Physics Practical Investigation Coursework
Investigating Simple Harmonic Oscillations

This investigation aims to explore the nature of different oscillating systems, including the factors upon which the oscillation depends and the energy transfer involved .

Preliminary Experiment

A pendulum was made using a bob hanging, by a piece of string, from a standing clamp. Experiments were carried out, recording the time taken for ten complete cycles from angles of displacement ranging from 5 to 30° in 5° interva ls. In separate experiments, the mass and string length were changed as the independent variables in order to investigate the effect they had upon the period of oscillation. The mass of the bobs used were 100, 200 and 300g; the length of the string varyin g between 15cm and 30cm. For each experiment, three trials were completed in order to allow identification of anomalous results and enable the calculation of an average time – this value was then divided by ten in order to work out the average time of one oscillation.

Length of string: 0.15m		Average time for 1 oscillation (s)		
	Amplitude: Angle of initial displacement (degrees)	100g	200g	300g
	5	1.08	1.08	1.09
	10	1.08	1.09	1.09
	15	1.09	1.09	1.09
	20	1.08	1.09	1.08
	25	1.09	1.10	1.09
	30	1.09	1.10	1.09
Length of string: 0.3m		Average time for 1 oscillation (s)		
	Amplitude: Angle of initial displacement (degrees)	100g	200g	300g
	5	1.31	1.32	1.31
	10	1.32	1.33	1.32
	15	1.32	1.33	1.32
	20	1.32	1.33	1.33
	25	1.33	1.33	1.33
	30	1.33	1.34	1.33

For complete table of data, see appendix.

It can be observed that the period of oscillation is independent of both mass and initial displacement, but does depend on length.

According to the equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The time period for an oscillation in a pendulum of length 30cm should be:
2π √ (0.3/9.81) = 1.098767 ≈1.10

and for 15cm
2π √ (0.15/9.81) = 0.776946 ≈ 0.78

Taking the observed period to be 1.33 with length 30cm, and the observed period with length 15cm to be 1.09 there is a difference between observed and expected results of 0.23 and 0.31 respectively. The pendulum exhibits simple harmonic motion, the energy being transferred between potential (at the extremes of the oscillation) and kinetic energy. However, resistive forces from friction between the string and the clamp and also the between the bob and the air, cause an exponential decrease in energy in the system. This results in a loss in amplitude and also increases the time for each oscillation. As is true for all systems showing simple harmonic motion, it could be observed that the acceleration of the mass is greatest as it began to move back towards its equilibrium position from the stationary extremes of the oscillation, the velocity being greatest as the equilibrium position was passed.

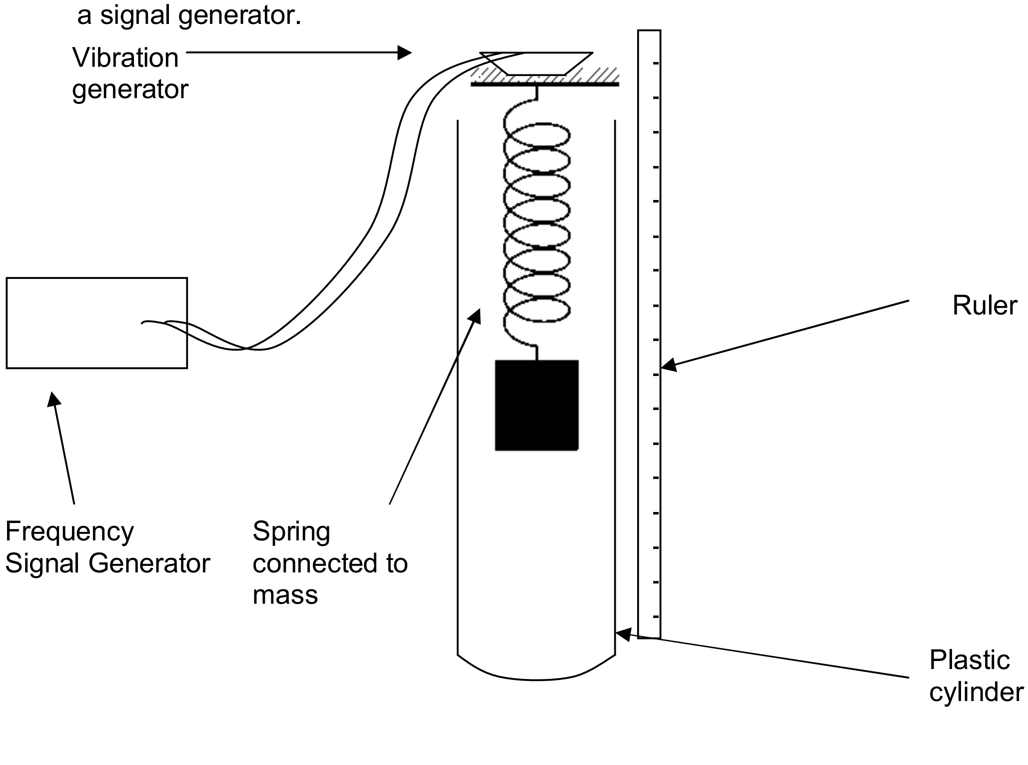
<u>Oscillation in a driven system</u>

The loss of energy through friction (not measured in the preliminary experiment), which was independent of mass, led to the investigation of another system for which the energy losses would be compensated by the input of additional energy.

