* (16) + (17) are needed
* (18) + (19) are not needed

Maximum speed of an oscillator

If an oscillator is executing simple harmonic motion, it has maximum speed when it passes through its equilibrium position. This is when its displacement x is zero. The maximum speed v_{max} of the oscillator depends on the frequency f of the motion and on the amplitude A. The maximum speed is given by:

$$v_{max} = (2\pi f)A$$

According to this equation, for a given oscillation:

$$v_{max} \propto A$$

A simple harmonic oscillator has a period that is independent of the amplitude. A greater amplitude means that the oscillator has to travel a greater distance in the same time – hence it has a greater speed.

The equation also shows that:

$$v_{max} \propto f$$

Increasing the frequency means a shorter period. A given distance is covered in a shorter time – hence it has a greater speed.

Have another look at Figure 5.15. The period of the motion is 0.40 s and the amplitude of the motion is 0.02 m. The frequency f can be calculated as follows:

$$f = \frac{1}{T} = \frac{1}{0.40} = 2.5\,\text{Hz}$$

We can now use the equation $v_{max} = (2\pi f)A$ to determine the maximum speed v_{max}:

$$v_{max} = (2\pi f)A = (2\pi \times 2.5) \times 2.0 \times 10^{-2}$$

$$v_{max} \approx 0.31\,\text{m s}^{-1}$$

This is how the values on Figure 15.5b were calculated.

SAQ

16 A mass secured at the end of a spring moves with s.h.m. The frequency of its motion is 1.4 Hz.

a Write an equation of the form $a = -(2\pi f)^2 x$ to show how the acceleration of the mass depends on its displacement.

b Calculate the acceleration of the mass when it is displaced 0.050 m from its equilibrium position.

Answer

Show calculations please (book answer is $-3.9\ \text{ms}^{-2}$)

17 A short pendulum oscillates with s.h.m. such that its acceleration a (in m s^{-2}) is related to its displacement x (in m) by the equation $a = -300x$. Determine the frequency of the oscillations.

Answer

18 The pendulum of a grandfather clock swings from one side to the other in 1.00 s. Calculate:

a the period of its motion

b the frequency

c the angular frequency.

d Write an equation of the form $a = -(2\pi f)^2 x$ to show how the acceleration of the pendulum weight depends on its displacement.

Answer

19 A trolley of mass m is fixed to the end of a spring. The spring can be compressed and extended. The spring has a force constant k. The other end of the spring is attached to a vertical wall. The trolley lies on a smooth horizontal table. The trolley oscillates when it is displaced from its equilibrium position.

a Show that the motion of the oscillating trolley is s.h.m.

b Show that the period T of the trolley is given by the equation

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Answer

Big ideas in physics

This study of simple harmonic motion illustrates some important aspects of physics.

- Physicists often take a complex problem (such as, how do the atoms in a solid vibrate?) and reduce it to a simpler, more manageable problem (such as, how does a mass–spring system vibrate?). This is simpler because we know that the spring obeys Hooke's law, so that force is proportional to displacement.

continued