SAQ

14 The vibration of a component in a machine is represented by the equation:

$$x = 3.0 \times 10^{-4} \sin(240\pi t)$$

where the displacement x is in metres. Determine the **a** amplitude, **b** frequency and **c** period of the vibration.

Answer

15 A trolley is at rest, tethered between two springs. It is pulled 0.15 m to one side and, when time $t = 0$, it is released so that it oscillates back and forth with s.h.m. The period of its motion is 2.0 s. Write an equation for its displacement x at any time t (assume that the motion is not damped by frictional forces). Sketch a displacement against time graph to show two cycles of the motion, giving values where appropriate.

Answer

Acceleration and displacement

In s.h.m., an object's acceleration depends on how far it is displaced from its equilibrium position. The greater the displacement x, the greater the acceleration a. In fact, a is proportional to x. We can write the following equation to represent this:

$$a = -(2\pi f)^2 x$$

This equation shows that a is proportional to x; the constant of proportionality is $(2\pi f)^2$. The minus sign shows that, when the object is displaced to the *right*, the direction of its acceleration is to the *left*. The acceleration is always directed towards the equilibrium position.

It should not be surprising that frequency f appears in this equation. Imagine a mass hanging on a spring, so that it can vibrate up and down. If the spring is stiff, the mass will be accelerated more for a given displacement and its frequency of oscillation will be higher. The equation:

$$a = -(2\pi f)^2 x$$

helps us to define simple harmonic motion. The acceleration a is directly proportional to displacement x; and the minus sign shows that it is in the opposite direction.

Simple harmonic motion is defined as follows:

> A body executes simple harmonic motion if its acceleration is directly proportional to its displacement from its equilibrium position, and is always directed towards the equilibrium position.

If a and x were in the same direction (no minus sign), the body's acceleration would increase as it moved away from the fixed point and it would move away faster and faster, never to return.

Figure 5.20 shows the acceleration a against displacement x graph for an oscillator executing s.h.m. Note the following:

- The graph is a straight line through the origin ($a \propto x$).
- It has a negative slope (the minus sign in the equation $a = -(2\pi f)^2 x$). This means that the acceleration is always directed towards the equilibrium position.
- The magnitude of the gradient of the graph is $(2\pi f)^2$.
- The gradient is independent of the amplitude of the motion. This means that the frequency f or the period T of the oscillator is independent of the amplitude and so a simple harmonic oscillator keeps steady time.

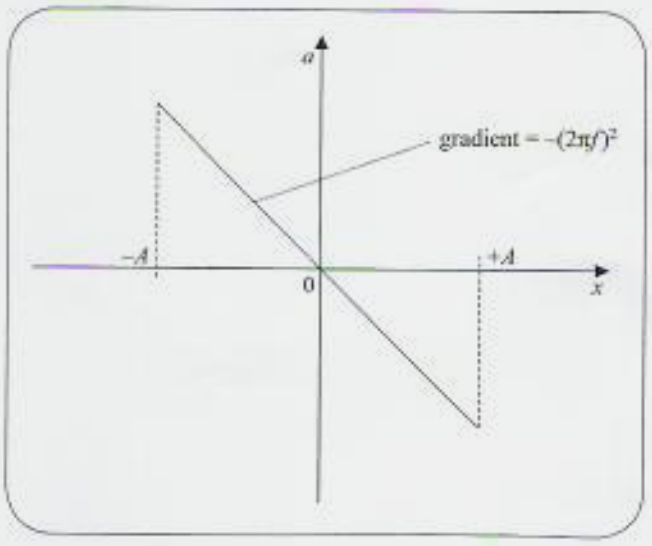

Figure 5.20 Acceleration a against displacement x graph for an oscillator executing s.h.m.