A2 Physics Coursework

Aim: To investigate a Torsional Pendulum.

Research and equations:

As we are working in circular motion, rather than linear motion, the equations that will help me investigate the Torsional pendulum will have to be derived. Here is how it is derived.

Using Force= Mass x Acceleration which is what you use for linear motion, this becomes Torque=Moment of Inertia x Angular acceleration. Using Force= -kx from a simple pendulum, this becomes Force=- Torsional Constant x Angular displacement

Therefore $I\ddot{\theta} = -k\theta$ This can definitely be compared to a=-ω^2x and becomes $\omega = \sqrt{\frac{k}{I}}$ However $T = \frac{2\pi}{\omega}$ therefore $T = 2\pi\sqrt{\frac{I}{K}}$ I then found out the exact expression which allowed me to directly work out I and K. The moment of inertia was simply $I = \frac{1}{12}mL^2$ However for the Torsional constant I first found the formula for the polar moment of inertia which was $I_p = \pi d^4/32$ and the angle of twist $\varphi = TL/GI_p$ this was rearranged to $T = GI_p/L$ where T is the Torsional constant, then substituting in I_p I got Torsional constant= $\frac{G\pi d^4}{32L}$ Using the equation $T = 2\pi\sqrt{\frac{I}{K}}$ I can now substitute in expressions for I and K to get an overall equation which came out to be: $\mathbf{T=2\pi}\sqrt{\frac{I32L}{G\pi d^4}}$ T=Time Period I=Moment of Inertia of the bar L=Length of wire G= Shear Modulus of material d= diameter of wire

The following web pages were used to help me derive these equations:

http://www.engin.umich.edu/students/ELRC/me211/me211/flash/tors_derivation15.swf

http://farside.ph.utexas.edu/teaching/301/lectures/node139.html

Preliminary Experiment:

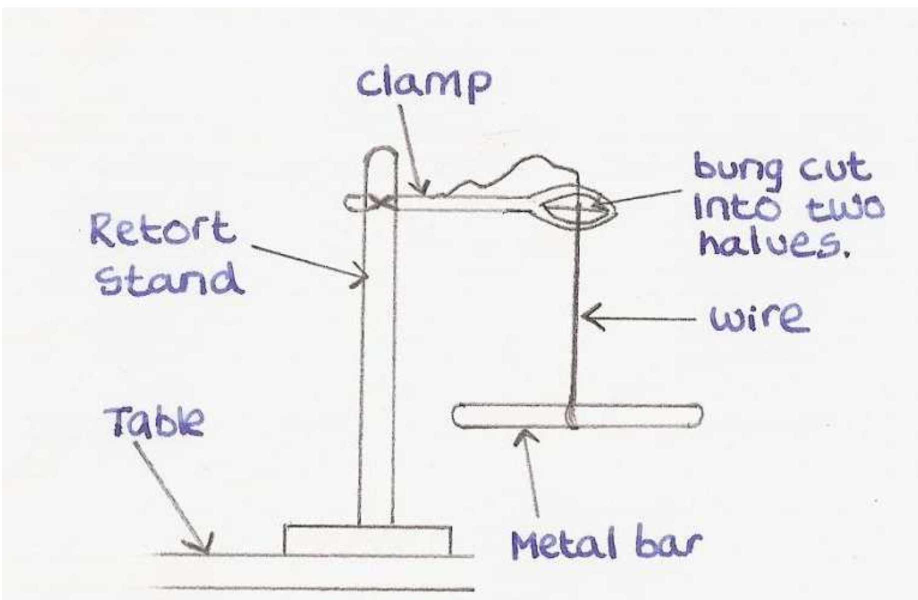
Aim: To investigate the relationship between Time period and the length of the wire on a Torsional pendulum.

Factors to vary and control

To ensure a fair test I must make sure that only factors that want to vary will change, therefore as I am investigating the effect of changing the length of wire on the time period I will only vary the length of wire. This means that the following must stay the same:

- Mass of the metal bar, including same diameter and length each time, these are all related to the moment of inertia.
- Type of wire (material) and its diameter, these are related to the Torsional constant.

