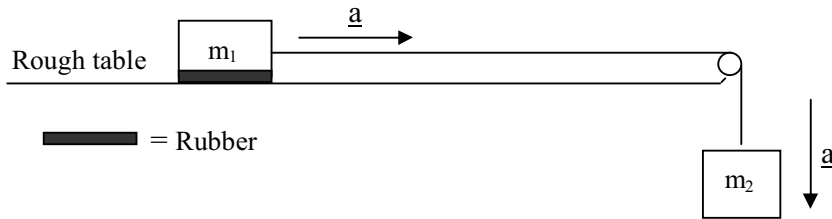Mechanics Coursework

This is experiment is designed to approximate μ, the coefficient of friction, for two surfaces. We used a piece of rubber on the bottom of a wooden block on a wooden table.

Assumptions

To undertake this experiment, first I developed an equation allowing a straight line to be plotted from our results. This will test if Coulomb's law is correct. The main assumptions made in this experiment are:

1. Acceleration is constant
2. The strings are light and inextensible, and the pulley is light and smooth
3. g, gravity is 9.8
4. Coulomb's law is correct
5. There is no air resistance
6. F=ma

These assumptions have different weights in affecting our results. Some are more likely to affect our results, whilst others will only have a small effect.

Assumption	How strong the effect is on results
Acceleration is constant	This will have a moderate effect on the results. We can see that the acceleration is not constant. This is because friction does change slightly with velocity, and we are ignoring air resistance
Light, inextensible strings	In fact the strings do stretch slightly, and do have a small weight. This will affect the results as forces will not be constant during the experiment, and therefore produce irregular acceleration.
Light, smooth pulley	This will affect the results in the same way as above. We attempted to use the smoothest pulley available, but obviously it can never be perfectly smooth. That the pulley has mass does not affect our results, as we are not dependent on the pulleys mass for our results.
Gravity is 9.8	This will introduce a systematic error into our calculations, but as we will use this assumption for both the results and the model, it will not make a difference for our experiment.

Coulomb's law is correct	This will make a big difference to our results. By assuming that $F_{max} = \mu R$, our calculations are fairly simple. However, this linear model is incorrect. This will affect our results considerably, and may results in a curve being plotted from our results instead of a straight line.
No air resistance	This will have a mild effect on the results. Because the block of wood and the weights are fairly small, the effects of air resistance are negligible. However there will be a small effect, which again may force the results into a curve rather than a straight line.
$f = ma$	This assumption will not impact our results very much at all. For macroscopic objects, such as we are dealing with, this law is a very good model. However if we were to repeat the experiment using single atoms for the objects, this would be a bad model.

To help overcome these systematic errors, we insured that we had the smoothest pulley, and that the surfaces that we used were consistent. In other words, that the table was the same roughness all the way along it, and that the rubber was the same all over.

Manipulating the model

When m_2 is released m_1 accelerates. These two objects can be looked at separately.
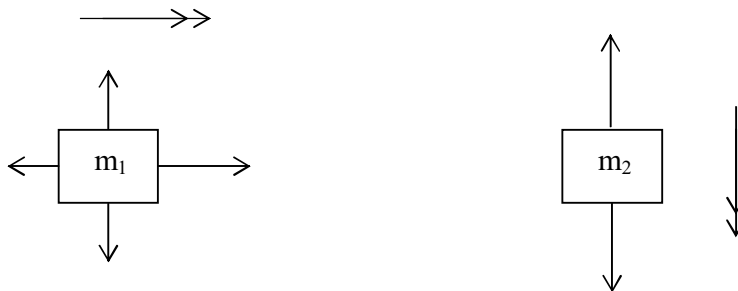
Using $f = ma$.

(1) $T - F = m_1 a$ $\qquad\qquad$ (2) $m_2 g - T = m_2 a$

$\quad m_1 g - R = 0$

(3) $m_1 g = R$

Using $F = \mu R$ (Coulomb's Law)

(4) $F = \mu m_1 g$

Substituting equation 4 into equation 1

$$T - \mu m_1 g = m_1 a$$
$$T = m_1 a + \mu m_1 g$$

Substituting into equation 3
$$m_2 g - (m_1 a + \mu m_1 g) = m_2 a$$
$$m_2 g - \mu m_1 g = m_2 a + m_1 a$$
$$m_2 g - \mu m_1 g = a(m_1 + m_2)$$
$$\therefore$$
$$(5)\ g(m_2 - \mu m_1) = a(m_1 + m_2)$$

