Student Name: WONG Wing Yan
Class and class no. : 6G2 (22)
Date of experiment: 25th March, 2009

Experiment 5 – SHM: determining acceleration due to gravity

Objective

In this experiment, we are going to study the simple harmonic motion of a simple pendulum. The acceleration due to gravity (g) can be estimated by the following set-up.

Apparatus

- Retort stand and clamp
- Protractor
- G-clamp
- Metre ruler
- String 1.5m long
- Pendulum bob
- Stop watch

Setup

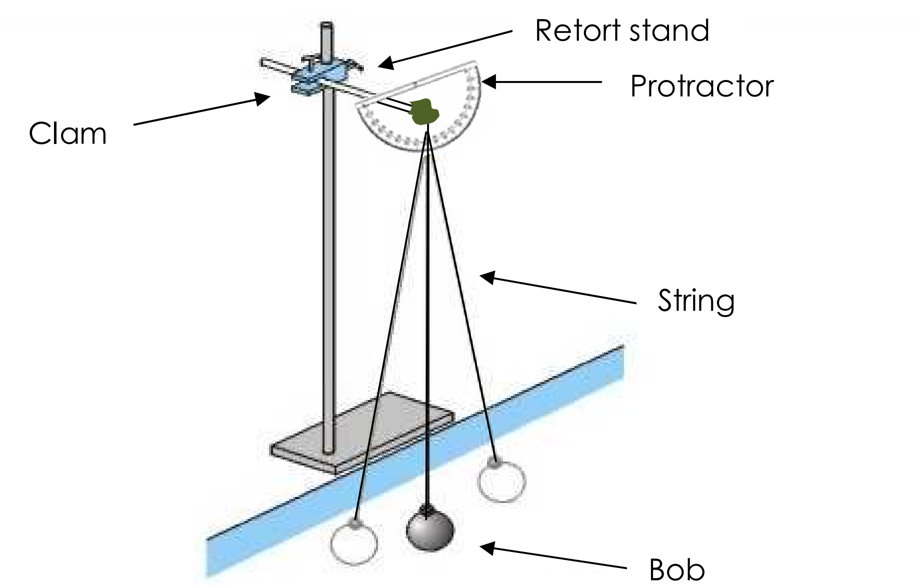


Figure 1

Theory

A simple pendulum can perform a simple harmonic motion as shown in figure 2. The acceleration due to gravity can be determined if we know the period (T) of the SHM and the length of the string (ℓ).

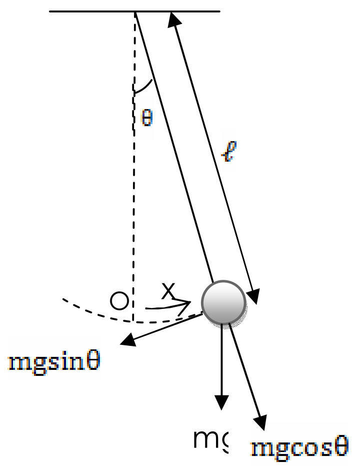


Figure 2

A light string with its upper end fixed and lower end attached to a pendulum bob mass m is shown. When the string is held to make and angle θ with the vertical, the bob displaces an arc length $(x = \ell\theta)$. The restoring force acting perpendicular to the bob is $F = -mg\sin\theta$

If the bob is released, it will move with an acceleration (a) towards the equilibrium position O. By Netwon's second law of motion, we have:

$$ma = -mg\sin\theta$$

For small angle θ (e.g. < 10°), we have $\sin\theta \approx \theta$ and therefore $\sin\theta \approx \frac{x}{\ell}$. Hence, it becomes:

$$a \approx -\frac{g}{\ell}x$$

As the bob continues to move, it performs a simple harmonic motion with angular velocity (ω) and it has an acceleration ($a = -\omega^2 x$). By comparing it with the equation, we have:

$$\omega \approx \sqrt{\frac{g}{\ell}}$$

However, the period can be shown as follows:

$$T = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = \left(\frac{4\pi^2}{g}\right)\ell$$

Note that period of the simple harmonic motion is independent of the mass.