A spring with an attached mass was connected to a vibration generator, causing it to oscillate at frequencies selected using the signal generator. At a certain frequency, the driving force adds energy at just the right moment during the cycle so that the oscillation is reinforced and the spring oscillates with maximum amplitude. This is the resonant frequency - the natural frequency of the system.

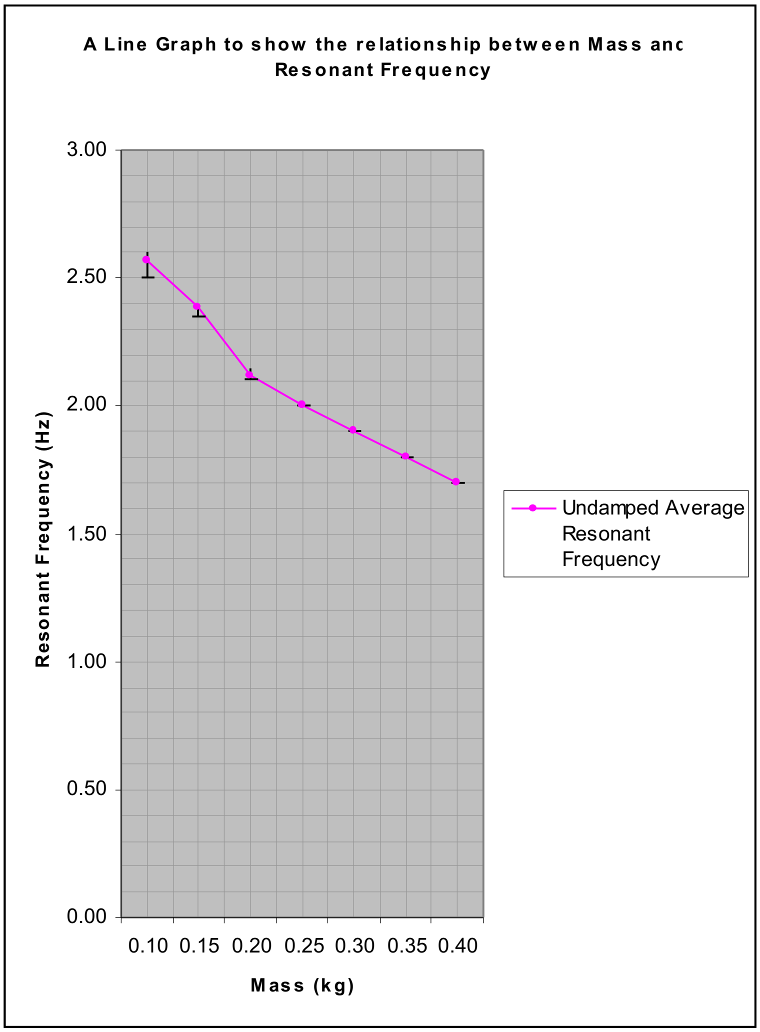
Masses ranging from 100g to 400g at 50g intervals were attached to the spring and both the resonant frequency and amplitude were recorded. The amplitude was measured by subtracting the increase in extension whilst at maximum amplitude, (at resonant frequency) from initial spring extension without any oscillation (using the ruler). Each experiment using a different mass was completed three times as the actual point of resonant frequency was a little subjective. Again, this allowed the identification of anomalous data and the calculation of average values.

<u>Safety:</u> A plastic cylinder surrounded the mass and spring to stop the spring swinging violently and the mass becoming detached. Mains electricity was used safely through a signal generator.



Mass (kg)	Average Resonant Frequency (Hz)	Average Amplitude of Resonant Frequency (Hz)
0.10	2.57	5.33
0.15	2.38	9.17
0.20	2.12	10.27
0.25	2.00	10.63
0.30	1.90	11.12
0.35	1.80	11.23
0.40	1.70	11.32

Full table of results in appendix.



Like the preliminary experiment it was observed that the acceleration of the mass was greatest as it began to move back towards equilibrium and the velocity is greatest as it passes this point. Unlike the preliminary experiment, the frequency does depend upon mass. The graph shows negative correlation between mass and resonant frequency - as the mass was increased, the resonant frequency decreased.

This can be seen in the following equation:

T= 2π √(m/k)

Therefore, as k and 2π are constants:

T ∝ √m

(1/f) ∝ √m

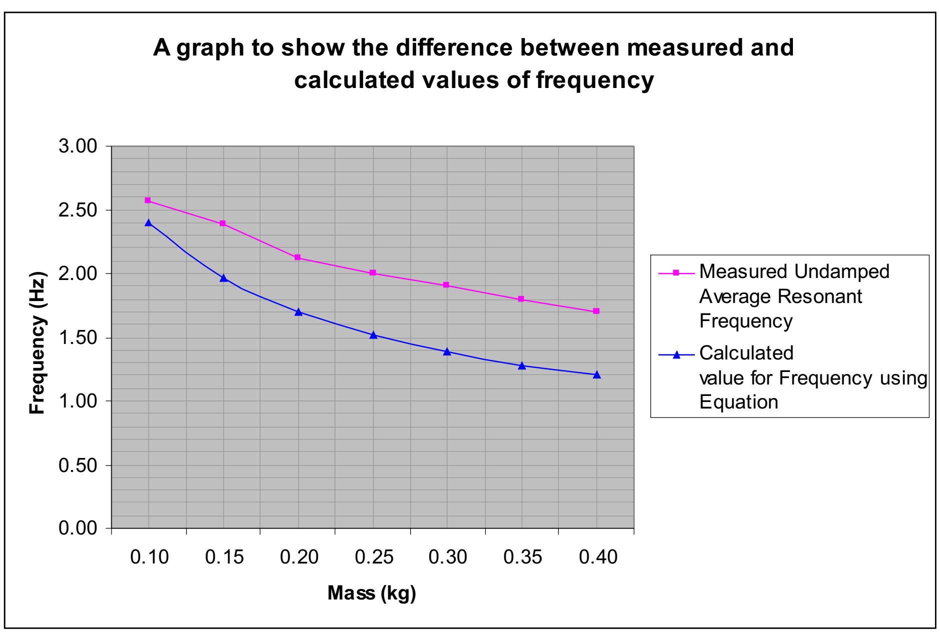
As frequency increases the value of 1/f becomes smaller, which as it is proportional to the square root of m, means that m also decreases.