Diagram:



Equipment:

- Retort Stand with clamp to hold the wire and bar when oscillating.
- Bung cut into two halves so I can change length of string easily.
- Metal Bar.
- Approximately a meter long wire.
- Stopwatch to record the Time periods.

- Micrometer to measure diameter of the wire and the metal bar
- Meter long ruler to measure out correct lengths of wire and measure length of the bar.

Method:

- Set up the apparatus as shown in the diagram above.
- Ensure the wire is fixed firmly around the centre of the bar, so that when left freely it rests in its equilibrium position.
- Using 0.1 meters as the starting point, make the length 0.1m using a meter rule, measuring from the base of the bung to the top of the bar at the knot.
- Turn the bar 90 degrees anticlockwise and release it, start the stopwatch at the same time of release.
- The time period for one complete oscillation is; for the end of the bar to go around clockwise once and changes direction then anticlockwise until it changes again, the moment it stops just before changing direction for a second time is one oscillation. Allow 5 complete oscillations for once length and divide the end time by five.
- Record the time period on a suitable table.
- Loosen the clamp and increase the length by 0.1m and repeat above steps until approximately 8 results are complete.
- Now measure the length of the bar using a meter ruler, and the diameter of the bar using a micrometer. Also measure the length of the wire using a meter ruler and its diameter using a micrometer. Record all these results.

To ensure that the experiment is carried out in safe environment I will make sure that I have plenty of space around me, with any obstacles removed to ensure the experiment can run smoothly.

Theory:

If simple harmonic motion applies, which I am assuming it does as shown in the equations above, also there is a clear similarity between the time period for a

Torsional pendulum and for a mass spring system which is simple harmonic motion, as shown in these 2 equations. $T = 2\pi\sqrt{\frac{I}{K}}$ and $T = 2\pi\sqrt{\frac{m}{K}}$

Simple harmonic motion is defined as; an oscillation in which the acceleration of an object is directly proportional to its displacement from equilibrium and has a restoring force directed back towards equilibrium.

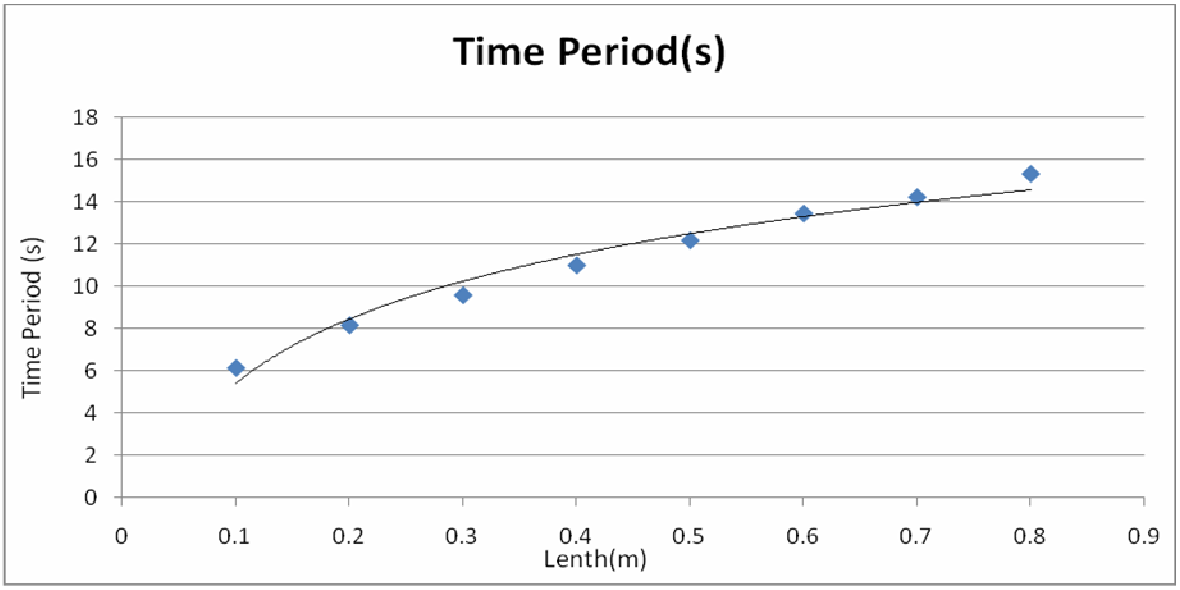
I am investigating for the preliminary experiment the effect on time period when the length of wire is changed. From the equation derived $\mathbf{T=2\pi}\sqrt{\frac{I32L}{G\pi d^4}}$ I can see that theoretically the relationship between time period and length should be $T \propto L^{0.5}$. Therefore an increase in length will increase the time period.

