, and so on into infinity, as the following table shows. Another argument he gives is the one of the arrow in flight. He said, "The tip of an arrow is in one and only one position at each and every instance of time; in other words, at every instance of time, it is at rest. Hence it never moves." Zeno assumes that a finite part of time consists of a finite series of successive instances.

Throughout an instance, he says, the tip of the arrow is at one point. Imagine a period consisting of 1,000,000 small instances, and picture the arrow in flight during the period. At each of the 1 million instances, the arrow is where it is, and at the next instance, it is somewhere else. It never moves, but somehow accomplishes the change of position. Thus, motion is an illusory, irregular sort of thing-a succession of stills, like a movie-not the smooth sort of transition our senses picture. All of these examples are that Cantor attempted to disprove by forming his own infinity theories. As of now, infinity is a tentative area in mathematics, because certain concepts involved with it have not of yet been proven to everyone's satisfaction. This is one of the few areas that mathematics and science may never be able to explain completely, because infinity can not be measured in the classic sense.