## Introduction

The trifilar suspension is used to determine the moments of inertia of a body about an axis passing through its mass center. The apparatus consists of a circular platform suspended by three equi-spaced wires of equal length. The body under consideration is placed with its mass centre exactly in the middle of the circular platform. The platform is given a small circular displacement about the vertical axis through its center, and is released. The periodic time of the subsequent motion is obtained by measuring the time taken to complete a definite number of oscillations. Then from the formula the moment of inertia of the body can be calculated.

In Addition the moment of inertia of a connecting rod was found. The apparatus was a knife edge to suspend the rod and record the oscillations. This was the second part of the experiment.

Where L=length of suspension wires.

r=radius from center to attachment points of the platform.

 $I_0$ =Moment of inertia of the platform.

M<sub>0</sub>=Mass of platform

I=Moment of inertia of body

M=Mass of body

g=Acceleration due to gravity

#### Results

	Mass of bar	1.95 kg
	Mass of rod	1.85 kg
		period (s)
	Platform	1.73
	Platform with the bar	2.14
	Platform with the rod	2.99
	Knife edge	
wide side down	length (m)	0.238
	period (s)	1.09
wide side up	length (m)	0.098
	period (s)	0.97

Theoretical moment of Inertia	•
(bar)	0.045 kg/m
	9.98x10 <sup>-3</sup>
Inertia of the platform	kg/m
Inertia of the platform and the bar	0.029 kg/m
Inertia of the platform and the rod	0.064 kg/m
Inertia of rod (hanging method)	0.013 kg/m
Inertia of rod (Hanging method)	0.042 kg/m

## Discussion

In this experiment two different sets of readings where gathered and recorded. Both the bar and the rod where tested by the trifilar method. However the rod was placed under one more test, the knife edge, results for these experiments can be seen on table one.

We also noticed that there was a great difference in our results, that is the ones obtained experimentally differed from the theoretical ones by about 0.015.For instance the result that was gotten by the trifilar method for the rod was not similar to the knife edge method, in addition theoretical values where lower that experimental values.

The discrepancy in the results could be in the swinging method, as the person holding the platform in the trifilar method may have given it some extra moment by a push as it can be the case in the knife edge experiment.

Further more, the connecting rod was tested two ways when in knife edge, hanging from the smaller joint and hanging by the larger joint. On both these cases the measurement for L changed as the distance from the hanging point to the centre of mass also changed.

The rod had inertia of 0.064 in the trifilar method, but on the hanging method the value for inertia came down to 0.042, this difference of 0.022 is our error. This discrepancy could be due to a miscount, problems with the timing, extra moment, human error etc.

## Conclusion

During this experiment we discovered that there is a big difference in theory and practice. And not only that but some methods can give discrepancies between themselves.

It was also worth noticing the values of inertia these moving parts had, as it gives the designer and the manufacturer a good idea of the performance of the part. In this case a connecting rod was considered but another example would be a helicopter blade, or rotor blade. The blades of the helicopter are under constant movement once the engine is on and good idea on the inertia can aid with the performance of the helicopter as well as make the machine more efficient.

# **Appendix**

When the platform is rotated

$$r\theta = L\phi$$
, or  $\phi = r\theta / L$ 

If tension in each wire is T, then its vertical component is  $T \cdot \cos \theta$ .

As angle  $\phi$  is small

$$3T = M_0 \cdot g$$
 or  $T = M_0 \cdot g / 3$ 

The horizontal component  $T \cdot \sin \phi$  is tangential to 'r' and if  $\alpha$  is the angular acceleration of the platform, from equation  $\sum M_G = I_G \cdot \alpha$  ( $\sum M_G$ , moment of the rotation axis through the centre of mass G)

$$-3r \cdot T \cdot \sin \phi = I_0 \cdot \alpha$$

As angle  $\phi$  is small,

$$-3r \cdot T \cdot \phi = I_0 \cdot \alpha$$

$$-3r \cdot (M_o \cdot g / 3) \cdot (r\theta / L) = I_o \cdot \alpha$$

Finally,

$$-(g \cdot r^2 \cdot M_o / L) \cdot \theta = I_o \cdot \alpha$$

With the body on the platform, the total mass is  $M_o$  + M and moment of inertia is  $I_o$  + I, so the equation is

$$-[g \cdot r^2 \cdot (M_o + M) / L] \cdot \theta = (I_o + I) \cdot \alpha$$

As 
$$T_n = 2\pi / \omega_n$$

$$T_n = 2\pi \cdot \sqrt{[L \cdot (I_o + I) / g \cdot r^2 \cdot (M_o + M)]}$$

From equation  $\sum M_o = I_o \cdot \alpha$ 

$$-M \cdot g \cdot \cdot \theta = I_1 \cdot \alpha$$

Where  $I_1$  is the moment of inertia about the fixed axis through the point of suspension parallel with the axis through the centre of mass G.

$$\label{eq:omega_n} \begin{split} {\omega_n}^2 &= M \cdot g \cdot \ \ / \ \, I_1 \\ & T_n = 2\pi \cdot \sqrt{(M \cdot g \cdot \ \ / \ \, I_1)} \\ & I_1 = (T^2 \ / \ \, 4\pi^2) \cdot M \cdot g \cdot \\ from the parallel theorem \quad I_{Y1Y1} = I_{YY} + M \cdot ( \ \ / \ \, 2)^2 \end{split}$$

For the theoretical work

 $I_1 = I_G + M^{-2}$ 

$$I_2 = (M / 12) \cdot (^2 + W^2)$$

Where  $I_2$  is the moment of inertia of a rectangular bar about the axis YY through the centre of mass G