Procedure

1. The apparatus was set up as shown in Figure 1.
2. The bob was moved to a height so that the attached string was taut and an angle of 10° was made with the vertical.
3. The period (t_1) for 20 complete oscillations was measured and it was recorded in the following table.
4. Steps 2 and 3 were repeated for another measurement of period (t_2).
5. Steps 2 to 4 was repeated by shortening the length of the string by 5 cm each time for measuring further 7 sets of data.
6. The graph of the square of the period (T^2) against the length of the string (ℓ) was plotted.
7. A best-fitted line was drawn on the graph and the slope was measured.
8. The acceleration due to gravity (g) was calculated from the equation, its slope is given by:

$$\text{slope} = \left(\frac{4\pi^2}{g}\right)$$

$$g = \frac{4\pi^2}{\text{slope}}$$

Precautions

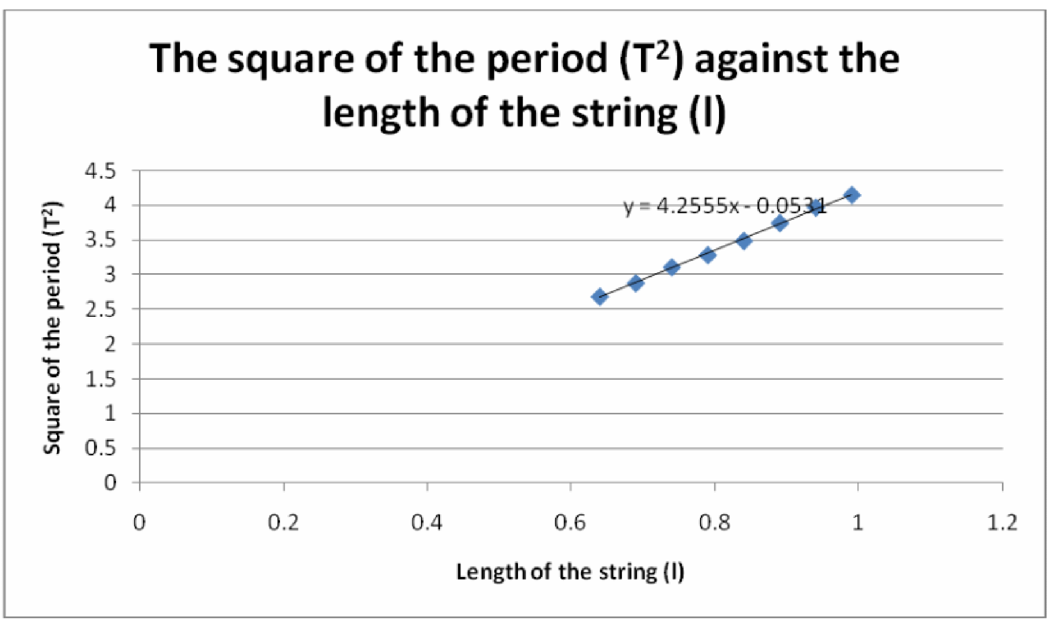
1. The angle of oscillation should be small enough. It is because when $\theta < 10°$, we have $\sin\theta \approx \theta$ and therefore $\sin\theta \approx \frac{x}{\ell}$.
2. A longer string can be used in order to have a less significant measuring error.

Data analysis and Results

Length of the string (ℓ) / m	20 period / s			One period (T) / s	T^2/s^2
	t_1	t_2	Mean$=\frac{t_1+t_2}{2}$		
0.99	40.66	40.87	40.765	2.03825	4.154
0.94	39.78	39.94	39.86	1.993	3.972
0.89	39.09	38.35	38.72	1.936	3.748
0.84	37.53	37.19	37.36	1.868	3.489
0.79	36.38	36.12	36.25	1.8125	3.285
0.74	35.63	34.91	35.27	1.7635	3.110
0.69	33.93	33.94	33.935	1.69675	2.879
0.64	33.03	32.50	32.765	1.63825	2.684