Results:

Length (mm)	Time Period (s)	Log l	log T
100	6.16	-1.00	0.79
200	8.18	-0.70	0.91
300	9.59	-0.52	0.98
400	11.00	-0.40	1.04
500	12.18	-0.30	1.09
600	13.45	-0.22	1.13
700	14.22	-0.16	1.15
800	15.32	-0.10	1.19

Extra Results:

Measurement	
Diameter of wire	0.42mm
mass of bar	201.1grams
length of bar	204mm



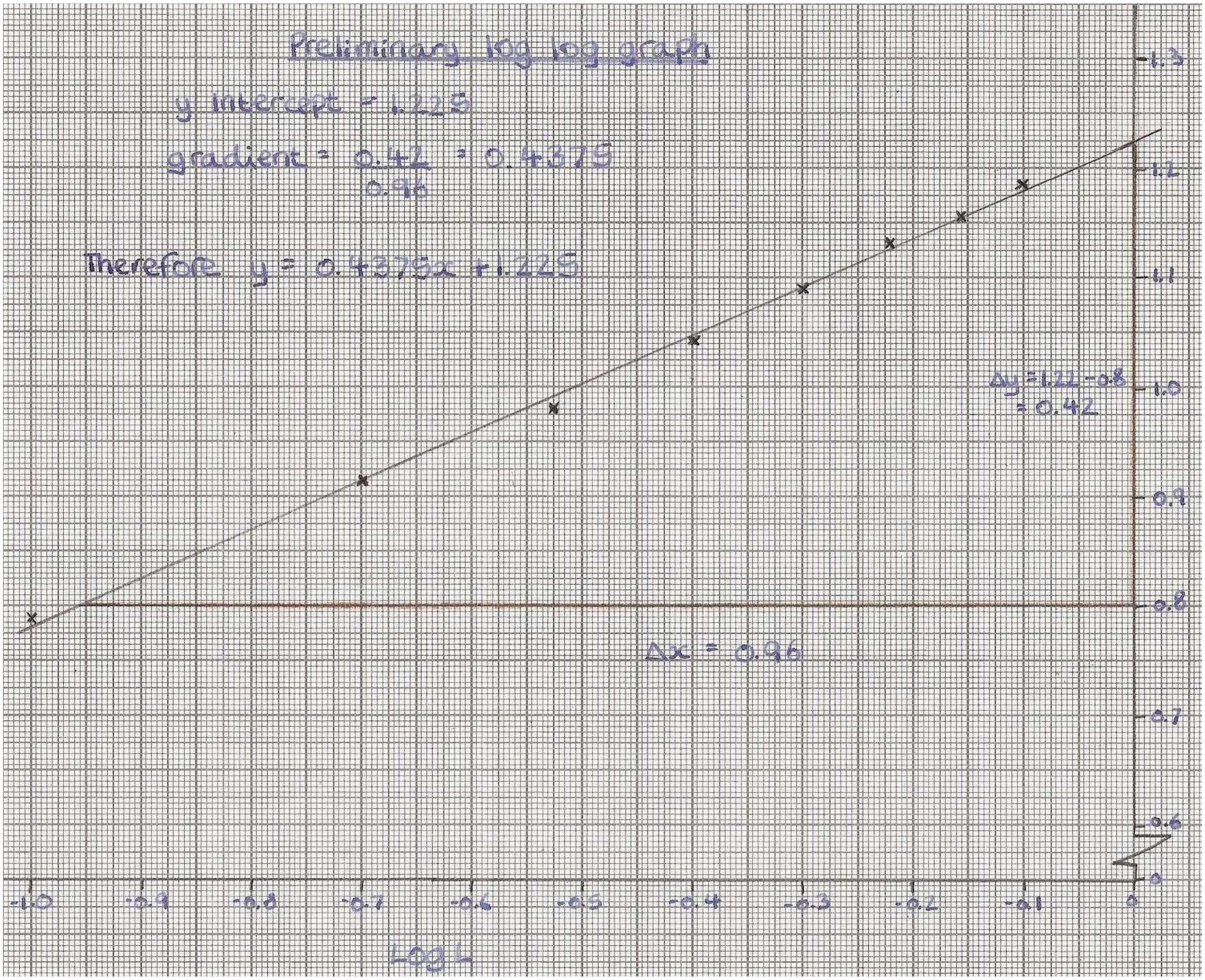
Conclusion:

