(13) is needed to be done

$v_{max} = (2\pi f)A$

13 An atom in a crystal vibrates with s.h.m. with a frequency of 10^{14} Hz. The amplitude of its motion is 2.0×10^{-12} m.

a Sketch a graph to show how the displacement of the atom varies during one cycle.

b Use your graph to estimate the maximum velocity of the atom.

Hint

Answer

Equations of s.h.m.

The graph of Figure 5.15a shown earlier represents how the displacement of an oscillator varies during s.h.m. We have already mentioned that this is a sine curve. We can present the same information in the form of an equation. The relationship between the displacement x and the time t is as follows:

$$x = A\sin(2\pi ft)$$

where A is the amplitude of the motion and f is its frequency. Sometimes the same motion is represented using a cosine function, rather than a sine function:

$$x = A\cos(2\pi ft)$$

The difference between these two equations is illustrated in Figure 5.19. The sine version starts at $x = 0$; i.e. the oscillating mass is at its equilibrium position when $t = 0$. The cosine version starts at $x = A$, so that the mass is at its maximum displacement when $t = 0$.

Note that, in calculations using these equations, the quantity $(2\pi ft)$ is in radians. Make sure that your calculator is in radian mode for any calculation (see Worked example 2). The presence of the π in the equation should remind you of this.

Worked example 2

A pendulum oscillates with frequency 1.5 Hz and amplitude 0.10 m. If it is passing through its equilibrium position when $t = 0$, write an equation to represent its displacement x in terms of amplitude A, frequency f and time t. Determine its displacement when $t = 0.50$ s.

Step 1 Select the correct equation. In this case, the displacement is zero when $t = 0$, so we use the sine form:

$$x = A\sin(2\pi ft)$$

Step 2 Substitute values using the information given in the question: $A = 0.10$ m, $f = 1.5$ Hz. So:

$$x = 0.10\sin(2\pi \times 1.5 \times t)$$

$$x = 0.10\sin(3.0\pi t)$$

Step 3 To find x when $t = 0.50$ s, substitute for t and calculate the answer:

$$x = 0.10\sin(2\pi \times 1.5 \times 0.50) = 0.10\sin(4.713)$$
$$= -0.10\text{ m}$$

This means that the pendulum is at the extreme end of its oscillation; the minus sign means that it is at the negative or left-hand end, assuming you have chosen to consider displacements to the right as positive.

(If your calculation above went like this: $x = 0.1\sin(2\pi \times 1.5 \times 0.5) = 0.1\sin(4.713)$ $= -8.2 \times 10^{-3}$ m, then your calculator was set to work in degrees, not radians.)

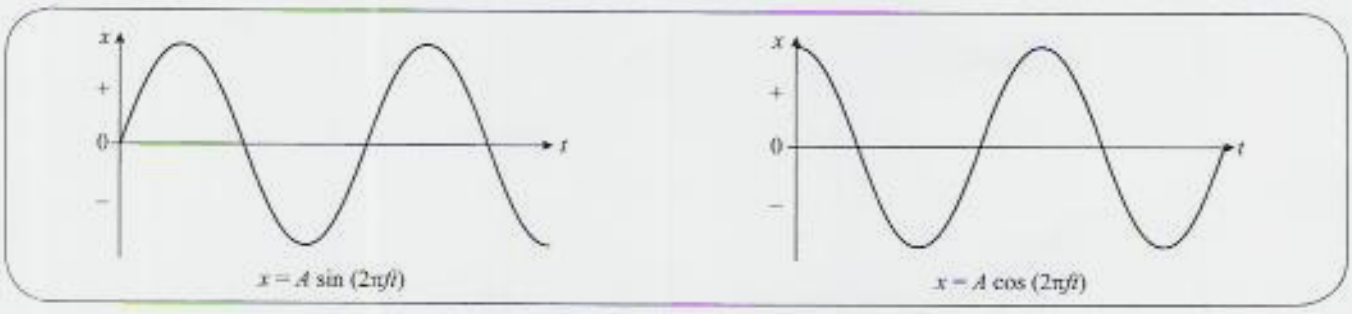

Figure 5.19 These two graphs represent the same simple harmonic motion. The difference in starting positions is related to the sine and cosine forms of the equation for x as a function of t.