The results obtained experimentally indicate a slightly more linear relationship than expected which is likely to be caused by measurement error.

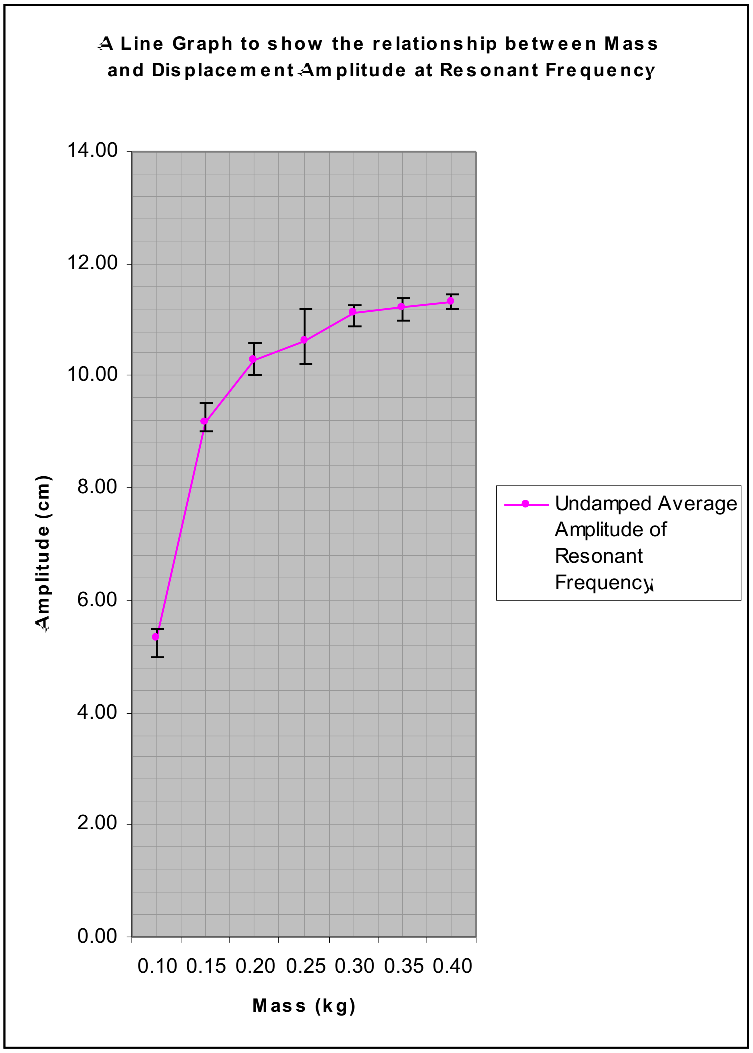
<u>Calculation of error:</u>
With the mass values and spring constant a value for frequency can be calculated using the equation:

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{M}{k}}.$$

Mass (kg)	Measured Undamped Average Resonant Frequency (Hz)	Calculated Value for Frequency Using Equation (Hz)	Difference
0.10	2.57	2.40	0.17
0.15	2.38	1.96	0.42
0.20	2.12	1.70	0.42
0.25	2.00	1.52	0.48
0.30	1.90	1.38	0.52
0.35	1.80	1.28	0.52
0.40	1.70	1.20	0.50



The previous graph shows the discrepancies between observed and expected values, with a difference of around 0.5 Hz. A difference of this nature possibly indicates that the signal generator was producing frequencies approximately 0.5Hz higher than measured.



The graph shows that the amplitude at the resonant frequency increases with mass. This is because there is more energy in the oscillating system with greater mass resulting in larger displacement. The amplitude seems to plateau, possibly because increasing amounts of energy are required to extend the spring when stretching it further– in accordance with the square relationship $E=1/2\ kx^2$.

A source of error in this experiment is due to the sometimes subjective nature to recognising the point when the system was actually at its resonant frequency. However, the frequency generator was not particularly sensitive and the pointer was a little unreliable, so error bars on the first graph are small. The errors are larger for the amplitude measurements - simply because the system had so much energy that the maximum vertical displacement was difficult to measure before it started swinging.