Mean of T^2 = 3.415125

Standard deviation σ = 2.65



The slope: 4.2555

$$g = \frac{4\pi^2}{\text{slope}}$$

$g = 9.277$

Discussion

In this experiment, we assume that the air resistance is negligible, it is because there is air resistance acting on the weight and the friction acting on the string may affect the motion of the weight and hence the period of the motion ;

We also assume there is a small displacement x from the equilibrium position because the angle of oscillation should be small enough. E.g. $\theta < 10°$, then we can perform the SHM

$\therefore \sin\theta \approx \theta, \quad \sin\theta \approx \frac{x}{\ell}.$

We should also assume there is no damping force, hence there is no energy lost in this experiment. Thus, the amplitude of the oscillation has remained constant.

The percentage error between the theoretical value of the acceleration due to gravity (g) and the experimental value:

$$\frac{9.81 - 9.277}{9.81} \times 100\% = 5.43\%$$

The experimental value g is smaller than theoretical value, since there is some source of error in this experiment.

Firstly, the angle of oscillation may be not small enough, it may be larger than 10°, small oscillation approximation is not appropriate. If $\theta > 10°$,

$$\sin\theta \neq \theta, \quad \sin\theta \neq \frac{x}{\ell}.$$

and this may affect the experimental result.

Secondly, there is a air resistance existing on the weight. Some of the restoring force is lost as friction instead of resorting the weight to the extreme position. The result is then affected.

Thirdly, the length of the string may not be long enough, the measuring error is quite significant.

Fourthly, the weight may not move in a constant speed. This affect the time for the experiment, and cause a error on period T.

Fifthly, there is an error on our reaction time to start and stop the timer. Although we take 2 times of the measurement of the 20 periods, the reaction time of humans directly affect the experimental results.

Sixthly, the experiment involves measurement error. The apparatus for measurement is not enough, for example, the metre rule can just measure the length of string which is correct to the nearest 0.1 cm. By the following equation,

$$\frac{\delta g}{g} = \pm\left[2\frac{\delta T}{T} + \frac{\delta m}{m}\right]$$

The largest possible error:

$$\delta g = \pm\left[2\frac{\delta T}{T} + \frac{\delta m}{m}\right] \times g$$

$$\delta g = \pm(2\frac{0.001}{3.415125} + \frac{0.0001}{0.815}) \times 9.277$$

$$\delta g = 0.00657$$

From this experiment, we can also know that period is independent of the mass of the body since the period of a system is depends only on the force constant of the spring and the mass of the body. A pendulum with a longer string or is at a place with smaller acceleration due to gravity will have a longer period.

Conclusion

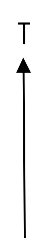
In this experiment, we try to study the simple harmonic motion of a simple pendulum. By looking for the value $\frac{4\pi^2}{\text{slope}}$, we can estimate the acceleration due to gravity (g).

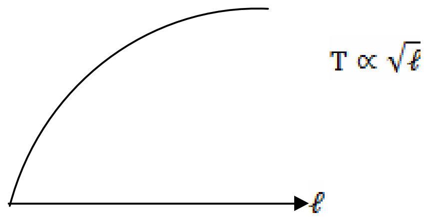
We compare the experimental value to theoretical value and find that the experimental one is a bit smaller than the theoretical value, which is due to the error of this experiment.

It is also clear to period is independent of the mass of the body. From the equation :

$$T^2 = 4\pi^2\sqrt{\frac{\ell}{g}}$$

we can know the relationship between T and ℓ, which is $T \propto \sqrt{\ell}$.





We can improve the experiment by measuring the angle carefully but not larger than 10°. We can also reduce the surface area of the weight, then we can reduce the air resistance. The time for more periods of oscillations (like 40 periods) could be taken for minimizing the measurement error. A more precise result could be obtained.

Reference

- New Way Physics for Advance Level – Book1(Mechanics) ; Manhattan Press (H.K.) LTD, page 263-265
- http://en.wikipedia.org/wiki/Simple_harmonic_motion