From the graph I can come to a simple conclusion that as the length increases the time period increases. However it is obvious that this is not a linear relationship, therefore I need to use log log graphs to help me get the relationship.

As I originally worked out that there is a relationship between time period and length for the Torsional pendulum, I can therefore say that T is proportional to l ($T \propto l$). However I can change this to $T=al^b$ where a and b are constants to be determined.

I can determine these using a log log graph where $\log T=b\log l+\log A$ which is in the form $y=mx+c$

Using the values of logT and logl in the results table above, I produced the log log graph below.



From the graph you can see that the gradient which is b = 0.4375

Ʌnd that logɅ=1.225, therefore to get Ʌ I would unlog it, $10^{1.225}$= 16.788 (3.d.p)

So if Ʌ= 16.788 and b=0.4283 then the relationship becomes **$T=16.788 \times l^{0.4375}$**

Evaluation:

There are clearly difficulties and problems with this method which will cause inaccuracies in the results. I will now outline the problems and estimate a percentage error for each one.

- Measurement of the length of the wire. The meter ruler is accurate to ± 0.5mm, as the smallest division is 1cm. The measurements I made were 10cm to 80cm, therefore maximum error is (0.5mm/100mm) x100= 0.5%, and the minimum error is (0.5mm/800mm) x100= 0.0625% error. Therefore average error is approximately (0.5+0.0625)/2=0.28125%.

- There is also error in the time periods as it's difficult to know exactly when to stop the stopwatch. You must stop it when it stops and is just about to change direction. However I may stop to early or too late, this causes random error, and therefore time period will be higher or lower than the true value. I predict that this will cause a maximum error of ± 0.5 seconds, this includes the error for a human reaction time, which can only react as fast as 0.1 seconds. Max error for the results I obtained would be (0.5/6.162)x100=8.114% and minimum error (0.5/15.318)x100=3.264%. These errors are very significant and will definitely cause inaccuracies in my results. The time I recorded was accurate to 0.05seconds, therefore maximum reading error was (0.005/6.162)x100=0.081%, this is however a lot less significant than experimental error.

- The scale is accurate to 0.05 grams. Therefore maximum error is (0.05/201.1) x 100 = 0.0249%, therefore this error was not so significant.

- The micrometer is accurate to ±0.005mm, as smallest division is 0.01mm, therefore error for my reading was (0.005/0.41) x 100 = 1.219%, this error was quite significant and a lot larger than I expected.

  The value for the gradient I obtained was 0.4375, however I was expecting 0.5, therefore there is clearly errors in the time period and length, which is what determined the gradient, with reasons for these errors stated above. The error for the gradient will be the total error of the time and length, therefore approximately 6% error, when adding average most significant error of the time period and length.

Using the Equation **T=2π** $\sqrt{\frac{I32\,L}{G\pi d^4}}$ I can work out the overall error of my experiment. As **2π** $\sqrt{\frac{I32\,L}{G\pi d^4}}$ x $\sqrt{L}$ =T and as I found out that **T=16.788 x l$^{0.4375}$**

Therefore 2π $\sqrt{\frac{I32}{G\pi d^4}}$ should be equal to 16.788 if my experiment had no errors. I will now work out how close to this value I actually got.

$$=2\pi\sqrt{\frac{\frac{1}{12}x0.2011\ x0.204^{2}x32}{44.7x10^{9}x\pi x0.0042^{4}}} = 14.12$$

Therefore the total error from what the true value should be is [(16.788-14.12)/16.788] x 100= 15.89%

From all the percentage errors above I can see that there are clearly issues with this preliminary experiment and that changes will have to be made for the final experiment to increase accuracy and reduce errors.