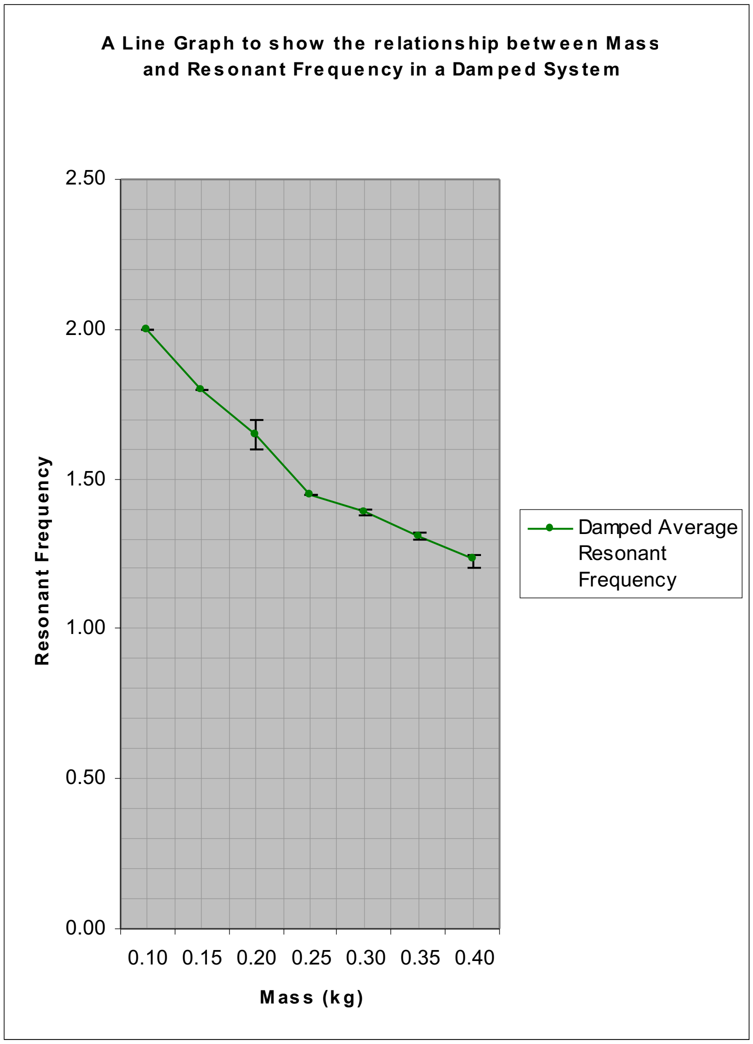
Damped Oscillations

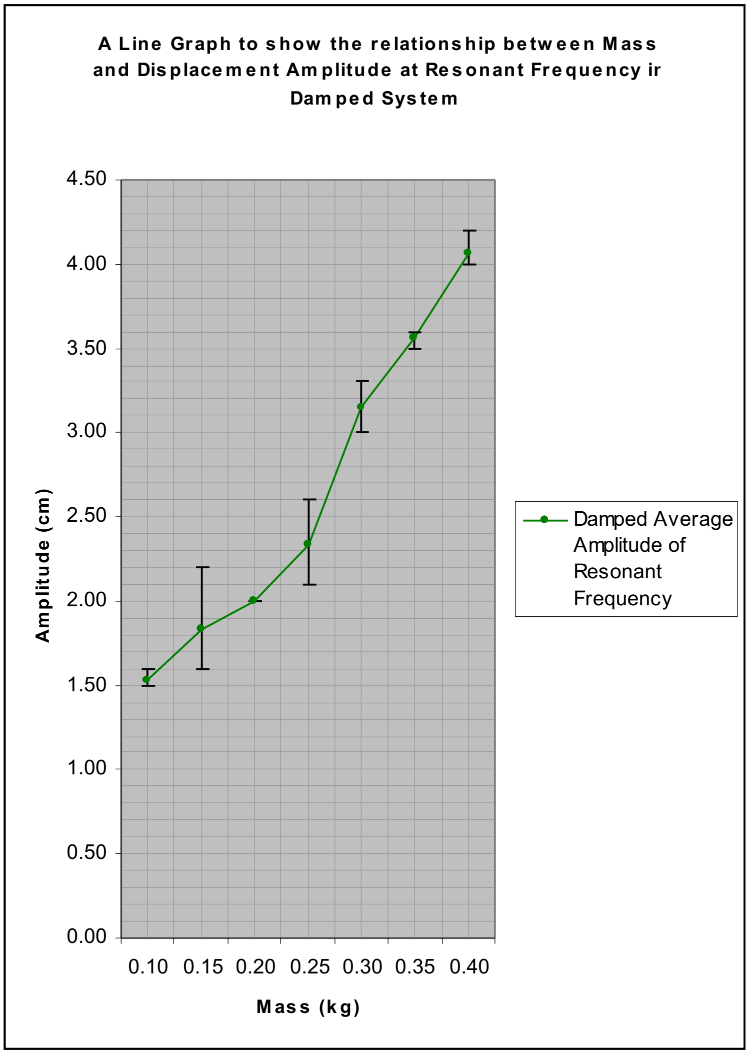
The above experiment was repeated with a constant volume of water in the plastic cylinder, covering the mass, acting as a damping force. As above, both the maximum amplitude and resonant frequency were recorded in three trials and the average calculated, removing any anomalous results.

Safety: Wires were kept away from water at all times.

Mass (kg)	Damped Average Resonant Frequency (Hz)	Damped Average Amplitude of Resonant Frequency (Hz)
0.10	2.00	1.53
0.15	1.80	1.83
0.20	1.65	2.00
0.25	1.45	2.33
0.30	1.39	3.15
0.35	1.31	3.57
0.40	1.23	4.07

See appendix for full table of data, including anomalous results.


