

# The Acceleration of a Freely Falling Body

To study the motion of a freely falling body, an object is allowed to fall and its position after successive equal time intervals is recorded on wax-coated paper by means of electric sparks. From these data, graphs of distance vs. time and velocity vs. time are plotted. The acceleration due to gravity is found by determining the slope of the velocity vs. time graph.

## Theory

In one dimension, an object's average velocity over an interval is the quotient of the distance it travels and the time required to travel that distance:

$$(1)$$

where  $v$  and  $t$ . The instantaneous velocity at a point is defined as the limit of this ratio as the time interval is made vanishingly small:

$$(2)$$

Hence, the velocity is given by the slope of the tangent to the distance vs. time curve. If the velocity were constant the slope would be constant, and the curve would be a straight line. This is evidently not the case for a freely falling body, since it is at rest initially but has nonzero velocities at later times.

When the velocity of a body varies, the motion is said to be accelerated. The average acceleration over an interval is the quotient of the change of the instantaneous velocity and the time required for that change:

where  $a$ . The instantaneous acceleration is defined analogously to the instantaneous velocity:

$$(3)$$

If a body moves in a straight line and makes equal changes of velocity in equal intervals of time, the body is said to exhibit uniformly accelerated motion. This type of motion is produced when the net force upon a body is constant. An example of this is the motion of a body falling freely in a vacuum. The acceleration of the body is called the acceleration due to gravity,  $g$ , and has the approximate value of  $9.81 \text{ m/s}^2$  ( $= 981 \text{ cm/s}^2 = 32.2 \text{ ft/s}^2$ ) near the surface of the earth.

For uniformly accelerated motion ( $a = \text{constant}$ ), the instantaneous acceleration is given by (3), which can be rearranged to give

When this equation is integrated from time  $t_0$  to  $t$  where the respective velocities are  $v_0$  and  $v$ , the result is

(4)

or

The graph of velocity vs. time is thus a straight line, the slope of which is the acceleration,  $a$ . In the experiment the value of  $g$  will be determined using this fact.

When (4) is substituted into (2) and the resulting equation is rearranged, the result becomes

The value of this expression when integrated from  $t_0$  to  $t$ , where the respective displacements are  $s_0$  and  $s$ , is

This equation shows that the distance vs. time graph is parabolic.

An important fact which will be used in graphing velocity vs. time is that for motion with constant acceleration, the average velocity between two displacements equals the instantaneous velocity at the midpoint in time of the interval. That is, the average velocity between  $t_0$  and  $t$  equals the instantaneous velocity at the midpoint in time,  $(t_0 + t)/2$ . This fact is important because only the average velocities are determined experimentally.

## Apparatus

o freefall apparatus o two meter long meter stick

o sparker o wax-coated paper strip

o power supply o masking tape

o switch o leads for sparker and power supply

A rod fitted with a metal disk falls freely between two vertical parallel conductors composed of a rail and a wire. A high voltage pulse across the conductors is supplied at fixed time intervals by a "sparker." While the body is falling, the high voltage causes sparks to jump between both the wire and the body and the body and the rail, so that marks are recorded on a heat-sensitive, wax-coated paper strip placed on the rail.

The body is suspended by means of an electromagnet which is connected to a power supply and is released by opening a switch in the circuit.

## Procedure

1) Select a sparker setting of 1/20 or 1/30 second, and record this value as  $\Delta t$ . These settings will yield about 12 to 18 data points.

2) Carefully attach the wax-coated paper strip to the rail with the wax-coated side facing outward. The top of the strip is secured under a plastic pressure strip, and the bottom under a rubber band.

- 3) Close the switch that allows current to flow through the electromagnet, and suspend the body from the center of the electromagnet. Mark the initial location of the body's metal disk on the wax-coated strip.
- 4) Steady the body to eliminate any swinging motion, engage the sparker, and open the switch in the electromagnet circuit. Keep the sparker engaged until the body reaches the bottom of the freefall apparatus.
- 5) Carefully remove the paper strip from the freefall apparatus, and tape the strip to a flat surface.
- 6) Circle and number each data point with the first dot after the starting position numbered as 1. Check that the distance between dots steadily increases. If this is not the case, then contact the instructor.
- 7) Place the meter stick on its edge along the strip such that the graduations touch the dots. Line up dot number 1 with some convenient position on the meter stick.
- 8) Leave the meter stick stationary and record the meter stick reading of each dot to a fraction of a millimeter, and record in the distance column in a table similar to the one shown in Figure 1.

data	time, in	distance	change in distance	average velocity	change in velocity	average acceleration
1						
2						
3						

4							
16							
17							
18							

Figure 1. An example of a format for the data table.

### Analysis

The data for time and distance are recorded in columns two and three in the table. Calculate the change in distance for the  $i$ th data point as  $\Delta d_i$  and the average velocity of the  $i$ th data point as  $\bar{v}_i$ . Compute the average velocity for intervals between every pair of data points separated by a time of 2, and enter the values in a data table. Remember that the average velocity over an interval is identical to the instantaneous velocity at the time midpoint of the interval. That is, the average velocity between data points 1 and 3 is identical to the instantaneous velocity at point 2, and so on. As an exercise only, carry the calculations one step further: compute the average acceleration between pairs of velocity values separated by a time of 2. The average acceleration of the  $i$ th data point is  $\bar{a}_i$ .

Plot distance vs. time and draw the best smooth curve through the data points. On a second sheet of graph paper, plot (instantaneous) velocity vs. time and draw the best straight line through the data points. In both graphs, the abscissa is the time axis. For accuracy, scale each graph so that it fills as much of the page as possible.

Determine the acceleration due to gravity by drawing the best eyeballed straight line through the data points on the velocity vs. time graph. (A transparent ruler is convenient for this purpose.) By constructing a large triangle, the slope of this line can be accurately determined.

Two additional methods of determining the acceleration due to gravity are:

A) Use the method of least squares (discussed in Appendix I) to find the slope of the line that best fits the points. This method yields the slope and intercept of the line that minimizes the sum of the squares of the ordinate deviations of the data.

B) Another computational technique is to average the value of the slopes that are found between appropriate pairs of points. For example, if there are 10 data points, compute the slope of the line passing through points 1 and 6, then points 2 and 7, etc., ending with the slope of the line passing through 5 and 10. The average of these 5 values yields a fairly reliable value of the acceleration.

Show all the relevant calculations of that were used for the methods used to find  $g$ .

Report all the calculated values of  $g$  and the accepted value of  $g$  in a table of results and calculate the percentage errors between all calculated values and the accepted value.

## Conclusions

Indicate what the major sources of error are in the experiment and explain how the experimental values are affected by the sources of error. Explain whether or not your experimental values reflect the effects of your sources of error.