Assuming constant acceleration (see note above)

$$s = ut + \frac{1}{2}a\,t^2$$
$$a = \frac{2s - 2ut}{t^2}$$
As $u = 0$
$$a = \frac{2s}{t^2}$$

Substituting into equation 5
$$g(m_2 - \mu m_1) = \frac{2s}{t^2}(m_1 + m_2)$$

The two variables will be m_1 and t. By putting $m_1 + m_2 = k$, we can cancel m_2 from the equation.
$$g(k - m_1 - \mu m_1) = \frac{2s}{t^2}(m_1 + k - m_1)$$

$$g(k - m_1 - \mu m_1) = \frac{2s}{t^2}k$$

$$9.8(k - m_1 - \mu m_1) = \frac{2s}{t^2}k$$

Setting k to be 1022g (this is the lowest weight for which it will still move)

$$1031.8 - 9.8m_1 - 9.8\mu m_1 = \frac{2044\ s}{t^2}$$
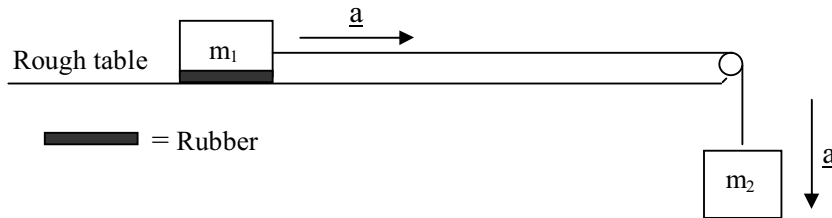
$s = 0.75$. This is because otherwise m_2 hits the floor.

$$1031.8 - 9.8m_1(1 + \mu) = \frac{1533}{t^2} \quad \ldots \ldots \ldots 1031.8 \text{ should be } 9.8*1022$$

$$0.673 - \frac{9.8m_1(1 + \mu)}{1533} = \frac{1}{t^2}$$

This allows us to plot a graph with m_1 on the x-axis, and $\frac{1}{t^2}$ on the y-axis.

Conducting the experiment

Again, this diagram shows the layout of the experiment. m_2 weighs more than m_1.

When m_2 is released m_1 accelerates. In my model I assumed that this acceleration is constant. However in reality this is not true. As mentioned previously, this will affect my results.
The weight of m_1 presses down on the table as the block accelerates. This causes frictional resistance between the block and the bottom of the table. The coefficient of friction between the rubber on the bottom of the block and the table is denoted by μ. By measuring the time taken to move over a specified distance, and by knowing the weight of the two masses it is possible to calculate μ.

Following on from the assumptions above, it is possible to attempt to minimise the effects of these modelling approximations. For instance, if it were possible to ensure constant acceleration then doing this would reduce errors, and make the results more accurate.

1. Ensuring that the pulley was as smooth as possible.
2. Ensuring that the table was of equal roughness all the way along it.
3. Making sure that there is no wind or draughts on the block.
4. Making sure that the string is as inextensible as possible.

Variation in experimental results

As in every experiment, there is variation in the results. This is because in, real life, there are more than two variables changing. For example, random draughts, inaccuracy in timing, inaccuracy in measuring distance, inaccurate weights and differences in the table surface.

The error bars on the graph show this error, with the central line showing an average.

Comparison between experimental data and predictions of the model

The predictions of the model give a line of the equation y = -1.37x + 13.8. This gives μ to be 1.14. This is a lot bigger than our original estimate. In addition, 13.8 is

approximately twice as big as 6.53. This shows that our experiment does not follow our original model. This is probably due to our assumption that Coulomb's Law is true. This model will produce a straight line. Our results show a subtle curvature, implying a non-linear relationship between F and μr. Although random variation could have created this curve, it seems that it is due to the inaccurate model of Coulomb's Law. If the experiment had been based on a non-linear law then a different set of equations would have been derived which would result in a linear function being plotted.

Revision of the process

To improve the experiment, it would be better to use light gates to improve accuracy for timing. On the results that we collected, the higher speeds resulted in wider error bounds and thus less accuracy. This is because it was a lot harder to accurately measure these fast times. Measuring the distances we were moving over more accurately would also increase accuracy, as would more accurate measurement of mass. However, accuracy is not the main issue in this experiment. For this experiment, it seems that our model is incorrect, rather than the results.

To improve this experiment, our assumptions need to be changed. Most assumptions are fine, but the assumption of Coulombs law is very inaccurate. This assumption of a linear relationship forced our results into a curve. To achieve a more meaningful analysis of these results, we need a better model on which to compare them. However, more accurate models for friction are much more complex and far less simple to use.