A Line Graph to show the relationship between Mass and Resonant Frequency in a Damped System
2.50
2.00
1.50
1.00
0.50
0.00
Resonant Frequency
0.10 0.15 0.20 0.25 0.30 0.35 0.40
Mass (kg)
Damped Average Resonant Frequency



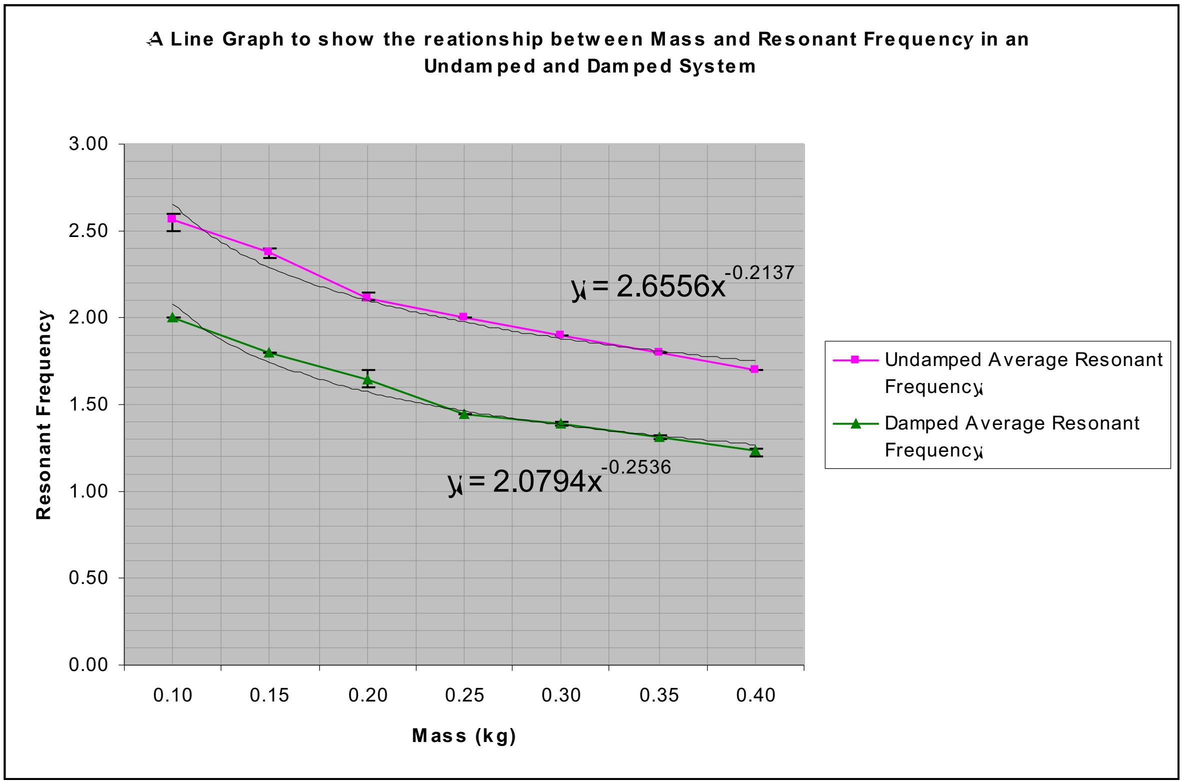
Exactly the same relationships are true as in the undamped system , the values are just lower. The damping effect of water is due to the removal of energy from the system and is caused by friction between the mass and damping fluid. Kinetic energy is lost as heat to the water.

The error range of amplitude is however larger due to additional difficulty in recognising the point of resonant frequency, caused by the large damping effect. It is presumed that a similar error in measured frequency values, as in the undamped experiment, is also present in this experiment.

The amplitude showed a more linear increase. This is probably because the spring extension between different mass values was that much smaller due to damping. If the energy of the system is proportional to the amplitude ^{2}, a system with decreased energy causes amplitude to be reduced considerably (as a consequence of the square relationship).

A source of error could be that the surface area of the hangi ng mass does not remain constant – meaning there could be additional friction between mass and fluid with larger masses and consequentially a greater loss of energy.

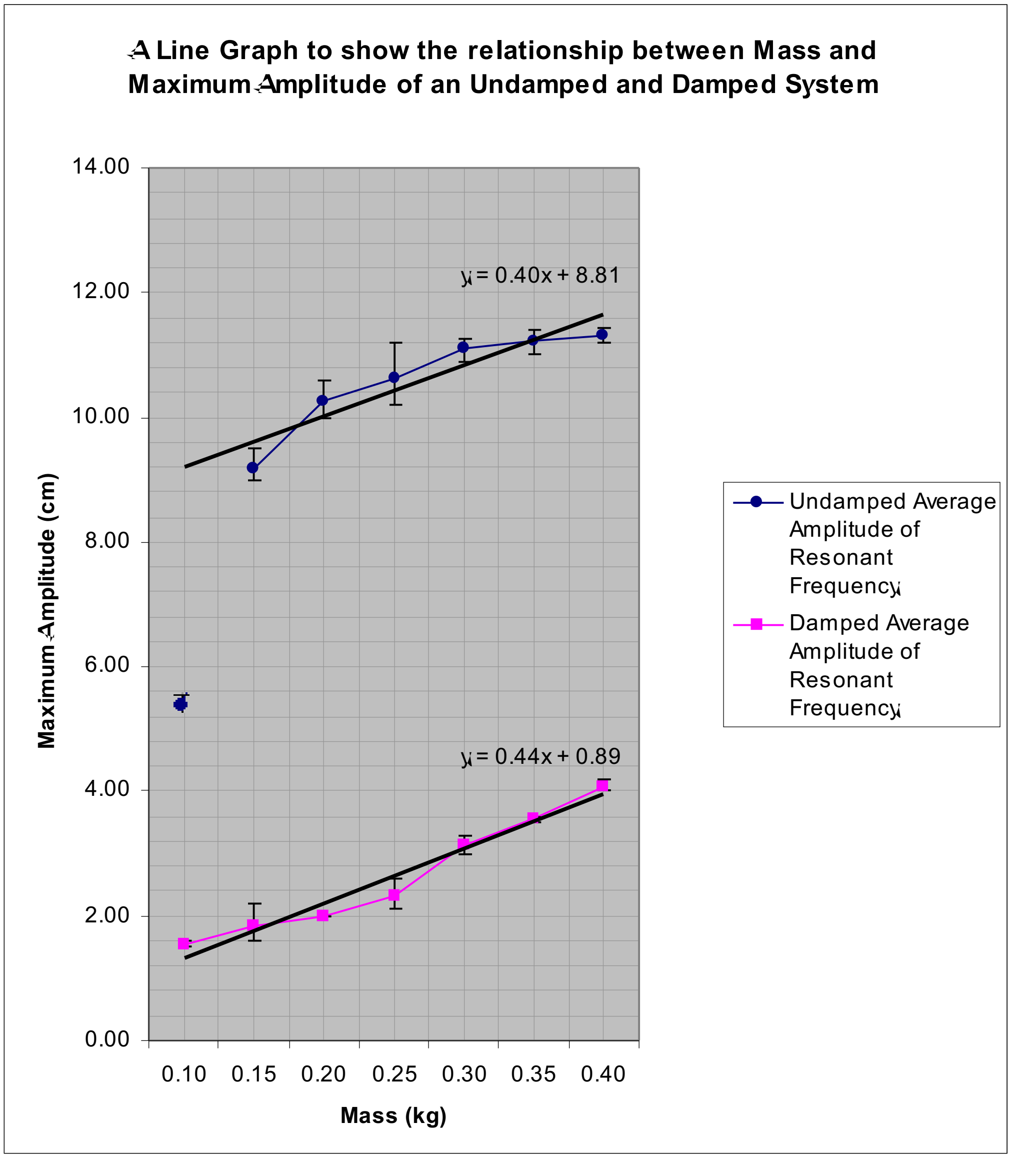
The effect of the damping can be seen more easily by comparison between the two experiments.



As can be seen in the graph, the resonant frequency is consistently lower in the damped system than in the undamped system.

Mass (kg)	Undamped Average Resonant Frequency (Hz)	Damped Average Resonant Frequency(Hz)	Difference (Hz)	% Difference
0.10	2.57	2.00	0.57	22.08
0.15	2.38	1.80	0.58	24.48
0.20	2.12	1.65	0.47	22.05
0.25	2.00	1.45	0.55	27.50
0.30	1.90	1.39	0.51	26.67
0.35	1.80	1.31	0.49	27.41
0.40	1.70	1.23	0.47	27.45

The damping effect reduces the resultant frequency by an average o f 0.52Hz. The percentage decrease in resonant frequency increases slightly with mass – possibly due a greater surface area of larger masses as discussed above.



To calculate an *approximate* percentage change, where the lines of best fit are almost equal in gradient (0.4 and 0.44), the first point was removed. The intercepts differ by 7.92 meaning that, over this range of values, the amplitude is reduced by approximately 8cm by the damping effect of the water. As a percentage change this is: (8/8.81) x 100 = 91%.

The reason that, in reality, the difference is not linear (especially in the undamped) system is the square relationship between amplitude and energy ($E=1/2\ kx^2$).

The energy of the system is equal to the sum of the kinetic energy and potential energy at any point. The easiest way of calculating the total energy is when the kinetic energy is zero and all the energy is stored as potential energy in the stretched spring. This allows the use of the previously mentioned equation:

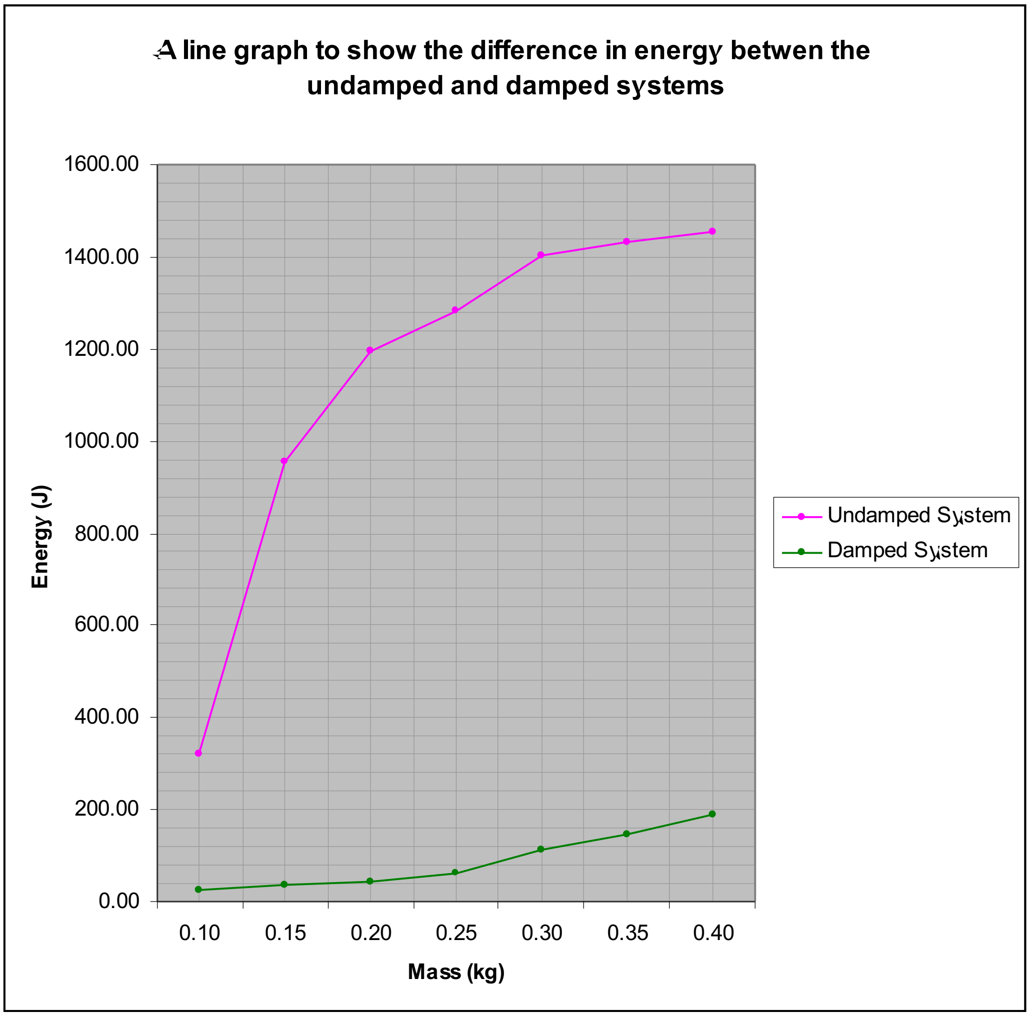
$E=1/2\ kx^2$ (x being the amplitude of oscillation)

Calculation of Spring Constant:

Mass (kg)	Weight (N)	Extension (m)	k
0.10	0.98	0.03	32.67
0.20	1.96	0.09	21.78
0.30	2.94	0.13	22.62
0.40	3.92	0.17	23.06
0.50	4.90	0.21	23.33

Taking the first value of k to be anomalous, the average spring constant is 22.70 which will be used in following equations to calculate the energy.

Mass (kg)	Energy in Undamped System (J)	Energy in Damped System (J)
0.10	322.44	26.69
0.15	953.72	38.01
0.20	1196.34	45.40
0.25	1283.32	61.79
0.30	1402.64	112.62
0.35	1432.23	144.38
0.40	1453.56	187.70



The graph shows, as expected, that system energy increases with mass whether it is damped or not, but the increase is much larger in the undamped system. The damped system has lower energy due to the fluid density providing a buoyancy force acting upwards against the weight of the masses – causing a smaller downwards resultant force and meaning a large amount of energy is lost as heat to the water through friction. As mentioned before, an uncontrolled variable in mass surface area could add to this effect, the larger the mass, the greater the surface area for loss of energy through friction.

It was planned to further the investigation with a more viscous damping fluid, but the large effect of water damping (water being a particularly non-viscous liquid) caused the belief that the system would then be damped to a critical point, or possibly over-damped. This would result in near impossible measurements of frequency as the frequencies were already very low with water.

The spring constant, k, could be changed and would affect the energy and therefore the whole oscillation of the system. Increasing k would mean a larger amount of energy could be stored in the spring – increasing displacement and frequency and decreasing the period, in accordance with the equation:

$T = 2\pi \sqrt{(m/k)}$

Changing the diameter of the cylinder containing a constant volume of water may also have an effect upon how easy it is for the water to be displaced sideways by the masses – increasing the diameter could potentially make the displacement easier, losing less energy and having less damping effect.

If this experiment was to be repeated I would ensure that the mass was of constant surface area. The effect of damping with different shaped masses, or those with different surface areas could also be investigated further.

Summary/Conclusion

In a system for which frequency is dependent upon mass, increasing the mass decreases the frequency. The amplitude is increased as the weight force acting upon the spring is larger and the system has more energy.

In both an undriven and damped system showing simple harmonic motion, energy is lost as heat through friction between moving parts and the fluid surroundings – whether it be air or water. The damping effect in water is great as it is much denser than air. This means there are more particles in a constant volume which cause frictional resistance against a moving mass, causing the system to lose energy as heat to the fluid. The loss of energy is apparent in the loss of amplitude, or spring extension.

Despite having relatively small error bars on graphs, the reliability of the results is questionable, mainly due to dubious frequency values measured experimentally. However the majority of the relationships and trends can still be seen. More accurate frequency values would have allowed a more accurate calculation of damping effect on resonant frequency.

Appendix

Preliminary Data

Length 30cm		Time taken for 10 oscillations				
Mass 100g	Angle of initial displacement (degrees)	Trial 1	Trial 2	Trial 3	Average	Average time for 1 oscillation
	5	13.15	13.14	13.13	13.14	1.31
	10	13.18	13.18	13.14	13.17	1.32
	15	13.22	13.27	13.23	13.24	1.32
	20	13.18	13.26	13.23	13.22	1.32
	25	13.27	13.27	13.27	13.27	1.33
	30	13.32	13.32	13.37	13.34	1.33
Length 30cm		Time taken for 10 oscillations				
Mass 200g	Angle of initial displacement (degrees)	Trial 1	Trial 2	Trial 3	Average	Average time for 1 oscillation
	5	13.16	13.18	13.20	13.18	1.32
	10	13.26	13.30	13.24	13.27	1.33
	15	13.27	13.30	13.26	13.28	1.33
	20	13.33	13.27	13.30	13.30	1.33
	25	13.32	13.32	13.36	13.33	1.33
	30	13.32	13.41	13.41	13.38	1.34
Length 30cm		Time taken for 10 oscillations				
Mass 300g	Angle of initial displacement (degrees)	Trial 1	Trial 2	Trial 3	Average	Average time for 1 oscillation
	5	13.11	13.12	13.15	13.13	1.31
	10	13.20	13.27	13.26	13.24	1.32
	15	13.14	13.15	13.18	13.16	1.32
	20	13.23	13.36	13.32	13.30	1.33
	25	13.27	13.30	13.34	13.30	1.33
	30	13.30	13.40	13.28	13.33	1.33
Length 15cm		Time taken for 10 oscillations				

Mass 100g	Angle of initial displacement (degrees)	Trial 1	Trial 2	Trial 3	Average	Average time for 1 oscillation
	5	10.71	10.80	10.82	10.78	1.08
	10	10.82	10.80	10.92	10.85	1.08
	15	10.86	10.87	10.86	10.86	1.09
	20	10.83	10.82	10.87	10.84	1.08
	25	10.88	10.90	10.90	10.89	1.09
	30	10.87	11.00	10.93	10.93	1.09
Length 15cm		Time taken for 10 oscillations				
Mass 200g	Angle of initial displacement (degrees)	Trial 1	Trial 2	Trial 3	Average	Average time for 1 oscillation
	5	10.74	10.84	10.89	10.82	1.08
	10	10.90	10.93	10.95	10.93	1.09
	15	10.89	10.84	10.90	10.88	1.09
	20	10.84	10.93	11.01	10.93	1.09
	25	10.98	11.00	10.93	10.97	1.10
	30	11.02	11.02	11.02	11.02	1.10
Length 15cm		Time taken for 10 oscillations				
Mass 300g	Angle of initial displacement (degrees)	Trial 1	Trial 2	Trial 3	Average	Average time for 1 oscillation
	5	10.76	10.80	11.01	10.86	1.09
	10	10.84	10.79	10.92	10.85	1.09
	15	10.82	10.90	10.86	10.86	1.09
	20	10.80	10.89	10.85	10.85	1.08
	25	10.84	10.90	11.01	10.92	1.09
	30	10.89	11.00	10.90	10.93	1.09

Data from Experiment 2&3

			Trial 1		Trial 2		Trial 3			
Undamped		Mass (kg)	Resonant Frequency	Amplitude of Resonant Frequency	Resonant Frequency	Amplitude of Resonant Frequency	Resonant Frequency	Amplitude of Resonant Frequency	**Average Resonant Frequency**	**Average Amplitude of Resonant Frequency**
		0.10	2.60	5.00	2.60	5.50	2.50	5.50	**2.57**	**5.33**
		0.15	2.40	9.50	2.40	9.00	2.35	9.00	**2.38**	**9.17**
		0.20	2.10	10.00	2.15	10.20	2.10	10.60	**2.12**	**10.27**
		0.25	2.00	10.20	2.00	10.50	2.00	11.20	**2.00**	**10.63**
		0.30	1.90	10.90	1.90	11.20	1.90	11.25	**1.90**	**11.12**
		0.35	1.80	11.00	1.80	11.40	1.80	11.30	**1.80**	**11.23**
		0.40	1.70	11.20	1.70	11.30	1.70	11.45	**1.70**	**11.32**
			Trial 1		**Trial 2**		**Trial 3**			
Damped		Mass (kg)	Resonant Frequency	Amplitude of Resonant Frequency	Resonant Frequency	Amplitude of Resonant Frequency	Resonant Frequency	Amplitude of Resonant Frequency	**Average Resonant Frequency**	**Average Amplitude of Resonant Frequency**
		0.10	2.00	1.50	2.00	1.50	2.00	1.60	**2.00**	**1.53**
		0.15	1.80	2.20	1.80	1.60	1.80	1.70	**1.80**	**1.83**
		0.20	1.70	3.00	1.60	2.00	1.65	2.00	**1.65**	**2.00**
		0.25	1.45	2.30	1.45	2.60	1.45	2.10	**1.45**	**2.33**
		0.30	1.40	3.00	1.40	3.30	1.38	3.70	**1.39**	**3.15**
		0.35	1.32	3.60	1.30	3.60	1.30	3.50	**1.31**	**3.57**
		0.40	1.25	4.00	1.20	4.20	1.25	4.00	**1.23**	**4.07**

Anomalous results (in red) are not included in averages.